Robust Control with Adaptive Law for the System Including Unobserved States

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1 Introduction

Model Reference Adaptive Control (MRAC) is consists of a reference model, a plant model, and adaptive law. It is made to follow the target system to the reference model by fixed gain. As study example, Yang developed an adaptive control for SISO state space system[1]. Yang's method[1] was not discussed about the performance of the reference model. MRAC depends on performance of the reference model. It was important to consider the reference model. We adopted closed-loop system that was stabilized by Robust LQ Control for the reference model[2]. We extended Yang's method to descriptor system, and verified by 2-DOF helicopter[2]. The system depends rationally on uncertain parameters, it is difficult to deal with the system. We adopted redundant descriptor representation to linear uncertainty.

If a system has unobserved states, robust controller can't be synthesized. Therefore, a reference model and an plant model are designed separately. Error between the actual plant and the reference model is canceled by adaptive law. The proposed method is applied to helicopter. For example problem, mass of helicopter is changed by luggage or people. Helicopter is oscillated by the weight. As a result, Crash accident is caused by this problem. It is deal with in the framework of MRAC. 3-DOF helicopter is used as experimental machine[3], [4], [5]. The effectiveness of our approach is verified by 3-DOF helicopter.

2 Proposed Method

If a system has unobserved states, robust controller can not be synthesized. Therefore, an plant model and a reference model are designed separately. Proposed system shows Fig.1.

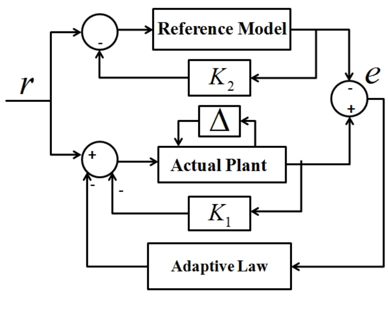

Fig. 1 Proposed System

Proposed method is consists of 3 approach: 1.Controller K_1 is designed by observed states in state feedback. 2.Reference model including unobserved states, controller K_2 is synthesized for reference model. 3.Error of the actual plant and the reference model is canceled by adaptive law. If the system depends rationally on uncertain parameters, the expression is not a polytopic type. Redundant descriptor representation and Linear Fractional Transformation (LFT) are adopted to reduce rational uncertainty to linear one.

2.1 Robust LQ Controller Synthesis

If state space system depends rationally on uncertain parameters, redundant descriptor system depends linearly on uncertainties. It is difficult to deal with uncertainties in state space representation whose dependency is not affine. In this paper, It is avoided this difficultly by using redundant descriptor representation. A continuous time multi-input multi-output system is described by Eq.(1).

$$\left(E_p + \sum_{i=1}^{k} \delta_i E_i\right) \dot{x}_p = \left(A_p + \sum_{i=1}^{k} \delta_i A_i\right) x_p + B_p u_p$$
$$y = C_p x_p \qquad (1)$$

$$E(\delta) = E_p + \sum_{i=1}^{k} \delta_i E_i, \quad A(\delta) = A_p + \sum_{i=1}^{k} \delta_i A_i$$

Where E_p, E_i, A_p, $A_i \in \Re^{n\times n}$, $B_p \in \Re^{n\times m}$, $C_p \in \Re^{p\times n}$. Eq.(1) has affine perturbation in each coefficient matrix. Additionally, $\delta_i \in \Re$ is perturbation elements which satisfy $|\delta_i| \leq 1$. If Eq.(1) is transformed to state space system, it depends on rational uncertain parameter. Generally, it is difficult to analyze the system stability directly whose $E(\delta)$ matrix has uncertain parameters. When the system depends rationally on uncertain parameters, the expression is not a polytopic type. Redundant descriptor representation is adopted to reduce rational uncertainty to linear one. Through adopting descriptor variables as $x_d := [x_p^T \ \dot{x}_p^T]^T$, uncertainties in each coefficient matrices are integrated into matrix $A(\delta)$.

