Gain Scheduled Control of ABS Based on Friction Coefficient Estimation

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Abstract

Anti-lock Brake System(ABS) prevents cars from slip by locks of wheels in brake operation at low friction road surface. The ABS are designed to optimize braking effectiveness while maintaining car controllability. The ABS is controlled by slip rate. In this paper, a friction coefficient between a road surface and the wheels of a vehicle is estimated and a reference slip rate is set as a function of the friction coefficient based on the estimation method. The gain scheduling control based on lyapunov function is applied to the ABS. Friction coefficient is estimated by unscented Kalman filter. Finally, validity of the proposal method is illustrated by simulations.

1 Introduction

In the case of breaking such as hitting brakes, the Anti-lock Brake system which is a device prevents the lock of the wheel. It is controlled by slip rate, and it is set to follow the reference slip rate. A lot of studies have considered to improve ABS control methods. Examples include sliding mode control[1] and nonlinear control[2]. The slip rate of the control target is set at 0.2[3]. But the slip rate depends on the friction coefficient. So it is considered that the reference slip rate has the most suitable value depending on friction coefficient. In this paper, a friction coefficient between a road surface and the wheels of a vehicle is estimated and a reference slip rate is set as a function of the friction coefficient based on the estimation method. The state of the road surface is assumed to be first frozen, then wet and the last dry. This paper design the controller in consideration of an reference slip rate depending on these road surfaces state. Unscented Kalman filter(UKF) is used to estimate the friction coefficient. Kalman Filter is a filter to estimate the state of a certain dynamic system using an observed value with the noise. In this paper, the friction coefficient is estimated by UKF. Then scheduling parameter of GS controller set friction coefficient, and the reference slp rate is set as a function of the friction coefficient the estimated by UKF. The effectiveness of the proposed method is illustrated by simulations.

2 Modeling

The model of the simplified ABS experimental device used in this study is shown in Figure1[4]. Table1 shows the physical constants and variables used in this study. The upper wheel simulates the car wheel, and the lower wheel simulates the road surface. The car velocity assumes to be between 10 to 50[km], and designs the control system that a friction coefficient let a slip rate follow the reference value within 0.1 to 0.7. It is assumed here that μ =0.7 is for dry road and μ =0.1 is frozen road. The reference slip rate can be obtained by friction coefficient.

The dynamical equations of the rotational motion of the upper wheel and lower wheel are shown in Eq.(1) and Eq.(2).

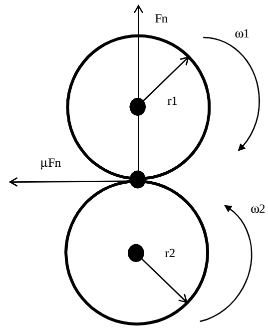

Figure 1 Simplified diagram of the ABS experimented device

Table 1 parameter

Radius of the upper wheel	$r_1 = 0.0995$	[m]
Radius of the lower wheel	$r_2 = 0.099$	[m]
Vertical force	$F_n = 58.234$	[N]
Moment of inertia of the upper wheel	$J_1 = 7.53 \times 10^{-3}$	[kgm^2]
Moment of inertia of the lower wheel	$J_2 = 25.60 \times 10^{-3}$	[kgm^2]
Angular Velocity of the upper wheel	ω_1	[rad/s]
Angular Velocity of the lower wheel	ω_2	[rad/s]
Braking torque	τ_1	[Nm]
Slip rate	λ	(−)
Friction coefficient between wheels	μ	[−]

$$J_1\dot{\omega}_1 = F_n r_1 \mu - \tau_1 \quad (1)$$

$$J_2\dot{\omega}_2 = -F_n r_2 \mu \quad (2)$$

The slip rate is defined in Eq.(3).

