Robust H_2 Control with Polytopic Observer for MIMO Descriptor System

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1 Introduction

In general, linear system is studied to include uncertain parameters[1],[2]. Recently, linear MIMO system which is described polytopic system is reported. Then in the real system, we cannot obtain the all states. The synthesis of an observer and the controller corresponding to uncertain parameter is necessary.

This paper presents a robust H_2 control system with polytopic observer for MIMO system which as described as descriptor form. Generally, the performance degradation is expected to happen in case that uncertainty which is considered in the robust observer for velocity estimate design[3],[4] and robust control synthesis process[5],[6],[7]. Robust observer theory have potential to improve performance and estimate velocity in control system. In this study, we focus on this characteristics of robust observer algorithms into usual robust control system. The proposed system is synthesized by two-step approach. First, polytopic observer is designed that minimizing an upper bound on a given quadratic cost function derived[3]. Second, robust H_2 controller is synthesized through solving some LMI conditions.

In this study, LMI based stability analysis method for the combined with system of attached minimal convex polytope algorithm[8],[9],[10],[11],[12] and MIMO system described as descriptor form. Our approach for MIMO descriptor systems are extension of polytopic observer which is designed minimizing an upper bound on a given quadratic cost function. In this study, we show that Lyapnov stability is guaranteed. Next, we synthesized the robust controller. Finally, the effectiveness of the proposed procedure is verified by simulations and experiments with 3-DOF helicopter.

We show the whole system which is combined with the full order observer in Fig.1.

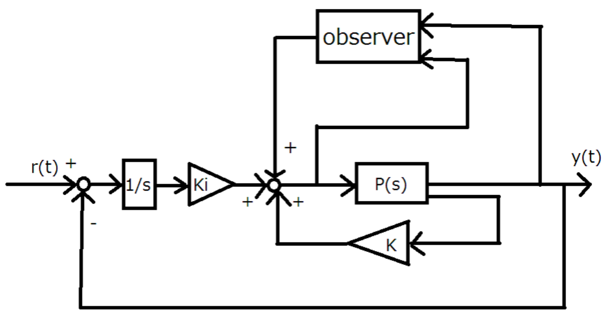

Figure 1 Robust H_2 control system with full order observer

2 Robust observer synthesis

It is difficult to deal with uncertainties in state space representation whose dependency is not affine. In this paper, we avoid this difficulty with using descriptor representation. Consider a continuous time multi-input multi-output system described by:

$$\Big(E+\sum_{i=1}^{k}\delta_i E_i\Big)\dot{x}(t) =$$
$$\Big(A+\sum_{i=1}^{k}\delta_i A_i\Big)x(t)+\Big(B+\sum_{i=1}^{k}\delta_i B_i\Big)u(t)$$
$$y = Cx(t) \tag{1}$$

where E,E_i,A,$A_i \in \Re^{n\times n}$,B,$B_i \in \Re^{n\times m}$,$C \in \Re^{l\times n}$.Eq(1) has affine perturbation in each coefficient matrices. Additionally, $\delta_i \in \Re$ is perturbation elements which satisfy $|\delta_i| \leq 1$. For simplicity $E(\delta)$,$A(\delta)$ and $B(\delta)$ matrices are defined as:

$$E(\delta) = E+\sum_{i=1}^{k}\delta_i E_i,\, A(\delta) = A+\sum_{i=1}^{k}\delta_i A_i$$
$$B(\delta) = B+\sum_{i=1}^{k}\delta_i B_i$$

In this section, polytopic observer and quadratic stability analysis for full order observer is discussed. Oya etal [3] developed a LMI-based stability analysis method that quadratic cost function for SISO system.

