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Abstract

We consider hierarchical facility location problems on a network called MLTP and FTPLP, where q facilities and p transfer points are located and each customer goes to one of the facilities directly or via one of the transfer points. In FTPLP, we need to find an optimal location of both the facilities and the transfer points while the location of facilities is given in MLTP. Although good heuristics have been proposed for the minisum MLTP and FTPLP, no exact optimal solution has been obtained due to the size of the problems. We show that the minisum MLTP can be formulated as the p-median problem, which leads us to obtain an optimal solution. We also present a new formulation of FTPLP and an enumeration-based approach to solve the problems with a single facility.

keywords: hierarchical facility location problem, p-median problem, enumeration approach

1 Introduction

There are various systems of facilities operated within hierarchical structures such as postal delivery systems, distribution systems, emergency aid systems and so on. These hierarchical systems are generally huge and complicated that consist of different types of interacting facilities. Therefore, it is significant to find a well-organized hierarchical structure and the establishment of interactive facilities in each hierarchy. However, studies on hierarchical facility location problems are scarce though a variety of facility location models have been studied so far [7].

Throughout this paper, we consider a system on a network that consists of facilities and transfer points. The facilities provide the same service to n demand nodes, and customers at each demand node can go to the facilities via a transfer point or directly to the facilities so as to receive the service. This can be regarded as a hierarchical facility location model with 2 levels, where facilities and transfer points are interacting each other. The travel time from the transfer points to the facilities is shorter than that of from the demand nodes to the transfer points or the facilities due to a rapid transportation system. This system can be applied to an emergency aid system where the transfer points and the facilities are associated with heliports and hospitals, respectively. The patients are transferred by an ambulance to one of the heliports then flown on a helicopter to one of the hospitals with faster speed. Another application could be a distribution system where the transfer points and the facilities play a role as local depots and distribution centers, respectively.

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There is some literature related to transfer point location models. Berman, Drezner and Wesolowsky [2] proposed TPLP (The Transfer Point Location Problem), where an optimal location of a single transfer point needs to be found on a plane under the condition that the location of the single facility is given. They also considered TPLP on a network and proposed an algorithm to solve the problems taking into account the continuously varying travel time discount rate between the transfer point and the facility. Berman, Drezner and Wesolowsky [3] proposed MTPLP (The Multiple Transfer Points Location Problem) as a natural extension of TPLP, where the establishment of multiple tranfer points is allowed. They mentioned that the minisum MTPLP on a network can be regarded as the p-median problem. Furthermore, Berman, Drezner and Wesolowsky [4] considered a generalized model called MLTP (Multiple Location of Transfer Points) and FTPLP (The Facility and Transfer Points Location Problem), where they allowed to locate multiple facilities. The location of facilities is given in MLTP though an optimal location of both facilities and transfer points need to be found in FTPLP. They proposed three heuristics to solve the minisum MLTP and FTPLP on a network and reported comprehensive computational results using the benchmark data set provided by OR-Library [1]. However, they didn't obtain any exact optimal solutions due to the size of the problems.

In this paper, we focus on the minisum version of MLTP and FTPLP on a network. We show that the minisum MLTP on a network can be formulated as the p-median problem, which enable us to obtain exact optimal solutions of the problem. We also propose a new flow-based formulation for FTPLP in which the number of variables and constraints has been reduced, and present an enumeration based approach for the minisum FTPLP with a single facility.

This paper is organized as follows. In Section 2, we explain the hierarchical location model. In Section 3, we propose new formulations of MLTP and FTPLP. In Section 4, we briefly explain an enumeration based approach for the minisum FTPLP. In Section 5, we show computational results using the benchmark data provided by OR-Library [1]. Finally, we give concluding remarks and mention our future work in Section 6.

2 Model Description

We consider a hierarchical system on a network composed of two levels in which transfer points and facilities are located in each level (See Figure 1). Customers go to one of the facilities directly or via one of the transfer points. They can travel between transfer points and facilities with faster speed (See the thick lines in Figure 1).

