Robust H_∞ Control for Active Magnetic Bearing System with Imbalance of the Rotor

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1 Introduction

In this paper, H_∞ controller is designed to suppress the vibration caused by the static and dynamic imbalanced rotor and gyroscopic effect. The purpose of this study is to control attitude of the rotor by state feedback. Mathematical model of AMB has the first and the second order terms of the angular velocity of the rotor. In attempt to guarantee the robust stability for a prescribed range of angular velocity with lower conservativeness, the second order terms are changed into first order terms by using linear fractional transformation (LFT) and descriptor representation. Polytopic representation is applied to the system matrices which has the first order terms of the varying parameter. H_∞ controller is derived by solving a finite set of LMI conditions at vertex matrices. Furthermore the effectiveness of the proposed controller is illustrated by the simulations comparing with robust linear quadratic controller (RLQ).

2 Modeling

AMB has four electromagnets and two gap sensors at the both ends of the rotor. The perturbations from equilibrium point are derived by the variable transformation after the models are derived at coordinates of X, Y and Z. The coordinates X, Y and Z are set as shown in Figure 1. The origin point of the coordinates is the geometric center of the rotor. $g_j[m]$ is the perturbation of gap from equilibrium point. Constant current of electromagnet and controlled input are represented by I_j and i_j, respectively ($j = \{lv, rv, lh, rh\}$). The suffices mean as follows, lv is vertical direction of the left side, lh is horizontal direction of the left side, rv is vertical direction of the right side, and rh is horizontal direction of the right side, respectively. To control attitude of the rotor, the current of the electromagnets are adjusted. Physical parameters of AMB are shown in Table 1. The rotor has both static and dynamic imbalance as shown in Figure 1. Here, distance between the geometrical center and the center of gravity of the rotor is represented as ε. Angle between the geometrical axis and inertia axis is represented as τ. The force and torque caused by the imbalance are occurred in equation of the motion as follows [1].

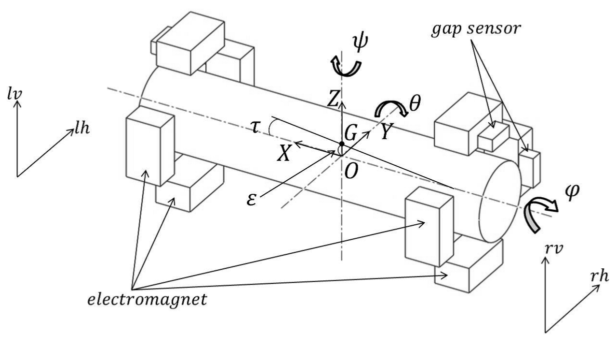

Figure 1 AMB system

Table 1 physical parameters

parameter	symbol	unit
Acceleration of gravity	g	[m/s^2]
Mass of rotor	m	[kg]
Length of rotor	l_m	[m]
Distance between the center and the center of gravity of the rotor	ε	[m]
Angle of the rotation axis to inertia axis	τ	[rad]
Distance between the center of gravity and the left side of the electromagnet	l_{ml}	[m]
Distance between the center of gravity and the right side of the electromagnet	l_{mr}	[m]
Moment of the X axis	J_x	[kgm^2]
Moment of the Y axis	J_y	[kgm^2]
Steady-state value of the sensor	G	[m]
Suction force constant	k	[Nm^2/A^2]
Radius of rotor	r	[m]
Constant current of vertical direction	I_{lv}, I_{rv}	[A]
Constant current of horizontal direction	I_{lh}, I_{rh}	[A]
Bias current	B	[A]

$$m\ddot{z} = f_{lv} + f_{rv} - mg + m\varepsilon p^2 \sin(pt) \quad (1)$$
$$m\ddot{y} = f_{lh} + f_{rh} + m\varepsilon p^2 \cos(pt) \quad (2)$$
$$J_y\ddot{\theta} = J_x p\dot{\psi} + f_{lv}l_{ml} - f_{rv}l_{mr} + (J_y - J_x)\tau p^2 \sin(pt) \quad (3)$$
$$J_y\ddot{\psi} = -J_x p\dot{\theta} - f_{lh}l_{ml} + f_{rh}l_{mr} + (J_y - J_x)\tau p^2 \cos(pt) \quad (4)$$

Here, y and z are the displacement in the direction of the axis Y and Z, respectively. θ and ψ are the rotation angle around the axis Y and Z, respectively. p and f_j are the angular velocity and the levitation force of the electromagnets, respectively ($j = \{lv, rv, lh, rh\}$). The angular velocity $p(t)$ is treated as a time varying parameter. z, y, θ and ψ can be approximated by assuming the displacement from steady gap $g_j(t)$ are small enough [2]. The levitation force of the electromagnet is given as Eq. (5) [3].