$$E_d\dot{x}_d = A_d(\Delta)x_d + B_d u_p, \ y = C_d x_d \qquad (2)$$
$$E_d = diag\{I, 0\}, \ C_d = [C_p \ 0]$$
$$A_d(\Delta) = \begin{bmatrix} 0 & I \\ A(\delta) & -E(\delta) \end{bmatrix}, \ B_d = \begin{bmatrix} 0 \\ B_p \end{bmatrix}$$

One integrator is added into the closed loop system. For the plant model Eq.(2), let y, r, $e_p := r - y$, $z = \int e_p dt$ are observable output, reference, error and integrated value of e_p, respectively. Letting state as $x_{dk} = [z^T \ \ x_d{}^T]^T$, we finally obtain Eq.(3) for the augmented system with integrator.

$$E_{dk}\dot{x}_{dk} = A_{dk}x_{dk} + B_{dk}u_p \qquad (3)$$
$$E_{dk} = \begin{bmatrix} I & 0 \\ 0 & E_d \end{bmatrix}, A_{dk} = \begin{bmatrix} 0 & -C_d \\ 0 & A_d \end{bmatrix}$$
$$B_{dk} = \begin{bmatrix} 0 & B_d{}^T \end{bmatrix}^T$$

To derive a stabilizing state feedback $u_p = K_1 x_p$, consider to minimize Eq.(4).

$$J = \int_0^\infty (x_{dk}{}^T Q x_{dk} + u_p^T R u_p)dt \qquad (4)$$

Where $Q \in \Re^{n\times n} > 0$ and $R \in \Re^{m\times m} > 0$ are given weighting matrices. For the redundant descriptor system, we have already obtained the following lemma in the previous research[6]. If there exist $X_{11} > 0$, X_d, Y_d such that Eq. (5) hold, then the closed loop system with the state feedback $u = K_1 x_p := Y X_{11}^{-1} x_p$ is stable.

$$\begin{bmatrix} \mathrm{He}[A_{dk}X_d - B_{dk}Y_d] & X_d^T(Q^{\frac{1}{2}})^T & Y_d{}^T(R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}}X_d & -I & 0 \\ R^{\frac{1}{2}}Y_d & 0 & -I \end{bmatrix} < 0 \quad (5)$$

$$X_d = \begin{bmatrix} X_{11} & 0 & 0 \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, Y_d = \begin{bmatrix} Y & 0 & 0 \end{bmatrix}$$

Furthermore, through maximizing the trace of X_{11}, J is guaranties $J < trace(X_{11})^{-1}$. Synthesized controller is divided into integration gain $K_{r1} \in \Re^{m\times m}$ and state gain $K_{x1} \in \Re^{m\times n}$ as $K_1 = [K_{r1} \;\; K_{x1}]$. Nominal input using robust LQ state feedback is given as follows.

$$u_{nom} = K_{x1}x_p + K_{r1}\int (r(t) - y)dt \quad (6)$$

2.2 Adaptive Law Synthesis

In this section, adaptive law and quadratic stability analysis for adaptive control loop are discussed. Yang et al [1] developed a LMI-based stability analysis method that employs σ-modification for SISO system. In this study, we expand Yang et al's method to MIMO descriptor system.

Consider the MIMO system described as descriptor system. Let descriptor variable is $\hat{x}_d = [\hat{x}_p \;\; \dot{\hat{x}}_p]^T$ then it is described as Eq(7).

$$\hat{E}_d\dot{\hat{x}}_d = \hat{A}_d\hat{x}_d + \hat{B}_d(u + W^T\phi(\hat{x}_d)),\ y = \hat{C}_d\hat{x}_d \quad (7)$$

$$\hat{E}_d = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \hat{A}_d = \begin{bmatrix} 0 & I \\ \hat{A}(\delta) & -\hat{E}(\delta) \end{bmatrix},$$
$$\hat{B}_d = \begin{bmatrix} 0 & \hat{B}_p^T \end{bmatrix}^T, \hat{C}_d = \begin{bmatrix} \hat{C}_p & 0 \end{bmatrix}$$