Throughout this paper, let N be the index set of demand nodes ($|N| = n$), P be the index

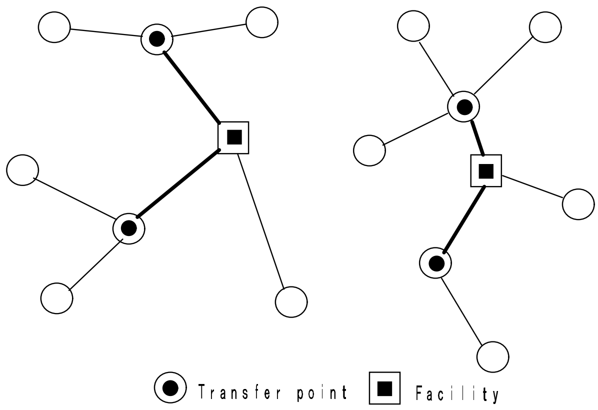

Figure 1: Hierarchical system on a network composed of two levels ($n = 16, p = 4, q = 2$). Customers go to one of the facilities directly or via one of the transfer points. Thick lines denote the rapid links.

set of transfer points ($P \subseteq N, |P| = p$) and Q be the index set of facilities ($Q \subseteq N, |Q| = q$). Let us denote t_{ij} be the travel time between node $i \in N$ and node $j \in N$, and d_i be the weight (demand) associated with node $i \in N$. We assume that the travel time on the links between transfer points and facilities is shorter than other links due to the rapid transportation system. We introduce a discount factor α ($0 < \alpha < 1$) to represent the time discount rate. Then, the travel time between transfer point j and facility k can be described as αt_{jk}.

In MLTP, we need to find an optimal $P \subseteq N$ under the condition that $Q \subseteq N$ is given. We also have to find optimal assignments of each customer to transfer points and each transfer point to the facilities. On the other hand, FTPLP is a natural extension of MLTP, where we need to find optimal $Q \subseteq N$ as well as $P \subseteq N$. Berman, Drezner and Wesolowsky [4] formulated the MLTP and FTPLP with the minisum and minimax criteria, respectively.

3 Formulation

3.1 Minisum MLTP and the $p-$median problem

Suppose that $Q \subseteq N$ is given in MLTP. Then the minimum travel time from node $i \in N$ via transfer point $j \in N$ is given by $\min_{k \in Q}(t_{ij} + \alpha t_{jk})$, while the minimum travel time from node $i \in N$ using nonstop route is given by $\min_{k \in Q} t_{ik}$. Therefore, the minimum travel time from node $i \in N$ is given by $\min(\min_{j \in N, k \in Q}(t_{ij} + \alpha t_{jk}), \min_{k \in Q} t_{ik})$. Here we introduce the following $n \times (n+1)$ matrix T as follows.

$$T_{ij} = \begin{cases} \min_{k \in Q}(t_{ij} + \alpha t_{jk}), & j = 1, \cdots, n, \\ \min_{k \in Q} t_{ik}, & j = n+1. \end{cases} \qquad (1)$$

In addition, let $N' = \{1, \cdots, n, n+1\}$, then the minisum MLTP can be formulated as the following p-median problem.

[Minisum MLTP]

$$\begin{aligned}
\text{minimize} \quad & \sum_{i \in N} d_i \sum_{j \in N'} T_{ij} x_{ij} && && (2)\\
\text{s.t.} \quad & \sum_{j \in N'} x_{ij} = 1, && i \in N, && (3)\\
& x_{ij} \leq y_j, && i \in N, j \in N, && (4)\\
& \sum_{k \in N} y_j = p, && && (5)\\
& x_{ij} \geq 0, && i \in N, j \in N', && (6)\\
& y_j \in \{0, 1\}, && j \in N. && (7)
\end{aligned}$$

Note that x_{ij} is a variable that specifies which node (a transfer point or a facility) is connected from node $i \in N$. More precisely, if $x_{ij} = 1$ for $j \in N$ then node $i \in N$ is connected to transfer point $j \in N$, and if $x_{ij} = 1$ for $j \in N' \backslash N$ i.e. $j = n + 1$ then node $i \in N$ is connected directly to the nearest facility. y_j is a binary variable such that $y_j = 1$ if node $j \in N$ is selected as a transfer point and 0 otherwise.

