Robust Stabilization for 3-DOF Helicopter Including Unmeasurable States using Output Feedback

M2013SC014 Takuma SUGINO
Supervisor : Isao TAKAMI

1 INTRODUCTION

When a person and a cargo are hanged by a wire and a rope from a helicopter, and the helicopter does a rescue operation, it is desirable to prevent swinging a suspended load for safety. About the oscillation suppression control of the suspended load, it is often the helicopter does not have a sensor to measure a swing angle of the load. We may not completely acquire a state because we are not able to use a sensor from reasons such as the mechanism of the system and cost in a real system. In this paper, we reproduce this situation using a 3-DOF helicopter which is experiment machine in imitation of a helicopter of tandem rotor type.

Some results are reported about such oscillation suppression control. Sonobe treats each state independently and suggests the control method to consider each to be plural SISO system[1]. On the other hand, there is the study using adaptive control method depending on a state estimate of the suspended load and the rope length by the image processing for the three-dimensional model[2]. Bisgaard considered a restriction condition of motion of the helicopter and the suspended load. A study using adaptive control method is reported about the oscillation suppression control in case of the swing angle of the suspended load is not able to measure using the 3-DOF helicopter[3].

The purpose of the study is designing a controller for attitude control of the 3-DOF helicopter and oscillation suppression control of the suspended load while being affected by the wind disturbance and motion of the helicopter. The state of the suspended load of the 3-DOF helicopter is not able to measure, but we pay attention to the point that is detectable and stabilizable. Using this, in this study, we consider stabilizing closed loop system by output feedback control theory. Also mass of the suspended load may not be the same all the time. Therefore we treat the mass of the suspended load as varying parameter and design a controller for robust stabilization in the range of the varying parameter. There is a study of the robust stabilization using polytope representation for the parameter that the range is decided on like this time[4]. And it is easy to treat the problem using polytope representation because it is relatively easy. However, we are not able to design a controller for robust stabilization by polytope representation using output feedback because its possible solution condition is not able to arrived at LMIs(Linear Matrix Inequalities)[5]. Therefore in this study, we treat the problem of the varying parameter as robust stabilization problem using H_∞ control theory[6]. And we design the controller which is guaranteed the stabilization of the helicopter and the suspended load for the parameters variation range. We design the H_∞ controller using output feedback by solving the LMIs using MATLAB[7]. Then, we choose penalty weights so that good control performance is provided by simulation using simulink.

2 MODELING OF THE PLANT

2.1 State equations

The schematic figure of the 3-DOF helicopter is illustrated in Figure 1. The support beam **AB** can rotate on point **O**. Let $\epsilon(t)$[rad] is the angle in vertical plane and $\lambda(t)$[rad] is the angle in horizontal plane. The support beam **CD** can rotate in vertical direction on point **B**. Let $\rho(t)$[rad] is the pitch angle. In addition, a suspended load of mass M_p[kg] is hung by a rope from the helicopter. Let l[m] is the rope length. Let $\theta(t)$[deg] is the swing angle of the suspended load. Now, 3-DOF helicopter has sensors to measure the states $\epsilon(t)$, $\lambda(t)$ and $\rho(t)$, but does not have a sensor for $\theta(t)$. So, the state $\theta(t)$ is not able to be measured directly.

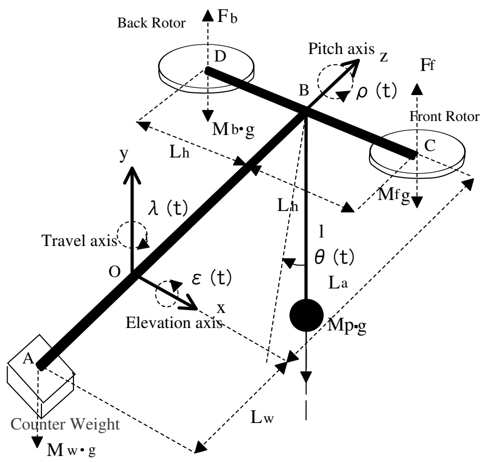

Figure 1 Schematic drawing of the 3-DOF helicopter

We derive the state equations that show the movement of the 3-DOF helicopter using Euler-Lagrange equation based on Figure 1. A state variable of the plant is defined as $x_p(t) = [\epsilon(t)\ \rho(t)\ \lambda(t)\ \theta(t)\ \dot{\epsilon}(t)\ \dot{\rho}(t)\ \dot{\lambda}(t)\ \dot{\theta}(t)]^{\mathrm{T}}$. An input is defined as $u(t) = [u_f(t)\ u_b(t)]^{\mathrm{T}}$. The state equation is given as Eq(1). Where $u_f(t)$[V] and $u_b(t)$[V] are each the input voltage of the front rotor and the back rotor.