$W^T(t)\phi(\hat{x}_d)$is a matched system uncertainty. Where $W(t) = [W_1(t)\cdots W_m(t)] \in \Re^{2n\times m}$, $W_i(t) \in \Re^{2n\times 1}$ is uncertain parameter matrix and $\phi(\hat{x}_d) \in \Re^{2n\times 1}$ is a known set of smooth basis functions. Actual input u for the argued system is described as Eq(8).

$$u = u_{nom} + u_{ad},\ \ u_{ad} = \hat{W}(t)^T\phi(\hat{x}_d) \quad (8)$$

Where u_{nom} is the nominal input for reference model derived in the previous section and u_{ad} is the adaptive signal. u_{ad} functions as canceling matched uncertainty $\hat{W}^T\phi(\hat{x}_d)$ through estimating the uncertain parameter matrix $W(t)$ with $\hat{W}(t) = [\hat{W}_1(t)\cdots\hat{W}_m(t)] \in \Re^{2n\times m}$, $\hat{W}_i(t) \in \Re^{2n\times 1}$. Reference model which generates ideal output for Eq.(7) is described as Eq.(9)

$$E_d\dot{x}_m = A_m x_m + B_m\int (r(t) - y)dt \quad (9)$$

Where A_m and B_m as $A_m = \hat{A}_d - \hat{B}_d[K_{x2} \;\; 0], B_m = \hat{B}_d K_{r2}$.

Let $e = x_m - \hat{x}_d$ is tracking error and $\tilde{W}(t) = \hat{W}(t) - W(t)$ $(\tilde{W}_1(t) = \hat{W}_1(t) - W_1(t)\cdots\tilde{W}_m(t) = \hat{W}_m(t) - W_m(t))$ is the estimation error. Let $\hat{B}_{di} = [\hat{B}_{d1}\cdots\hat{B}_{dn}] \in \Re^{2n\times m}$

Finally the error between Eq.(7) and Eq.(9) is obtained as Eq.(10).

$$\hat{E}_d\dot{e} = A_m e + \hat{B}_d\hat{W}(t)^T\phi(\hat{x}_d) \quad (10)$$

$\hat{W}(t)$ are updated using Eq. (11) as adaptive law with σ-modification[1], [8], [9].

$$\dot{\hat{W}}(t) = -\gamma\phi(\hat{x}_d)e^T\hat{P}\hat{B}_d - \sigma\hat{W}(t) \quad (11)$$

Where $\gamma > 0$ is adaptive gain and $\sigma > 0$ is σ-modification gain.

The matrix $\hat{P} > 0$ in Eq.(11) satisfies following LMI condition Eq.(12) by reference[2].

$$A_m^T\hat{P}^T + \hat{P}A_m + 2\rho\hat{E}_d\hat{P} < 0,\ \ \hat{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \quad (12)$$

Let $\zeta = [\tilde{W}_1^T\cdots\tilde{W}_m^T \;\; e]^T$ as error dynamics variables, the consolidated error dynamics whose descriptor variable is consist of the tracking error and weight estimation error is described as Eq.(13).

$$\bar{E}\dot{\zeta} = \bar{A}\zeta + \bar{B}\sigma W \quad (13)$$

$$\bar{A} = \begin{bmatrix} -\sigma I_N & 0 & 0 & -\gamma\phi(\hat{x}_d)\hat{B}_{d1}^T\hat{P} \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & -\sigma I_N & -\gamma\phi(\hat{x}_d)\hat{B}_{dn}^T\hat{P} \\ \hat{B}_{d1}\phi(\hat{x}_d) & \cdots & \hat{B}_{dn}\phi(\hat{x}_d) & A_m \end{bmatrix},$$
$$\bar{B} = \begin{bmatrix} -I_{N\times n} \\ 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} I & 0 \\ 0 & \hat{E}_d \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

Where $\phi(\hat{x}_d) = [\phi_1(\hat{x}_d), \ldots, \phi_N(\hat{x}_d)]^T$is a set of basis functions. Each vertex of the uncertainty region is defined as: $\phi_i \in [\underline{\phi}_i, \overline{\phi}_i]$

For the uncertainties, let polytope $\bar{A}$ as Eq. (14).

$$\bar{A} = \sum_{i=1}^{n} a_i\bar{A}_i,\ \ \sum_{i=1}^{n} a_i = 1,\ a_i \geq 0 \quad (14)$$

The following is already obtained for stability analysis of descriptor systems for Eq.(13) [1]. Quadratic stability is analyzed by solving Eq.(15) at each vertexes of $\phi(\cdot)$.