In a similar manner, we can formulate the minimax MLTP as the following p-center problem [5]. Note that x_{ij} for all $i, j \in N$ are binary variables.

[Minimax MLTP]

$$\begin{aligned}
\text{minimize} \quad & L && \\
\text{s.t.} \quad & T_{ij} x_{ij} \leq L, && i \in N, j \in N',\\
& x_{ij} \in \{0, 1\}, && i \in N, j \in N',\\
& (3) - (5), (7). &&
\end{aligned}$$

3.2 FTPLP

Berman et al. [4] formulated the minisum FTPLP as a 0-1 integer programming problem. In addition to location variables, they employ path-based 0-1 variables x_{ijk} taking 1 if node i is assigned to facility k via a transfer point j and 0 otherwise. As a result, the model has $n^3 + n^2 + 1$ binary variables and $2n^3 + n^2 + n + 2$ constraints with n nodes problem. Hence, even for a problem with $n = 100$, it has over a million variables and over 2 million constraints. In this section, we propose more compact model of FTPLP using flow-based formulation, which has $2n$ binary variables, $2n^2$ continuous variables and $6n + 2$ constraints. Narula and Obgu [6] also addressed a similar hierarchical location problem and proposed a flow-based formulation of the problem. Applying the same approach to the minisum FTPLP, the problem is reduced to that with $2n$ binary variables, $3n^2$ continuous variables and $5n+2$ constraints.

In addition to the notation defined in Section 3.1, we introduce M as a large value and employ the following decision variables.

- z_j: binary variable such that $z_j = 1$ if node $j \in N$ is selected as a facility or a transfer point, and 0 otherwise.
- w_k: binary variable such that $w_k = 1$ if node $k \in N$ is selected as a facility, and 0 otherwise.
- φ_{ij}: flow volume between node $i \in N$ and transfer point (or facility) $j \in N$.
- ψ_{jk}: flow volume between transfer point $j \in N$ and facility $k \in N$.

Then the minisum FTPLP can be formulated as follows:

[Minisum FTPLP]

$$\text{minimize} \quad \sum_{i\in N}\sum_{j\in N} t_{ij}\varphi_{ij} + \alpha \sum_{j\in N}\sum_{k\in N} t_{jk}\psi_{jk} \tag{8}$$

$$\text{s.t.} \quad \sum_{i\in N}\varphi_{ij} = \sum_{k\in N}\psi_{jk}, \qquad j\in N, \tag{9}$$