Consider the MIMO system described as descriptor system. Let descriptor variable is $\hat{x}(t) = [x^T \dot{x}^T]^T$ then Eq(1) is described as follows.

$$\hat{E}\dot{\hat{x}}(t) = \hat{A}(\delta)\hat{x}(t)+\hat{B}\hat{u}(t), y = \hat{C}\hat{x}(t) \tag{2}$$
$$\hat{E} = diag\{I,0\}, \hat{C} = \begin{bmatrix} C & 0 \end{bmatrix}^T$$
$$\hat{A}(\delta) = \begin{bmatrix} 0 & I \\ A(\delta) & -E(\delta) \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 \\ B(\delta) \end{bmatrix}$$

Where Eq(2) is redundant descriptor state space equation as estimated mechanism. The full order observer for argued system is describe as follows.

$$\hat{E}\dot{\hat{x}}(t) = \hat{A}(\delta)^T\hat{x}(t)+\hat{C}^T u(t)+H(y(t)-\hat{B}(\delta)^T\hat{x}(t)) \tag{3}$$

Matrix H is observer gain. We consider to minimizing the following cost function.

$$J = \int_0^\infty \hat{x}(t)^T Q\hat{x}(t)+u(t)^T Ru(t)dt \tag{4}$$

Where $Q \in \Re^{n\times n} > 0$,$R \in \Re^{m\times m}$ are given weighting matrices. The observer gain H is synthesized by optimazing Eq(4). Estimation error $e(t) \triangleq x-\hat{x}$ is defined. Estimation error system is as follow.

$$\begin{aligned}\dot{e}(t) &= (A(\delta)-HC)e(t) \\ &\quad +(A_e-HC_e-B_eK_x)\hat{x}\end{aligned} \tag{5}$$

Where $A_e = A(\delta) - A$,$B_e = B(\delta) - B$,$C_e = C(\delta) - C$. In this study, the system which omitted $\hat{x}$.

$$\dot{e}(t) = (A(\delta) - HC)e(t) \tag{6}$$

We consider an observer gain is guaranteed $e(t) \to 0$ of Eq(6).

Lemma1: Consider the following $e \triangleq x - \hat{x}$. Letting $x_e = [\hat{x}^T e^T]^T$ as extended system. then Eq(7) is described as follows.

$$\dot{x}_e = A_\phi(\delta)x_e \tag{7}$$

$$A_\phi = \begin{bmatrix} A_{\phi 11} & HC \\ A_{\phi 21} & A(\delta) - HC \end{bmatrix} \tag{8}$$

$$A_{\phi 11} = A + HC_e - BK_x$$

$$A_{\phi 21} = A_e + HC_e - B_e K_x$$

If there exists $\hat{P} > 0$, such that Eq(9) hold.

$$A_\phi^T \hat{P} + \hat{P}A_\phi + \Theta < 0 \ for \ \forall \in \delta$$

$$\Theta = \begin{bmatrix} Q + K_x^T R K_x & Q \\ Q & Q \end{bmatrix}, \hat{P} = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix} \tag{9}$$

In this approach, the full order observer which guaranteed asymptotically stability. The robust controller which is combined with robust observer is discussed. Then, Eq(9) satisfies K_x,H.If there exists $e(t) \to 0$,$\hat{x}(t) \to 0$, we obtain $x(t) \to 0$. The whole system is performed asymptotic stability.

We obtained following relationship:

$$\dot{V}(e) < 0 \to \lim_{t\to\infty} e(t) = 0 \tag{10}$$

2.1 Robust observer design

Eq(11) is the control plant for dual system.