Eq.(13) is quadratically stable for perturbation ϕ_i if there exists $X_{11} > 0$ such that

$$\bar{X}^T\bar{A}_n^T + \bar{A}_n\bar{X} < 0, \qquad n = 1, \ldots 2^n \quad (15)$$
$$\bar{X} = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}$$

Where σ and γ are designed whether satisfy the quadratic stability through solving above LMI at each vertices.

3 Applying to 3-DOF Helicopter

The effectiveness of the proposed method is verified with using test scale 3 DOF(Degree-Of-Freedom) helicopter. 3-DOF helicopter is shown Fig.2. The helicopter has parallel rotors at the front and back. The helicopter is able to control elevation and traveling by front and back propeller. Let, the elevation angle is $\epsilon(t)$[rad], the pitching angle is $\rho(t)$[rad], the traveling angle is $\lambda(t)$[rad]. The voltage of front rotor is $V_f(t)$[V], the voltage of back rotor is $V_b(t)$[V]. 3-DOF helicopter's model is shown Fig.3. Eq.(16), Eq.(17), Eq.(18), and Eq.(19) are obtained by lagrange equations of motion.

Fig. 2 3DOF-Helicopter

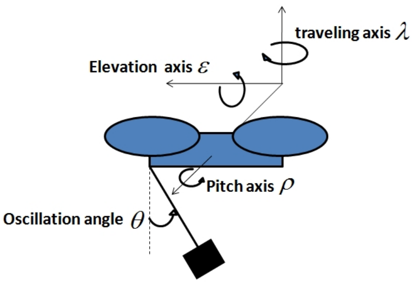

Fig. 3 3DOF-Helicopter model

$$(J_\epsilon+m_wL_a^2+m_wl^2)\ddot{\epsilon} = -B_\epsilon\dot{\epsilon}-m_wL_al\dot{\theta}+K_fL_a(V_f+V_b) \quad (16)$$

$$(J_\rho+m_wL_h^2+m_wl^2)\ddot{\rho} = -B_\rho\dot{\rho}+K_fL_h(V_f-V_b) \quad (17)$$

$$(J_\lambda+m_wL_a^2+m_wL_h^2)\ddot{\lambda} = -B_\lambda\dot{\lambda}+m_wL_al\dot{\theta}+U\rho \quad (18)$$

$$2m_wl^2\ddot{\theta} = m_wgl\dot{\theta}-m_wL_al\dot{\epsilon}+m_wL_al\dot{\lambda}-B_\theta\dot{\theta} \quad (19)$$

Let, inertia of helicopter body about elevation is J_ϵ[kg·m^2], inertia of helicopter body about pitching is J_ρ[kg·m^2], inertia of helicopter body about traveling is J_λ[kg·m^2], distance between travel axis to helicopter body is L_a[m], distance between pitch axis to helicopter body is L_h[m], propeller force-thrust constant is K_f[N/V], viscous friction of elevation is B_ϵ[N·m/V], viscous friction of pitching is B_ρ[N·m/V], viscous friction of traveling is B_λ[N·m/V], viscous friction of theta is B_θ[N·m/V], mass of the weight is m_w[kg], length of the string is l[m], lift to keep the helicopter body in a horizontal is U[N]. Therefore, controlled plant is described as the model which depends on m_w and l as not simply affine. Therefore m_w and l are considered as the uncertainty parameter in deriving robust controller. The uncertainty is assumed that true value of m_w exists in between 0 to 30[g] and l exists in between 0 to 7[cm].