$$\sum_{j\in N}\varphi_{ij} = d_i, \qquad i\in N, \tag{10}$$

$$\sum_{i\in N}\varphi_{ij} \le M z_j, \qquad j\in N, \tag{11}$$

$$\sum_{k\in N}\psi_{jk} \le M z_j, \qquad j\in N, \tag{12}$$

$$\sum_{j\in N}\psi_{jk} \le M w_k, \qquad k\in N, \tag{13}$$

$$w_j \le z_j, \qquad j\in N, \tag{14}$$

$$\sum_{j\in N} z_j = p+q, \tag{15}$$

$$\sum_{k\in N} w_k = q, \tag{16}$$

$$\varphi_{ij} \ge 0, \qquad i\in N, j\in N, \tag{17}$$

$$\psi_{jk} \ge 0, \qquad j\in N, k\in N, \tag{18}$$

$$z_j \in \{0,1\}, \qquad j\in N, \tag{19}$$

$$w_k \in \{0,1\}, \qquad k\in N. \tag{20}$$

The objective function (8) is the total sum of the travel time. Constraints (9) are the flow conservation constraints at node $j \in N$. Constraints (10) require all demand to be satisfied. Constraints (11) ensure that no customer travels to other than transfer points and facilities. Constraints (12) and (13) guarantee the hierarchical network structure, where customers should travel from their origin to one of the facilities directly or via a transfer point. Constraints (14), (15) and (16) require that exactly p transfer points and q facilities have to be selected. Constraints (11) and (14) allow customers to travel directly to facilities. Note that $M = \sum_{i \in N} d_i$ is large enough to solve the problem.

In the model proposed by Narula and Obgu [6], they employed three types of flow variables associated with flow from nodes to transfer points, flow from nodes directly to facilities and flow from transfer points to facilities. Each has n^2 variables then the total number of flow variables is $3n^2$. On the other hand, we employ two types of flow variables. One is associated with flow from demand nodes and the other is associated with flow from transfer points. Thus we can further reduce the number of flow variables to $2n^2$. For the constraints, we additionally incorporate (12) to make the flow conservation consistent at the selected facilities. Hence, our model has n more constraints compared to Narula et al.'s model.

In a similar manner, we can formulate the minimax FTPLP as follows. Note that φ_{ij} and ψ_{ij} for all $i, j \in N$ are binary variables.

[Minimax FTPLP]

$$
\begin{array}{lll}
\text{minimize} & L & \\
\text{s.t.} & t_{ij}\varphi_{ij} + \alpha t_{jk}\psi_{jk} \le L, & j \in N, j \in N, k \in N, \\
 & \varphi_{ij} \in \{0, 1\}, & i \in N, j \in N \\
 & \psi_{ij} \in \{0, 1\}, & i \in N, j \in N \\
 & (19), (20). &
\end{array}
$$

This model has $n^3 + 6n + 2$ constraints while the one proposed by Berman et al. [4] has $3n^3 + 2n^2 + n + 2$ constraints.

4 Enumeration approach for the minisum FTPLP with a single facility

We obtain a compact formulation of FTPLP in the previous section. However, it still requires a long time to solve the problems using a solver even when $q = 1$ from our preliminary computational experiments. In this section, we consider a straightforward enumeration based approach to solve the minisum FTPLP with a single facility ($q = 1$).

From the definition of MLTP and FTPLP, the minisum FTPLP can be reduced to the minisum MLTP if the set of facilities $Q \subseteq N$ is given. Moreover, there are n possible facility candidates for the minisum FTPLP with a single facility. Therefore, we can obtain an optimal solution of the problems by solving all the n minisum MLTP. To make this straightforward enumeration approach more efficient, we attempt to get a lower bound of the problem.

Suppose that the triangle inequality holds for travel time, and the facility is established. In the following discussion, we use $\hat{k} \in N$ as the index of the established facility. Let denote $j^{(i)}$ be the index of the transfer point that minimize the travel time from node $i \in N$ to facility $\hat{k}$, i.e $j^{(i)} = \arg\min_{j \in N}(t_{ij} + \alpha t_{j\hat{k}})$. According to the discussion in Section 3.1, the travel time between node $i \in N$ and the facility $\hat{k}$ is given by $\min(t_{ij^{(i)}} + \alpha t_{j^{(i)}\hat{k}}, t_{i\hat{k}})$, hence, the optimal value is given by

$$\sum_{i \in N} d_i \min\left(t_{ij^{(i)}} + \alpha t_{j^{(i)}\hat{k}}, t_{i\hat{k}}\right). \tag{21}$$

Since the triangle inequality holds, $t_{i\hat{k}} \leq t_{ij^{(i)}} + d_{j^{(i)}\hat{k}}$ holds. In addition, because $0 < \alpha < 1$, the following inequalities hold.