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t) \tag{1}$$

Now, A_p and B_p are given as follows.

$$A_p = E_0^{-1} A_0\ ,\ B_p = E_0^{-1} B_0$$

$$E_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_\epsilon + M_p L_a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_\rho & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_\lambda + M_p L_a^2 & M_p l L_a \\ 0 & 0 & 0 & 0 & 0 & 0 & M_p l L_a & M_p l^2 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -B_\epsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -B_\rho & 0 & 0 \\ 0 & U & 0 & 0 & 0 & 0 & -B_\lambda & 0 \\ 0 & 0 & 0 & M_p gl & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & K_f L_a & K_f L_h & 0 & 0 \\ 0 & 0 & 0 & 0 & K_f L_a & -K_f L_h & 0 & 0 \end{bmatrix}^T$$

Where J_ϵ, J_ρ and J_λ[kg·m^2] are each moment of inertia.

2.2 Servo system

We synthesis a controller that follows a provided reference. In order to remove a steady-state error, a servo system is used. The error between the observed output $y(t)$ and the reference $r(t)$ is $e(t)$. $\tilde{w}(t)$ is $[r_\epsilon(t)\ r_\lambda(t)]^T$. The state variable of the servo system satisfy $x(t) = [x_p(t)\ \int_0^t e_\epsilon(t)dt\ \int_0^t e_\lambda(t)dt]^{\mathrm{T}}$. The servo system is expressed as follows.

$$\begin{cases} \dot{x}(t) = Ax(t) + \tilde{B}_1\tilde{w}(t) + B_2u(t) \\ \tilde{z}(t) = \tilde{C}_1x(t) \\ y(t) = C_2x(t) \end{cases} \tag{2}$$

Where matrices $A, \tilde{B}_1, B_2, \tilde{C}_1$ and C_2 are given as follows.

$$A = \begin{bmatrix} A_0 & 0 \\ -C_e & 0 \end{bmatrix},\ C_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{B}_1 = \begin{bmatrix} 0 \\ I \end{bmatrix},\ B_2 = \begin{bmatrix} B_0 \\ 0 \end{bmatrix},\ \tilde{C}_1 = \begin{bmatrix} 0 & I \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2.3 Extended system

We consider to realize stability for the swing angle $\theta(t)$ of the suspended load that is not able to be measured directly. Where $w_\theta(t)$ is disturbance for $\theta(t)$. $w(t)$ is $[w_\theta(t)\ \tilde{w}(t)]$. The extended system is expressed as follows.

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) \\ z(t) = C_1x(t) + D_{12}u(t) \\ y(t) = C_2x(t) \end{cases} \tag{3}$$

Where matrices B_1, C_1 and D_{12} are given as follows.

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & W_\theta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_{e\epsilon} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_{e\lambda} \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} W_u \\ 0 \end{bmatrix}$$

Where W_θ, $W_{e\epsilon}$, $W_{e\lambda}$ and W_u are each weighting constants for the states and the input.

3 CONTROLLER SYNTHESIS

3.1 Robust stabilization problem

Mass of the suspended load may not be the same all the time. Where $\Delta_m(s)$ is uncertainty of the multiplication of the nominal plant and the perturbated plant. We apply a small gain theorem and derive a frequency weight W_t satisfying $\bar{\sigma}\{\Delta_m(j\omega)\} < |W_t(j\omega)|$. Where matrix W_t is given as follows.

$$W_t = \begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix}$$

Singular plots of the frequency weight and uncertainty of the multiplication are shown in Figure2. Now, the range of the varying parameter M_p is $2.85 \leq M_p \leq 3.15$. We choose the frequency weight to cover each uncertainty of the multiplication. In Figure2, a dotted line is the frequency weight W_t. The generalized plant $G(s)$ is expressed as follows.