$$\begin{cases} E_d \dot{x}_d(t) = A_d x_d(t) + B_d u_d(t) \\ y_d(t) = C_d x_d(t) \end{cases} \tag{11}$$

$$A_o = \begin{bmatrix} \hat{A}(\delta) & \hat{B}(\delta) \\ 0 & 0 \end{bmatrix}, B_o = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$C_o = \begin{bmatrix} C & 0 \end{bmatrix}$$

$$C_w = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}, D_w = \begin{bmatrix} 0 \\ R \end{bmatrix}$$

$$A_d = A_o^T \in \Re^{n\times n}, B_d = C_o^T \in \Re^{n\times m}, C_d = B_o^T \in \Re^{l\times n}$$

$$X = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}, Y = \begin{bmatrix} Y_{11} & 0 \end{bmatrix}$$

$$\text{minimize } \gamma$$

$$X_{11} > 0$$

$$\begin{bmatrix} He[A_{dmin}X + B_d Y] & (C_w X - D_w Y)^T \\ C_w X - D_w Y & -I \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} He[A_{dmax}X + B_d Y] & (C_w X - D_w Y)^T \\ C_w X - D_w Y & -I \end{bmatrix} < 0 \tag{13}$$

$$Y_{11} = K_d X_{11}$$

Observer gain H is obtained by Eq(12),Eq(13).

$$H = -K_d^T \tag{14}$$

3 Robust controller synthesis

Generally, it is difficult to analyze the system stability directly whose $E(\delta)$ matrix has uncertainty parameters.However through adopting descriptor variables as $\breve{x}(t) := [x^T \dot{x}^T u^T]^T$,uncertainties in each coefficient matrices are integrated into matrix $\breve{A}$.

$$\breve{E}\dot{\breve{x}} = \breve{A}(\delta)\breve{x} + \breve{B}u, y = \breve{C}\breve{x} \tag{15}$$

$$\breve{E} = diag\{I, 0, 0\}, \breve{C} = \begin{bmatrix} C & 0 & 0 \end{bmatrix}^T$$

$$\breve{A}(\delta) = \begin{bmatrix} 0 & I & 0 \\ A(\delta) & -E(\delta) & B(\delta) \\ 0 & 0 & -I \end{bmatrix}, \breve{B} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$$

Note that $\breve{E}$ is independent from uncertainty parameters and only $\breve{A}$ linearly depends on uncertainty.

3.1 Extended system

One integrator is added into the closed loop system. For the plant model Eq(15),let y and r are output, reference, respectively.Letting state as $\tilde{x}(t) = \begin{bmatrix} \int r - y & \breve{x} & u \end{bmatrix}^T$,We finally obtain Eq(16) for the augmented system with integrator.

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t) \tag{16}$$

$$\tilde{E} = \begin{bmatrix} I & 0 \\ 0 & \breve{E}(\delta) \end{bmatrix}, \tilde{A} = \begin{bmatrix} 0 & -C \\ 0 & \breve{A}(\delta) \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 \\ \breve{B}(\delta) \end{bmatrix}, B_r = \begin{bmatrix} I \\ O \end{bmatrix}$$

3.2 H_2 controller synthesis

The plant is described as follows:

$$\begin{aligned} \tilde{E}\dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t) + B_r r \\ z(t) &= \tilde{C}\tilde{x}(t) + \tilde{D}u(t) \end{aligned} \tag{17}$$

$$\tilde{C} = \begin{bmatrix} W_e & 0 & 0 \\ 0 & W_x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{D} = \begin{bmatrix} 0 \\ W_u \end{bmatrix}$$

where W_e,W_x and W_u are weights for the integration of error, the state, and the input. For the redundant descriptor system, we have already obtained the following lemma in the previous research[5],[6],[7].

Lemma2 If there exist $X_{11} > 0$, X, Y such that Eq(18),(19) hold, then the closed loop system with the state feedback $u = K_{11}x := Y_{11}X_{11}^{-1}x$ is stable.