$$\alpha t_{i\hat{k}} \leq \alpha t_{ij^{(i)}} + \alpha t_{j^{(i)}\hat{k}} \leq t_{ij^{(i)}} + \alpha t_{j^{(i)}\hat{k}}. \tag{22}$$

$$\alpha t_{i\hat{k}} \leq t_{i\hat{k}}. \tag{23}$$

From (22) and (23), we obtain

$$\alpha t_{i\hat{k}} \leq \min\left(t_{ij^{(i)}} + \alpha t_{j^{(i)}\hat{k}}, t_{i\hat{k}}\right), \tag{24}$$

which implies that $\alpha t_{i\hat{k}}$ is a lower bound of the travel time from node $i \in N$ to the facility $\hat{k}$. Thus, the following inequality holds with $d_i \geq 0$.

$$\alpha \sum_{i \in N} d_i t_{i\hat{k}} \leq \sum_{i \in N} d_i \min\left(t_{ij^{(i)}} + \alpha t_{j^{(i)}\hat{k}}, t_{i\hat{k}}\right). \tag{25}$$

From (21) and (25), we confirm that

$$L(\hat{k}) = \alpha \sum_{i \in N} d_i t_{i\hat{k}} \tag{26}$$

is a lower bound of the optimal value of MLTP with established facility $\hat{k} \in N$.

Another factor that can make the enumeration approach perform better is the order in which the candidates are selected as facilities. Our preliminary test has shown that the travel-time-based order produced a better upper bound in the early stage of iterations. Thus, for this paper, we sort the candidates in the ascending order of $\delta(k) = \sum_{i \in N} d_i t_{ik}$ because a node k

associated with smaller $\delta(k)$ is expected to be a facility. We denote that the resultant order as $s_1, s_2, \cdots, s_n$, which implies $\delta(s_1) \le \delta(s_2) \le \cdots \le \delta(s_n)$. In addition, we add an upper bound constraint that requires the objective value to be smaller than the best known upper bound. More precisely, the constraint can be written as

$$\sum_{i \in N} d_i \sum_{j \in N'} T_{ij} x_{ij} < u^*,$$

where u^* is the best known upper bound. Now we summarize the enumeration algorithm.

[Enumeration Algorithm for the Minisum FTPLP with a single facility]

Step 0: Compute $\delta(k) = \sum_{i \in N} d_i t_{ik}$ and sort them in the ascending order, $s_1, s_2, \cdots, s_n$, which implies $\delta(s_1) \le \delta(s_2) \le \cdots \le \delta(s_n)$. Set $u^* := \infty$ and $m := 1$.

Step 1: Compute lower bound $L(s_m)$ according to the equation (26). If $L(s_m) \ge u^*$, then go to Step 3.

Step 2: Solve the minisum MLTP with established facility s_m and obtain the optimal value $U(s_m)$. If $U(s_m) < u^*$, then set $u^* := U(s_m)$.

Step 3: If $m = n$ then terminate. u^* is the optimal value. Otherwise, let $m := m + 1$ and go to Step 1.

5 Computational Experiments

In this section, we report computational results of the proposed approach. All tests were carried out on a DELL DIMENSION 8300 computer with Intel Pentium 4 processor available in speeds of 3.0 GHz operated under Windows XP professional with 2.0 GB DDR-SDRAM memory. We used the 40 data sets for p-median problems provided by OR-Library [1] and solved them using AMPL and ILOG CPLEX 10.0. To compare our results with those presented by Berman et al. [4], we assume that the demand at each node is equal to 1, i.e. $d_i = 1$ for all $i \in N$, and $\alpha = 0.8$ throughout the entire tests.