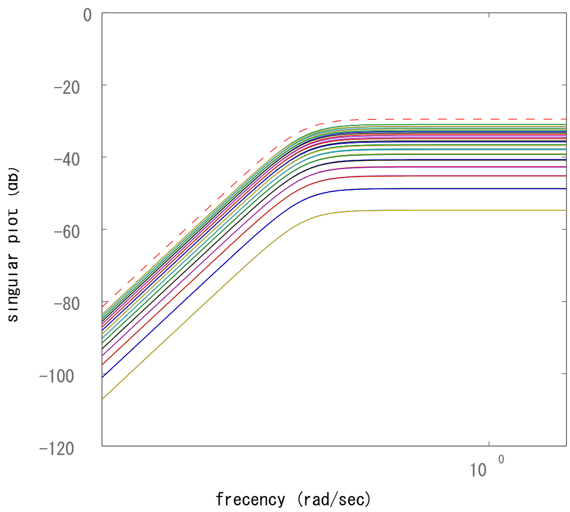

Figure 2 $\bar{\sigma}\{\Delta_m(j\omega)\} < |W_t(j\omega)|$

$$G(s) = \left[\begin{array}{cc|cc} A & 0 & B_1 & B_2 \\ B_tC_3 & A_t & 0 & 0 \\ \hline C_1 & 0 & D_{11} & D_{12} \\ D_tC_3 & C_t & 0 & 0 \\ C_2 & 0 & D_{21} & 0 \end{array}\right] \tag{4}$$

Where matrix C_3 is given as follows.

$$C_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3.2 H_∞ controller synthesis using output feedback

Now, Eq(4) is expressed as Eq(5). We design the output feedback controller K expressed as Eq(6) using H_∞ controller synthesis for the generalized plant Eq(5).

$$G : \begin{cases} \dot{x}(t) = A_Gx(t) + B_{G1}w(t) + B_{G2}u(t) \\ z(t) = C_{G1}x(t) + D_{G11}w(t) + D_{G12}u(t) \\ y(t) = C_{G2}x(t) + D_{G21}w(t) \end{cases} \tag{5}$$

$$K : \begin{cases} \dot{x}_K(t) = A_Kx_K(t) + B_Ky(t) \\ u(t) = C_Kx_K(t) + D_Ky(t) \end{cases} \tag{6}$$

There is a constant value $\gamma_\infty > 0$. If $X \in \mathbb{S}^n_{++}, Y \in \mathbb{S}^n_{++}, \hat{A}_K \in \mathbb{R}^{n\times n}, \hat{B}_K \in \mathbb{R}^{n\times n_y}, \hat{C}_K \in \mathbb{R}^{n_u\times n}$ and $\hat{D}_K \in \mathbb{R}^{n_u\times n_y}$ which satisfy following LMIs(7), (8) exist, the closed loop system is stable. Also the output feedback control gain which realize $\| G \|_\infty < \gamma_\infty$ is given as Eq(6).

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{7}$$

$$\begin{bmatrix} He(A_G X + B_{G2})\hat{C}_K & * & * & * \\ \hat{A}_K + (A_G + B_{G2}\hat{D}_K C_{G2})^T & He(YA + \hat{B}_K C_{G2}) & * & * \\ B_{G1}^T + D_{G21}^T \hat{D}_K^T B_{G2}^T & B_{G1}^T Y + D_{G21}^T \hat{B}_K^T & -\gamma_\infty^2 I & * \\ C_{G1}X + D_{G12}\hat{C}_K & C_{G1} + D_{G12}\hat{D}_K C_{G2} & D_{G11} + D_{G12}\hat{D}_K D_{G21} & -I \end{bmatrix} < 0 \tag{8}$$

We derive desired H_∞ controller using $X, Y, \hat{A}_K, \hat{B}_K, \hat{C}_K$ and $\hat{D}_K$ satisfying the LMIs(7), (8). Where matrices $I - XY$ and $M, N \in \mathbb{R}^{n\times n}$ are regular. They satisfy $I - XY = MN^T$.

$$\left.\begin{aligned} D_K &:= \hat{D}_K \\ C_K &:= (\hat{C}_K - D_K C_{G2} X)M^{-T} \\ B_K &:= N^{-1}(\hat{B}_K - YB_{G2}D_K) \\ A_K &:= N^{-1}(\hat{A}_K - NB_K C_{G2}X - YB_{G2}C_K M^T \\ &\quad -Y(A_G + B_{G2}D_K C_{G2})X)M^{-T} \end{aligned}\right\} \tag{9}$$

4 SIMULATION

The output feedback gain is obtained by solving the LMIs(7), (8). Then, $D_{11} = 0, D_{21} = 0$, and matrices M and N is derived by LU decomposition.

4.1 Simulation adding disturbance

The weighting constants $W_{e\epsilon}, W_{e\lambda}$ and W_u are chosen by trial and error. We set the reference of $\epsilon(t)$[deg] and $\lambda(t)$[deg] to 20[deg] and 200[deg] respectively. Then, we add the wind disturbance for the suspended load at 30[sec] to the simulation. The simulation results are shown in Figure3, Figure4, Figure5 and Figure6.