The oscillation angle θ can't be observed. A reference model is used to deal the problem. A reference model and an actual plant are designed separately. The actual plant is designed in six-dimensional state vector. And, the reference model is designed in eight-dimensional state vector. The actual plant follows the reference model.

3.1 The plant model

Let, $x_p := [\epsilon\ \rho\ \lambda\ \dot{\epsilon}\ \dot{\rho}\ \dot{\lambda}\ ]^T$ as state variable, then plant dynamics of the actual plant is Eq.(20). Where, $J_{\epsilon1} = J_\epsilon + m_wL_a^2 + m_wl^2$, $J_{\rho1} = J_\rho + m_wL_h^2 + m_wl^2$, $J_{\lambda1} = J_\lambda + m_wL_h^2 + m_wL_a^2$, $u_p = [V_f\ V_b]$.

$$E_p\dot{x}_p = A_px_p + B_pu_p \quad (20)$$

$$A_p = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -B_\epsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & -B_\rho & 0 \\ 0 & 0 & 0 & 0 & 0 & -B_\lambda \end{bmatrix}$$

$$E_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_{\epsilon1} & 0 & 0 \\ 0 & 0 & 0 & 0 & J_{\rho1} & 0 \\ 0 & U & 0 & 0 & 0 & J_{\lambda1} \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0 & 0 & K_fL_a & K_fL_h & 0 \\ 0 & 0 & 0 & K_fL_a & -K_fL_h & 0 \end{bmatrix}^T$$

In this study, we focus on m_w and l as uncertain parameters in deriving robust control low. By using redundant descriptor representation, it is enable to deal with the plant model uncertainties in polynomials and to express more naturally. Generally, it is not easy to directly analyze stability of the system whose matrix E_p contains uncertainty parameters in Eq.(20). To deal with this difficulty, redundant descriptor representation is adopted [7]. Let $x_d := [\epsilon\ \rho\ \lambda\ \dot{\epsilon}\ \dot{\rho}\ \dot{\lambda}\ \ddot{\epsilon}\ \ddot{\rho}\ \ddot{\lambda}\ ]^T$ as descriptor variable, then plant dynamics is Eq.(21).

$$E_d\dot{x}_d = A_dx_d + B_du_p \quad (21)$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -B_\epsilon & 0 & 0 & J_{\epsilon1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -B_\rho & 0 & 0 & J_{\rho1} & 0 \\ 0 & U & 0 & 0 & 0 & -B_\lambda & 0 & 0 & J_{\lambda1} \end{bmatrix}$$

$$E_d = \begin{bmatrix} I_6 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & K_fL_a & K_fL_h & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_fL_a & -K_fL_h & 0 \end{bmatrix}^T$$

3.2 The reference model

The model with state variables is designed that can't be observed the the reference model. Let $\hat{x}_p :=$

$[\epsilon\ \rho\ \lambda\ \theta\ \dot{\epsilon}\ \dot{\rho}\ \dot{\lambda}\ \dot{\theta}\ ]^T$ as state variable, then plant dynamics of the reference model is Eq.(22).

$$\hat{E}_p\dot{\hat{x}}_p = \hat{A}_p\hat{x}_p + \hat{B}_p u_p \tag{22}$$

$$\hat{A}_p = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -B_\epsilon & 0 & 0 & -m_w L_a l \\ 0 & 0 & 0 & 0 & 0 & -B_\rho & 0 & 0 \\ 0 & U & 0 & 0 & 0 & 0 & -B_\lambda & m_w L_a l \\ 0 & 0 & 0 & m_w g l & -m_w L_a l & 0 & m_w L_a l & -B_\theta \end{bmatrix}$$

$$\hat{E}_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_{\epsilon 1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_{\rho 1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{\lambda 1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2m_w l^2 \end{bmatrix}$$

$$\hat{B}_p = \begin{bmatrix} 0 & 0 & 0 & 0 & K_f L_a & K_f L_h & 0 & 0 \\ 0 & 0 & 0 & 0 & K_f L_a & -K_f L_h & 0 & 0 \end{bmatrix}^T$$