Table 1 and 2 show the results of the minisum MLTP with $q = 1$ and $q = 5$, respectively. In each data set, the first q nodes are given facilities as Berman et al. [4] did. The column labeled "CPLEX" shows the total CPU time required to solve the problem by our approach. Note that it is not including computational time to generate T_{ij}. The column labeled "DA", "SA" and "TS" show that the total CPU time required using descent approach, simulated annealing and tabu search presented by Berman et al. [4], respectively. Their heuristics were implemented in

Microsoft PowerStation Fortran 4.0 and run on a 2.8 GHz Pentium 4 computer with 256 MB RAM. The descent heuristic was run 100 times for each problem and the other two heuristics were run 10 times each, where each were run from a randomly generated feasible initial solution. The CPU times presented in the tables are average CPU times. It is difficult to estimate the computational speed of these different platforms, however, we are sure that we obtained exact optimal solutions within a reasonable time. We also make sure that all the best-known solutions found by Berman et al. [4] are exactly the same as the exact optimal solutions.

Table 3 shows the results of the minisum FTPLP with $q = 1$. The column labeled "EA" shows the total CPU time required to solve the problem using the enumeration based approach. It rather requires a long time especially for large problems, however, it still works well. In all 40 cases, we obtained exact optimal solutions and made sure that all the best-known solutions found by Berman et al. [4] are optimal. We also tried to solve the problems using directly our new formulation of the minisum FTPLP. However, it takes about 100 to 400 seconds for 100 nodes problem, and for a 200 nodes problem, we didn't get any solution even after 10 hours computation.

6 Conclusion

In this paper, we showed that the minisum MLTP can be formulated as the p-median problem and the minimax MLTP can be formulated as the p-center problem. We also proposed a new flow-based formulation for the minisum FTPLP and the minimax FTPLP. We solved the minisum MLTP exactly using proposed formulation and also solved the minisum FTPLP with $q = 1$ by an enumeration based approach. Finally, we found out exact optimal solutions of the 40 benchmark problems provided by OR-Library [1] within a reasonable time.

Since the proposed enumeration based approach is straightforward, it may not work well on large problems with $q \geq 2$. Sophisticated procedures to find better lower and upper bounds would be required to solve large problems.

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Table 1: Results for MLTP ($q = 1, \alpha = 0.8$)

				CPU sec.			
Data	n	p	Opt. val.	**CPLEX**	DA	SA	TS
pmed1	100	5	11827.8	0.30	0.00	0.65	0.03
pmed2	100	10	9279.2	0.38	0.01	0.98	0.10
pmed3	100	10	14137.6	0.38	0.01	0.98	0.11
pmed4	100	20	12956.8	0.33	0.03	1.36	0.29
pmed5	100	33	10887.6	0.33	0.06	1.77	0.57
pmed6	200	5	22588.4	1.89	0.01	2.80	0.14
pmed7	200	10	13603.8	1.48	0.04	3.73	0.40
pmed8	200	20	16412.8	1.67	0.14	5.19	1.33
pmed9	200	40	12503.2	1.91	0.41	7.16	4.09
pmed10	200	67	9588.0	1.91	0.81	10.51	8.14
pmed11	300	5	12639.6	4.20	0.03	5.88	0.31
pmed12	300	10	12760.2	4.36	0.10	8.26	0.96
pmed13	300	30	15848.8	4.80	0.71	14.54	6.71
pmed14	300	60	17035.2	5.11	2.21	22.90	21.10
pmed15	300	100	11046.4	6.00	4.15	30.24	39.19
pmed16	400	5	21194.8	7.69	0.08	17.57	0.85
pmed17	400	10	13240.6	8.31	0.23	23.31	2.29
pmed18	400	40	21685.6	10.52	2.80	46.93	29.30
pmed19	400	80	14053.4	13.55	9.00	76.81	93.88
pmed20	400	133	15670.0	11.95	17.82	105.30	184.00
pmed21	500	5	15889.8	13.33	0.14	32.40	1.43
pmed22	500	10	17449.4	25.44	0.38	45.96	3.67
pmed23	500	50	16100.8	18.17	7.25	111.37	70.71
pmed24	500	100	16404.6	20.75	22.84	183.13	224.95
pmed25	500	167	20121.8	21.86	45.72	263.11	451.95
pmed26	600	5	16314.0	21.95	0.19	50.62	1.88
pmed27	600	10	14778.4	28.77	0.55	73.22	5.42
pmed28	600	60	13542.6	44.44	14.78	211.51	150.78
pmed29	600	120	12741.0	35.66	46.56	342.14	461.68
pmed30	600	200	14353.8	35.39	93.99	485.87	968.43
pmed31	700	5	16479.8	34.73	0.28	73.80	2.78
pmed32	700	10	21265.2	31.45	0.78	105.47	7.67
pmed33	700	70	18595.0	70.58	27.22	351.74	268.71
pmed34	700	140	24277.0	52.28	85.40	570.89	897.70
pmed35	800	5	15779.0	43.58	0.48	96.89	4.83
pmed36	800	10	18981.2	43.09	1.01	141.36	10.25
pmed37	800	80	16987.6	73.67	47.25	540.05	482.11
pmed38	900	5	17900.2	93.16	0.54	131.78	5.35
pmed39	900	10	19485.0	88.73	1.34	186.15	13.20
pmed40	900	90	20289.0	97.73	77.22	768.53	777.52