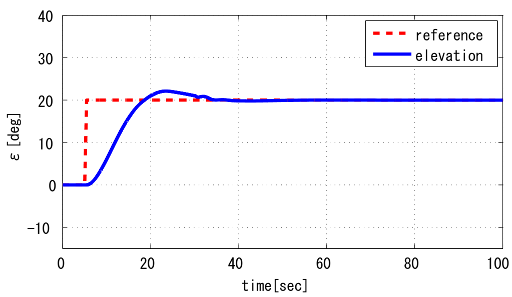

Figure 3 elevation(adding disturbance)

In Figure3 and Figure4, it can be seen that results of the simulation of the elevation and the traveling converge in the reference. Thus, we were able to design the controller for position control of the 3-DOF helicopter. In Figure5, it can be seen that the simulation of the swing angle of the suspended load suppresses the wind disturbance at 30[sec]. Thus, we were able to design the controller for stabilization of the state that is not able to be measured directly by output feedback.

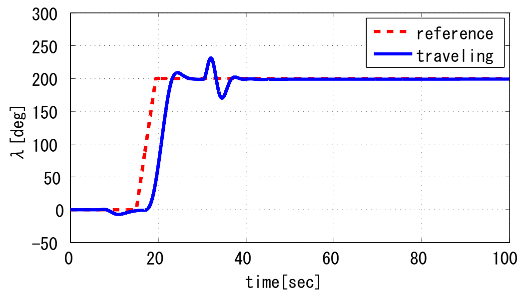

Figure 4 traveling(adding disturbance)

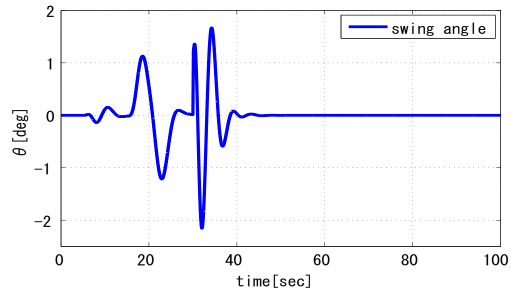

Figure 5 swing angle(adding disturbance)

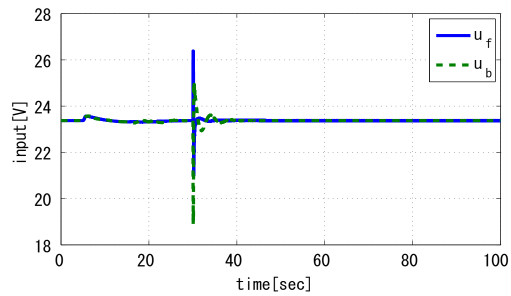

Figure 6 input(adding disturbance)

4.2 Simulation using nominal controller

We simulate to verify the robust stability. At first, we use the controller of not considering the robust stability. Then, mass of the suspended load M_p is 0.15[kg] heavier than the nominal load. The simulation results are shown in Figure7, Figure8, Figure9 and Figure10.

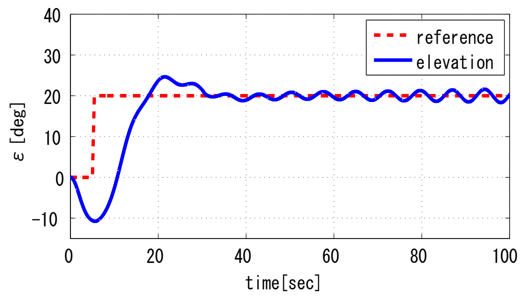

Figure 7 elevation(nominal controller)

In Figure7, Figure8, Figure9 and Figure10, the signals are not able to be stabilized when we used the nominal controller.

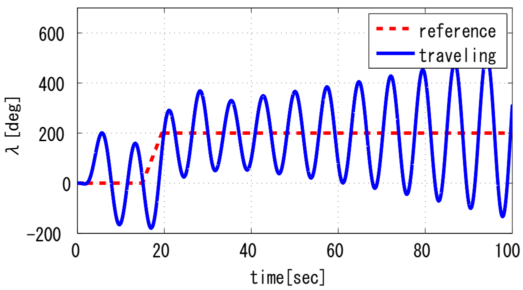

Figure 8 traveling(nominal controller)

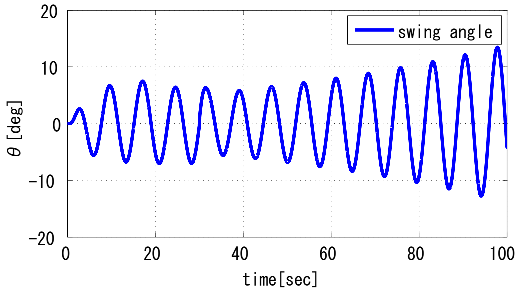

Figure 9 swing angle(nominal controller)