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} \quad (3)$$

The following equation is obtained from Eq.(1), Eq.(2) and Eq.(3).

$$\dot{\lambda} = -\frac{1}{\omega_2}\left(\frac{r_2 F_n \mu}{J_2}(1-\lambda) + \frac{r_1}{J_1 r_2}\right) + \frac{1}{\omega_2}\frac{r_1}{J_1 r_2}) \quad (4)$$

The behavior around equilibrium point $(\lambda^*, {\tau_1}^*)$ is considered. Here, λ^* is the reference slip rate, and ${\tau_1}^*$ is the equilibrium brake torque. Using Taylor expansion around the equilibrium point, nonlinear model Eq.(4) can be linearized.

$$\dot{\lambda} \simeq \dot{\lambda}(\lambda^*, {\tau_1}^*) + \frac{\partial}{\partial \lambda}\dot{\lambda}(\lambda - \lambda^*) + \frac{\partial}{\partial \tau}\dot{\lambda}(\tau - \tau^*) \quad (5)$$

The following expression are provided from Eq.(5).

$$\dot{\lambda} = -\sigma(C_1\lambda^* + C_2 - C_1\mu)(\lambda - \lambda^*) + \sigma C_3(\tau_1 - \tau_1^*) \quad (6)$$

$$\sigma = \frac{1}{\omega_2}, C_1 = \frac{r_2 F_n}{J_2}, C_2 = \frac{r_2 F_n}{J_2} + \frac{{r_1}^2 F_n}{J_1 r_2}, C_3 = \frac{r_1}{J_1 r_2}$$

3 Handling of the Slip Rate

It is widely accepted that a reference slip rate of 0.2 is convenient in conventional method. However it is shown in figure2[4] that the reference slip rate is suitable for each friction coefficient. Reference slip rate is different depending on the stare of the wheel. The slip rate(λ) corresponding to the maximum friction coefficient is called λ_{opt}. Slip rate has close relation to a braking friction coefficientμ_b and a slide slip friction coefficientμ_s. By keeping slip rate to optimal value, μ_s keeps very high value. Therefore, directional stability and maneuverability are both assured. Here, $0(\%) \leqq \lambda \leqq \lambda_{opt}$ is called stability region, $\lambda_{opt} < \lambda \leqq 100(\%)$ is called unstable region[5]. In this paper, a friction coefficient between road surface and the wheels of a vehicle is estimated and a reference slip rate is set as a function of the friction coefficient based on the estimation method. The reference slip rate can refer to the relations of the friction coefficient between a road surface and the wheels as follows. (α, β are constant)

$$\lambda^* = \alpha\mu + \beta \quad (7)$$

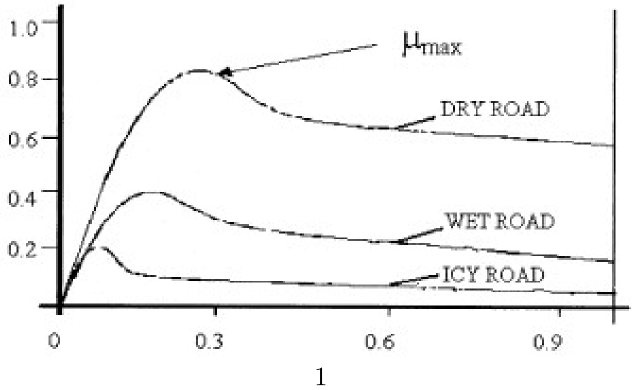

Figure 2 Comparison of slip rate

4 Control Design

In this paper, the state variable assumes it the deviation of the slip rate and the integral calculus. State equation is obtained from Eq.(8). Let state variable $x(t) = [x_1(t) \ \ x_2(t)]^T = [\int(\lambda - \lambda^*) \ \ \lambda - \lambda^*]^T$ and input $u(t) = \tau_1 - \tau_1^*$. The state equation is obtained as follows.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (8)$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\sigma(C_1\lambda^* + C_2 - C_1\mu) \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \sigma C_3 \end{bmatrix}$$

4.1 GS Control Design

This paper try to design the gain scheduling controller. The following equation is about the linear matrix inequallity.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$

The parameter box Θ is defined by vertexes which are upper bound and lower bound of μ. The scheduling parameter θ is the friction coefficient.

$$\Theta = \theta = \mu : \theta \in \{\overline{\theta}, \underline{\theta}\} \quad (10)$$

To derive a stabilizing state feedback controller $u = Kx$, the following cost function is considered.

$$J = \int_0^\infty (x(t))^T Q x(t) + u(t)^T R u(t))dt \quad (11)$$

Here $Q > 0$ is a weight matrix for state variables, and $R > 0$ is a weight matrix for inputs. By minimizing γ, the cost function J is minimized.

$$P(A + BK) + (A + BK)^T P + K^T R K + Q < 0 \quad (12)$$

$$x_0^T P x_0 \prec \gamma$$

LMI condition to derive the stabilizing state feedback GS controller is as follows. Let $X := P^{-1}, Y := KX$.