Table 2: Results for MLTP ($q = 5, \alpha = 0.8$)

				CPU sec.			
Data	n	p	Opt. val.	**CPLEX**	DA	SA	TS
pmed1	100	5	7888.8	0.16	0.00	0.55	0.03
pmed2	100	10	7075.4	0.14	0.01	0.86	0.09
pmed3	100	10	8415.0	0.16	0.01	0.88	0.11
pmed4	100	20	10064.4	0.17	0.03	1.31	0.28
pmed5	100	33	6932.6	0.14	0.05	1.70	0.52
pmed6	200	5	11491.0	0.73	0.01	2.22	0.13
pmed7	200	10	8856.4	0.73	0.04	3.33	0.42
pmed8	200	20	11270.0	0.69	0.13	4.87	1.34
pmed9	200	40	9105.4	0.69	0.40	6.86	3.94
pmed10	200	67	7509.4	0.67	0.77	10.19	7.93
pmed11	300	5	10073.8	1.86	0.03	5.42	0.30
pmed12	300	10	9647.0	1.94	0.09	7.70	0.93
pmed13	300	30	9656.0	1.78	0.66	13.76	6.66
pmed14	300	60	10393.4	1.77	2.05	22.07	20.44
pmed15	300	100	10033.4	1.64	3.89	29.69	38.69
pmed16	400	5	14813.2	3.42	0.07	16.29	0.74
pmed17	400	10	10963.2	3.48	0.23	22.16	2.26
pmed18	400	40	13325.0	3.45	2.85	46.03	28.14
pmed19	400	80	10113.2	3.22	8.85	74.23	88.64
pmed20	400	133	11351.8	3.03	18.15	102.47	183.83
pmed21	500	5	11894.4	5.66	0.12	29.63	1.26
pmed22	500	10	13654.0	5.78	0.37	43.12	3.70
pmed23	500	50	11735.2	5.50	7.01	109.78	69.27
pmed24	500	100	11552.2	5.30	22.51	181.66	230.65
pmed25	500	167	12345.0	5.19	45.26	258.00	454.84
pmed26	600	5	13665.6	8.17	0.19	48.28	1.88
pmed27	600	10	12966.4	7.88	0.55	69.99	5.43
pmed28	600	60	11730.8	7.78	14.73	212.96	144.63
pmed29	600	120	11155.6	7.70	46.93	341.59	469.4
pmed30	600	200	10671.6	7.91	93.29	475.33	938.12
pmed31	700	5	13185.2	11.11	0.27	70.38	2.63
pmed32	700	10	14560.0	11.03	0.75	102.43	7.62
pmed33	700	70	11963.4	11.17	27.11	343.67	273.27
pmed34	700	140	14519.2	10.77	85.53	564.56	879.42
pmed35	800	5	12574.6	15.14	0.37	92.76	3.73
pmed36	800	10	15909.6	14.94	0.99	136.25	10.04
pmed37	800	80	15188.2	14.72	45.95	534.63	468.89
pmed38	900	5	16562.6	20.14	0.37	92.76	3.73
pmed39	900	10	16710.2	19.42	0.99	136.25	10.04
pmed40	900	90	15750.8	18.59	45.95	534.63	468.89