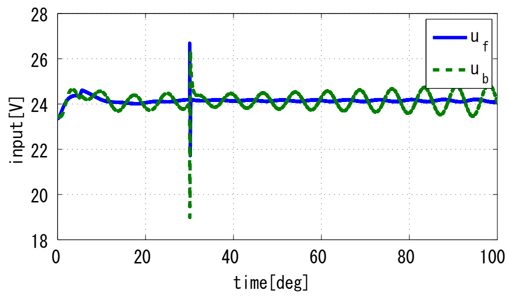

Figure 10 input(nominal controller)

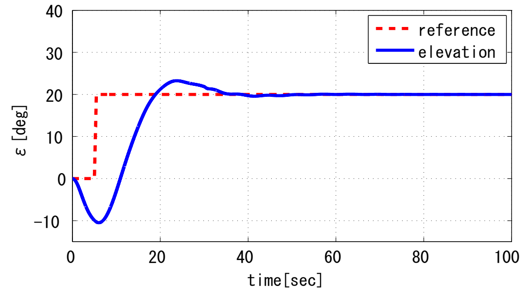

Figure 11 elevation(robust controller)

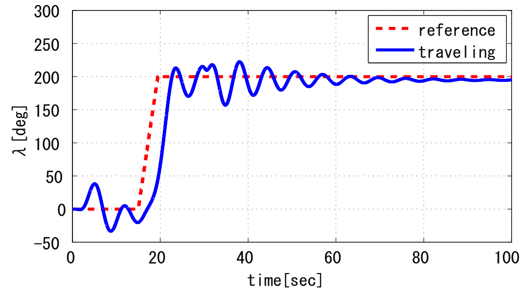

Figure 12 traveling(robust controller)

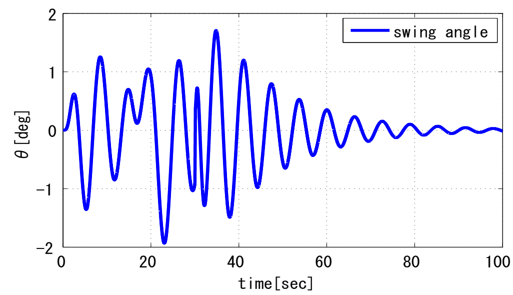

Figure 13 swing angle(robust controller)

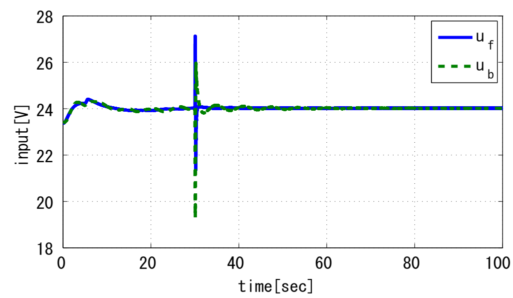

Figure 14 input(robust controller)

4.3 Simulation using robust controller

Next, we use the controller which guaranteed the robust stability on a condition same as 4.2. The simulation results are shown in Figure11, Figure12, Figure13 and Figure14.

In Figure11, Figure12, Figure13 and Figure14, the signals are able to be stabilized when we used the proposed controller. Thus, the controller which is guaranteed the robust stability for varying parameter by H_∞ control using output feedback is effective.

5 CONCLUSION

In this paper, we show the stabilization for the unmeasurable state using output feedback. In addition, we realized robust stability by H_∞ control using output feedback for the control object which has a varying parameter including the state space representation.

References

[1] Sonobe, M., Miwa, M., Hino, J.: Study on simple vibration control for small-size helicopter with slung load system (Experiment based studies of the vibration control on hovering), Transactions of the JSME (in Japanese), vo.78, no.789(2012)

[2] Bisgaard, M., Bendsten, J., and Cour-Harbo, A. : Modeling of generic slung load system, AIAA Modeling and simulation technologies conference and exhibit(2006)

[3] Katsurayama, N. : Robust Control with Adaptive Law for the System Including Unobserved States, Masters thesis, Nanzan University(2014)

[4] Sugino, T., Nagaya, S., Morikawa, T., Takami, I., Chen, G.:Robust stabilization of the 3DOF-helicopter using descriptor representation and polytope systems, SICE annual conference(2013)

[5] Asai, T.: Analysis and synthesis of linear robust control systems based on linear matrix inequalities, (2003)

[6] Yoshikawa, T., Imura, J.: Modern control theory, shokodo(2009)

[7] Ebihara, Y.: System control by linear matrix inequalities, Morikita(2012)