$$X = \begin{bmatrix} X_{11} & 0 & 0 \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, Y = \begin{bmatrix} Y_{11} & 0 & 0 \end{bmatrix}$$

$$\text{minimize } \gamma$$

$$X_{11} > 0$$

$$\begin{bmatrix} He[\tilde{A}_{min}X + \tilde{B}Y] & (\tilde{C}X - \tilde{D}Y)^T \\ \tilde{C}X - \tilde{D}Y & -I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} He[\tilde{A}_{max}X + \tilde{B}Y] & (\tilde{C}X - \tilde{D}Y)^T \\ \tilde{C}X - \tilde{D}Y & -I \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} \gamma^2 & B_r I \\ B_r^T & X_{11} \end{bmatrix} > 0 \tag{20}$$

$$K = \begin{bmatrix} K_{11} & 0 \end{bmatrix} \tag{21}$$

Furthermore, through maximaizing the trace of X_{11}. Synthesized controller is divided into integration gain $K_i \in \Re^{m\times m}$ and state gain $K_x \in \Re^{m\times n}$ as $K = [K_i K_x]$.

4 Applying to 3-DOF helicopter

The picture of the 3-DOF helicopter is put on Fig.2.It has two propellers drew by DC motors at front and back.The helicopter can move in horizontal and vertical direction.The simple figure of the helicopter is illustrated in Fig.3.The support beam AB can rotate on point O.Where $\epsilon(t)$[deg] is the angle in vertical plane and $\lambda(t)$ is the angle in horizontal plane.The support beam CD can rotate in vertical direction on point B.Where $\rho(t)$[deg] is the angle of this time.Where u_f[V] and u_b[V] are the input voltage of the front rotor and back rotor each.The counter weights is added to keep the balance of the helicopter and it saves the energy of the rotors.

Figure 2 3-DOF helicopter

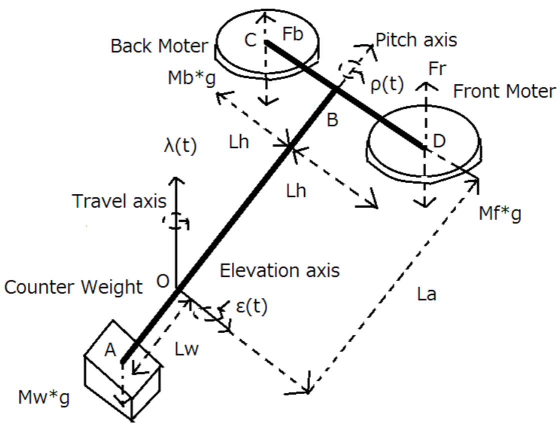

Figure 3 Schematic drawing

M_f	Mass of the front rotor	[kg]
M_b	Mass of the back rotor	[kg]
M_w	Mass of the counter weight	[kg]
g	Gravitation acceleration	$[m/s^2]$
L_a	Distance from O to B	[m]
L_h	Distance from B to C or D	[m]
L_w	Distance from O to A	[m]
K_f	Lift coefficient	[N/V]

Table 1 Physical parameters

4.1 State space equation

The state vector for the 3-DOF helicopter is defined as $x(t) = \begin{bmatrix} \epsilon(t) & \rho(t) & \lambda(t) & \dot{\epsilon}(t) & \dot{\rho}(t) & \dot{\lambda}(t) \end{bmatrix}^T$. Input is defined as $u(t) = \begin{bmatrix} u_f(t) & u_b(t) \end{bmatrix}^T$. The state space representation is given as follows:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + L \\ y(t) = Cx(t) \end{cases} \tag{22}$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_\epsilon & (M_f - M_b)L_aL_h & 0 \\ 0 & 0 & 0 & 2(M_f - M_b)L_aL_h & J_\rho & 0 \\ 0 & 0 & 0 & 0 & 0 & J_\lambda \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = K_f \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ L_a & L_a \\ L_h & -L_h \\ L_a \sin\rho(t) & L_a \sin\rho(t) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -(M_f + M_b)gL_a + M_wgL_w \\ -(M_f - M_b)gL_h \\ 0 \end{bmatrix}$$

where J_ϵ[kg$\cdot m^2$],J_ρ[kg$\cdot m^2$] and J_λ[kg$\cdot m^2$] are each moment of inertia. There are given as follows.