Table 3: Results for FTPLP ($q = 1, \alpha = 0.8$)

				CPU sec.			
Data	n	p	Opt. val.	**EA**	DA	SA	TS
pmed1	100	5	9470.8	8.0	0.01	0.96	0.14
pmed2	100	10	8397.8	9.0	0.03	1.67	0.63
pmed3	100	10	10088.2	19.0	0.03	1.7	0.82
pmed4	100	20	10230.2	5.0	0.04	3.1	1.40
pmed5	100	33	7226.0	2.0	0.14	5.11	3.22
pmed6	200	5	11346.0	120.0	0.03	3.9	0.50
pmed7	200	10	9325.0	54.0	0.05	6.69	1.81
pmed8	200	20	10325.0	45.0	0.32	12.3	8.54
pmed9	200	40	9871.4	39.0	0.54	25.28	18.41
pmed10	200	67	7520.0	41.0	2.11	66.62	79.26
pmed11	300	5	9854.2	223.0	0.13	8.94	1.64
pmed12	300	10	11851.0	516.0	0.15	15.49	4.14
pmed13	300	30	10049.8	174.0	1.55	41.29	40.90
pmed14	300	60	11320.2	255.0	4.69	109.01	147.74
pmed15	300	100	9230.6	71.0	5.35	282.66	253.81
pmed16	400	5	11067.0	1007.0	0.22	22.5	3.27
pmed17	400	10	11291.6	789.0	0.65	37	14.02
pmed18	400	40	12359.4	846.0	6.68	120.16	170.83
pmed19	400	80	11167.0	412.0	17.19	415.1	575.50
pmed20	400	133	11387.6	236.0	45.09	784.82	1438.48
pmed21	500	5	12471.6	1950.0	0.23	41.72	3.87
pmed22	500	10	13813.0	2672.0	0.53	68.15	13.51
pmed23	500	50	12135.2	559.0	8.09	269.78	252.88
pmed24	500	100	11947.2	473.0	26.26	914.99	1070.13
pmed25	500	167	10123.4	178.0	100.94	1807.43	3333.97
pmed26	600	5	13027.4	2824.0	0.78	63.97	8.45
pmed27	600	10	12350.4	1940.0	1.09	102.12	19.67
pmed28	600	60	11454.6	838.0	16.45	599.16	582.45
pmed29	600	120	11924.0	981.0	54.54	1749.4	2214.71
pmed30	600	200	12679.6	576.0	231.65	3546.75	7039.86
pmed31	700	5	13802.4	7167.0	0.75	88.92	8.40
pmed32	700	10	14320.2	5692.0	1.79	142.96	40.81
pmed33	700	70	13072.8	1445.0	33.93	1211.75	1286.50
pmed34	700	140	12358.4	1893.0	97.83	3059.34	4021.96
pmed35	800	5	13962.6	12437.0	0.69	119.55	13.10
pmed36	800	10	15655.0	14782.0	2.02	189.18	38.17
pmed37	800	80	14691.0	2442.0	117.56	1985.41	3315.35
pmed38	900	5	14355.6	13168.0	0.99	152.85	13.24
pmed39	900	10	14052.6	10336.0	1.73	241.57	44.36
pmed40	900	90	15304.2	8827.0	107.31	2992.48	5004.59

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