Theorem1

If there exists $X, Y(\Theta)$ satisfying the following LMI, the system is stable.

$$minimize : \gamma, subject \ \ to \ \ X > 0$$

$$\begin{bmatrix} He[AX + BY(\theta)] & X^T(Q^{\frac{1}{2}})^T & Y(\theta)^T(R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}}X & -I & 0 \\ R^{\frac{1}{2}}Y(\theta) & 0 & -I \end{bmatrix} < 0 \quad (13)$$

$$\begin{bmatrix} W & I \\ I & X \end{bmatrix} > 0 \quad (14)$$

$$trace(W) < \gamma \quad (15)$$

Finally, the gain scheduling controller K is given as follow.

$$K = Y(\theta)X^{-1} \quad (16)$$

5 Kalman Filter

A friction coefficient between a road surface and the wheels of a vehicle can be measured in real time. In this paper, a friction coefficient between a road surface and the wheels of a vehicle is estimated and a target slip rate is set as a function of the friction coefficient based on the estimation method. So, in this study, Unscented Kalman filter is used to estimate the friction coefficient. This paper designs UKF based on the theory of the referenced paper[6]. UKF can be applied for nonlinear system. Therefore, UKF can be applied to the estimate friction coefficient. The following equations can be used to design the UKF. (τ_g is Torque acting on the balance lever)

$$J_2\dot{\omega}_2 = -r_2\tau_g S - r_2 S\tau_1 \quad (17)$$

$$S = \frac{\mu}{L(sin\phi - \mu cos\phi)} \quad (18)$$

S is function of μ. So to estimate the friction coefficient, S is added to state variable. Let state variable $x_2(t) = [\omega_2(t) \quad S(t)]^T$, and input $u_2(t) = \tau_1$.

$$\dot{x}(t) = A_2 x_2(t) + B_2 S(t) u_2(t) \tag{19}$$

$$y(t) = C_2 x_2(t) \tag{20}$$

$$A_2 = \begin{bmatrix} 0 & \frac{\tau_g r_2}{J_2} \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} \frac{r_2}{J_2} \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

μ is varing parameter, and S is varying parameter, too. S is defined as follows.

$$S(k+1) = S(k) + v(k) \tag{21}$$

$v(k)$ is process noise. Let state variable $x_{2d}(t) = [\omega_2(k) \quad S(k)]^T$ and input $u_{2d}(t) = \tau_1$. The discretized nonlinear state space representation is as follows.

$$x_{2d}(k+1) = A_2 d x(k) + B_2 d S(k) u_2 d(k) + B_v v(k) \tag{22}$$

$$y(k) = C_2 x_{2d}(k) + \omega(k) \tag{23}$$

$$A_2 d = e^{A_2 Ts}, B_{2d} = (\int_0^{Ts} e^{A_2 \lambda} d\lambda) B_2$$

$$B_v = [0 \quad 1]^T$$

$\omega(k)$ is observation noise. Ts is sampling time. let $\hat{x}$ is a state estimate, and $g(k)$ is a Kalman gain. Optimal value of $\hat{x}(k)$ and $g(k)$ are as follows.

$$\hat{x}(k) = \hat{x}^-(k) + g(k)y(k) - \hat{y}^-(k) \tag{24}$$

$$g(k) = \frac{P^-_{xy}(k)}{P^-_{yy}(k) + \sigma^2_\omega} \tag{25}$$

$\hat{x}^-(k)$ is a priori state estimate. $\hat{y}^-(k)$ is a priori output estimate. $P^-_{xy}(k)$ is a priori state, output error covariance matrix. $P^-_{yy}(k)$ is a priori output error covariance matrix. σ^2_ω is of covariance of observation noise $\omega(k)$. The optimal value $\hat{x}(k)$ is decided by the Kalman gain $g(k)$. Therefore, the friction coefficient between a road surface and the wheels of a vehicle can be estimated by Eq.(24).