Redundant descriptor representation is adopted [7]. Let $\hat{x}_d := [\epsilon\ \rho\ \lambda\ \theta\ \dot{\epsilon}\ \dot{\rho}\ \dot{\lambda}\ \dot{\theta}\ \ddot{\epsilon}\ \ddot{\rho}\ \ddot{\lambda}\ \ddot{\theta}\ ]^T$ as descriptor variable, then plant dynamics is Eq.(23).

$$\hat{E}_d\dot{\hat{x}}_d = \hat{A}_d\hat{x}_d + \hat{B}_d u_p \tag{23}$$

$\hat{A}_d$ in Eq.(23) can't be deal in exact polytope representation, because it has uncertain parameters ml, ml^2. LFT enables us to extract high order terms of uncertainty as affine. $\hat{A}_d$ is transformed by LFT, then A_δ can be defined Eq.(24).

$$\hat{A}_d = A_n + A_\delta,\ A_\delta = B_\delta\Delta(I - D_\delta\Delta)^{-1}C_\delta(\delta) \tag{24}$$
$$\Delta = diag(l, l, l)$$

Eq.(25) is equivalent to Eq.(23).

$$\hat{E}_d\dot{\hat{x}}_d = A_n\hat{x}_d + B_\delta w_\delta + \hat{B}_d u_p \tag{25}$$
$$z_\delta = C_\delta\hat{x}_d + D_\delta w_\delta$$
$$w_\delta = \Delta z_\delta$$

Let descriptor variable $\hat{x}_{dl} := [\hat{x}_d\ z_\delta]^T$ then closed loop system is obtained as Eq.(26).

$$\hat{E}_{dl}\dot{\hat{x}}_{dl} = \hat{A}_{dl}\hat{x}_{dl} + \hat{B}_{dl}u_p \tag{26}$$

$$\hat{A}_{dl} = \begin{bmatrix} A_n & B_\delta\Delta \\ C_\delta(\delta) & -I + D_\delta\Delta \end{bmatrix}$$
$$\hat{E}_{dl} = \begin{bmatrix} \hat{E}_d & 0 \\ 0 & 0 \end{bmatrix}$$
$$\hat{B}_{dl} = \begin{bmatrix} \hat{B}_d \\ 0 \end{bmatrix}$$

Note that $\hat{E}_{dl}$ is independent from uncertainty parameters and only $\hat{A}_{dl}$ linearly depends on uncertainty.

The effectiveness of the proposed method is verified by some experiments. The step response of traveling is $r = -1.57$[rad](-90 degree). The robust LQ with adaptive law, the robust LQ, and the nominal LQ are compared. In this experiment, the helicopter has the oscillation angle of weight to verify the robust control performance. The weight $m_w = 110$[g] is larger than considered range ($0 \leq m_w \leq 30$). The length of the string $l = 8$[cm] is larger than considered range ($0 \leq l \leq 7$). Step responses are shown Fig.4. From Fig.4, response of the proposed method is the best in the response of three. Therefore, this results shows the effectiveness of robust LQ control with adaptive law in 3-DOF helicopter.

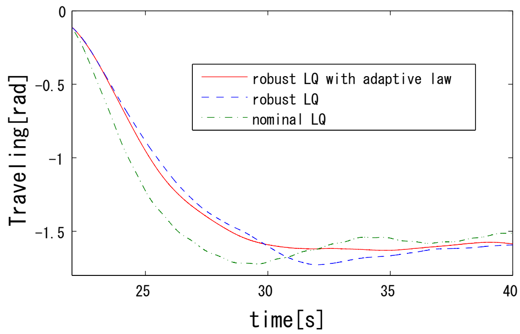

Fig. 4 Step response of traveling with load weight of 110g

4 Conclusion

In this study, robust LQ control with adaptive law was extend the system including unobserved states. The proposed method was applied to 3-DOF helicopter. The effectiveness of our approach was verified by experimental results of 3-DOF helicopter.

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