$$J_\epsilon = (M_f + M_b + M_g)L_a^2 + M_wL_w^2 \tag{23}$$

$$J_\rho = (M_f + M_b)L_h^2 \tag{24}$$

$$J_\lambda = (M_f + M_b)(L_a^2 + L_h^2) + M_wL_w^2 \tag{25}$$

Now take notice of $\sin\rho(t) = \rho(t)$ as $\sin\rho(t) = \rho(t)$,then

$$\ddot{\lambda}(t) = \frac{U(t)}{J_\lambda}\rho(t), (U(t) := K_fL_a(u_f(t) + u_b(t))) \tag{26}$$

Eq(26) is nonlinear.Matrices E, A and B are rewritten by linearization around equilibrium point, $U_0 = 0$ and $\rho_0 = 0$.

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (M_f + M_b)L_a^2 + M_wL_w^2 & (M_f - M_b)L_aL_h & 0 \\ 0 & 0 & 0 & 2(M_f - M_b)L_aL_h & (M_f + M_b)L_h^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{U_0}{J_\lambda} & 0 & 0 & 0 & 0 \end{bmatrix}, B = K_f \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ L_a & L_a \\ L_h & -L_h \\ 0 & 0 \end{bmatrix}$$

We verify the proposed method using the simulation and experiments in 3-DOF helicopter. In this simulations and experiments, we add a weight of 50.0[g] at the helicopter to verify the robust control and observer performance. W_e,W_x and W_u are chosen as follows.

$$W_e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W_u = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$W_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The simulation and experiment results are shown for the Fig.4, Fig.5 and Fig.6. The step response of 3-DOF helicopter is verified by simulations and experiments. The step response of elevation which is given 30 seconds later is 0.26[rad](15[deg]).

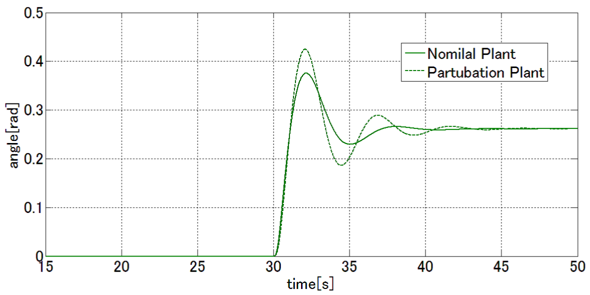

Figure 4 positioning of elevation(simulation)

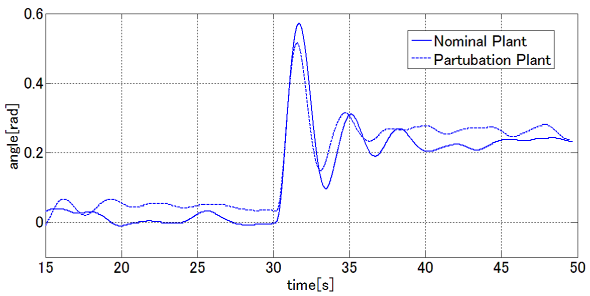

Figure 5 positioning of elevation(experiment)

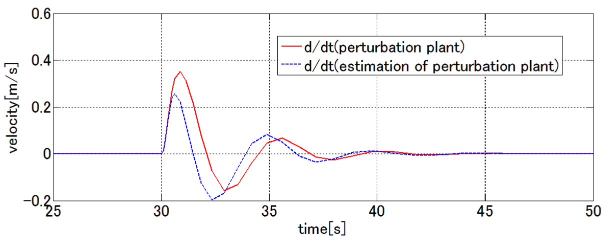

Figure 6 velocity estimation

5 Conclusion

In this study, we designed robust H_2 control system which is combined with robust observer. In addition, we show the method to deal with uncertain parameters which is described in rational function by redundant descriptor system. The effectiveness of proposed system is verified by the experiments. It can be said that the proposed method, that is the combination of robust controller and robust observer theory, is able to improve the robust control performance.

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