6 Simulation

In this section, the effectiveness of the designed controller is verified by simulations. Simulations are conducted on frozen road, wet road and dry road. Simulations are shown the slip rate, comparison of the car velocity and comparison of the friction coefficient. The relations of reference slip rate and friction coefficient is shown in Table2. The initial speed of the car is 50[km/h].

Table 2 Relations of a friction coefficient and the reference slip rate

Road	Friction coefficient	Reference slip rate
Frozen	0.1	0.3
Wet	0.4	0.25
Dry	0.7	0.2

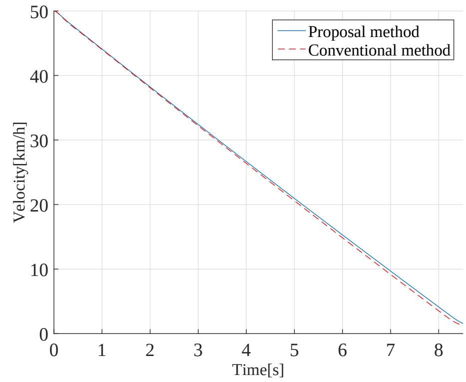

Figure 3 Comparison of the car velocity at the frozen road

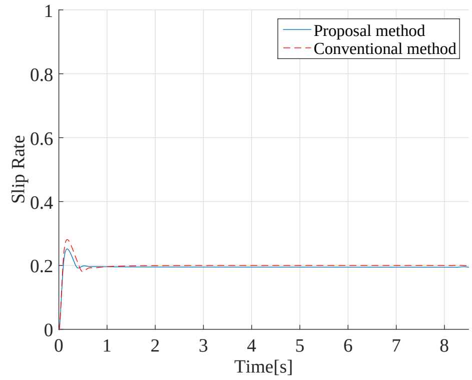

Figure 4 Slip Rate at the frozen road

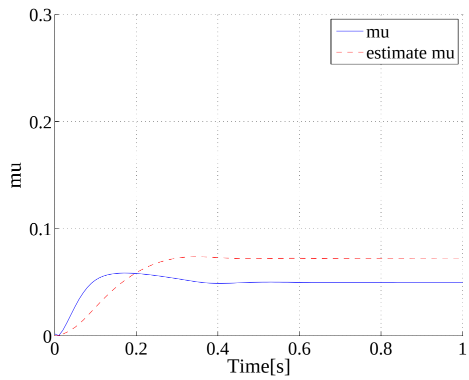

Figure 5 Comparison of the friction coefficient

Figure.3,6,9 show the proposal method can stop than conventional method. From the simulation result, proposal method has a shorter time to stop than conventional method. Figure.4,7,10 show the reference slip rate can change by the friction coefficient. In Figure5,8,11 shows the comparison of the friction coefficient. Solid line shows the friction coefficient value that really to gives, and dot line shows friction coefficient value that to estimate by UKF.

7 Conclusion

In this paper, a friction coefficient between a road surface and the wheels of a vehicle is estimated and a reference slip rate is set as a function of the friction coefficient based on the estimation method on ABS. Therefore, the

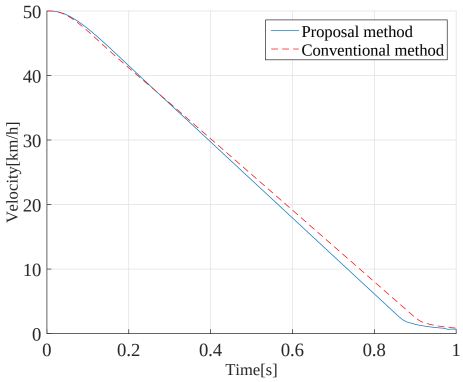

Figure 6 Comparison of the car velocity at the wet road

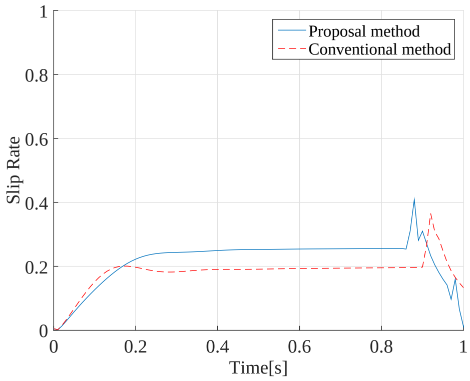

Figure 7 Slip rate at the wet road

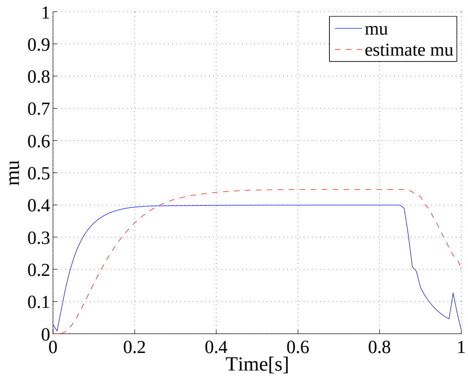

Figure 8 Comparison of the friction coefficient

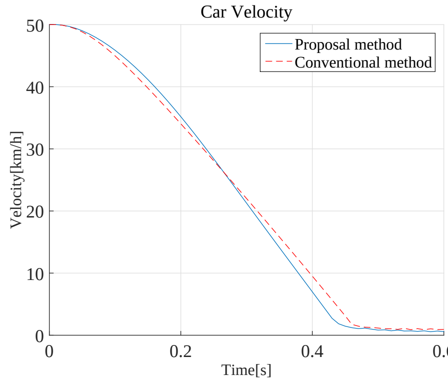

Figure 9 Comparison of the car velocity at the dry road

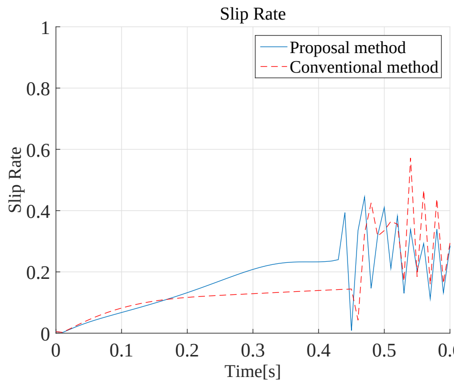

Figure 10 Slip rate at the dry road

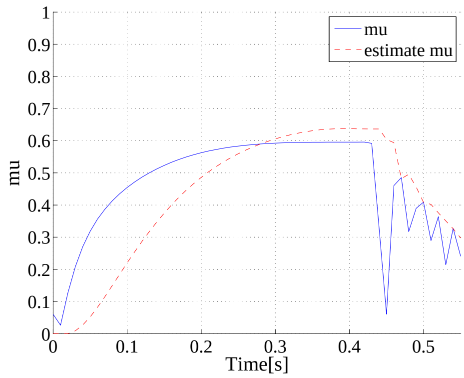

Figure 11 Comparison of the friction coefficient

effectiveness of the proposed method is verified.

References

[1] Makoto Yokoyama, Yosiaki Iwata, Sinji Katayori, Masamiti Imamura, Makoto Niibe. "Sliding Mode Control of Anti-lock Brake System.", The Japan Society of Mechanical Engineers. Vol.63. No611. 1997. (in Japanese).

[2] Tor A.Johasen, Idar Petrersen, Jens Kalkkuhl and Jens Ludemann. " Gain-scheduled Wheel Slip Control in Automotive Brake Systems.", IEEE Transaction on Control System Technology, Vol11, 2003.

[3] Yuichi IKEDA,Takashi NAKAJIMA,Yuichi CHIDA. "Vehicular Slip Ratio Control Using Nonlinear Control Theory. ", Journal of System Design and Dynamics Vol6,2012.

[4] R.Khatun, C. M. Bingham, N. Schofield, and P. H. Mellor. "Application of Fuzzy Control Algorithms for Electric Vehicle Antilock Braking/Traction Control Systems." IEEE Transactions on Vehicular Technology, Vol.52, No.5, September 2003.

[5] NihonABScorporation "Study of ABS for car use." Sankaishacorporation. 1993.

[6] Hiroshi Kataoka. "Gain Scheduling Control for ABS with Estimation of Friction Coefficient by Unscented Kalman Filter.", 2015.