$$f_j = k\left\{\frac{(B + (I_j + i_j))^2}{(g_j - G)^2} - \frac{(B - (I_j + i_j))^2}{(g_j + G)^2}\right\} \quad (5)$$

The state variable $x(t)$ and the input variable $u(t)$ are defined as follows.

$$x(t) = [g_{lv}\ g_{rv}\ g_{lh}\ g_{rh}\ \dot{g}_{lv}\ \dot{g}_{rv}\ \dot{g}_{lh}\ \dot{g}_{rh}]^T \quad (6)$$
$$u(t) = [i_{lv}\ i_{rv}\ i_{lh}\ i_{rh}]^T \quad (7)$$

The state equation of AMB is derived as follows.

$$\dot{x}(t) = A(p)x(t) + Bu(t) + D(p^2)w(t) \quad (8)$$

LFT is applied to disturbance matrix $D(p^2)$. The descriptor representation is derived as state-space representation as follows.

$$E\dot{x}_l(t) = A_l(p)x_l(t) + B_l u(t) + D_l w(t) \quad (9)$$

Eq. (9) is equivalent to Eq. (8) but has no second order terms of angular velocity. The range of time varying parameter p is defined as $p \in [\underline{p}, \overline{p}] = [p_1, p_2]$.

Matrix $A_l(p)$ is represented by the following matrix polytope.

$$A_l(p) = \alpha A_l(p_1) + (1-\alpha)A_l(p_2), \alpha \in [0,1] \quad (10)$$

Robust H_∞ controller is designed for the system (9) with matrix polytope (10).

3 Controller design

The output $z_l(t)$ is defined to design H_∞ controller as follows.

$$z_l(t) = W_x x_l(t) + W_u u(t) \quad (11)$$

For the obtained state-space representation Eq. (9), we design the state feedback H_∞ controller. The LMI conditions to derive the state feedback H_∞ controller stabilizing the system (8) are as given as follows.

Theorem 1 : If there exist matrices X and Y satisfying the following LMI conditions, the system (9) is stabilized by $u = Kx_l = YX^{-1}x_l$ and the system (8) is stabilized by $u = \tilde{K}x = Y_{11}X_{11}^{-1}x$. Furthermore, H_∞ norm $||T_{wz_l}||_\infty$ is less than γ_∞.

$$\begin{bmatrix} He[M(p)] & D_l & (W_x X + W_u Y)^T \\ D_l^T & -\gamma_\infty^2 I & O \\ W_x X + W_u Y & O & -I \end{bmatrix} \prec 0 \quad (12)$$

$$M(p) = A_l(p)X + B_l Y, Y = KX \quad (13)$$

$$X = \begin{bmatrix} X_{11} & 0 \\ X_{12} & X_{22} \end{bmatrix}, X_{11} \succ 0, Y = [Y_{11} \quad 0] \quad (14)$$

To guarantee the stability of the system (9), Eq. (12) have to be satisfied for all $p \in [p_1, p_2]$. However, inequality (12) has only first order terms of p. If Eq. (12) is satisfied by common solution at the both vertex matrices $A_l(p_1)$ and $A_l(p_2)$, the stability is guaranteed for all angular velocity. Common solution is obtained by solving the following set of LMI conditions shown in Corollary 1.

Corollary 1 : If there exist matrices X and Y satisfying (14), (15), the system (9) is stabilized by $u = Kx_l = YX^{-1}x_l = \tilde{K}x = Y_{11}X_{11}^{-1}x$ for the prescribed range of angular velocity p and $||T_{wz_l}||_\infty$ is less than γ_∞.

$$\begin{bmatrix} He[M(p_i)] & D_l & (W_x X + W_u Y)^T \\ D_l^T & -\gamma_\infty^2 I & O \\ W_x X + W_u Y & O & -I \end{bmatrix} \prec 0 \quad (15)$$

$$(i = 1, 2)$$

The feedback gain K stabilizing the system (8) is derived from obtained matrices X and Y.

4 Simulation

In this section, the effectiveness of proposed method is illustrated by simulations using mathematical model of MBC 500 [3]. The angular velocity of the rotor is increased to 25,000 [rpm] in 10 seconds. In this study, the rotor has both static and dynamic imbalance. Therefore, the vibration increase with p^2. Note that the disturbance itself can not be controlled. In this situation, we aim to suppress vibration as much as possible. The simulation results of the displacements from the equilibrium point and input current on the vertical direction of the left side are shown in Figure 2 and Figure 3, respectively. From Figure 3, the input currents of H_∞ and

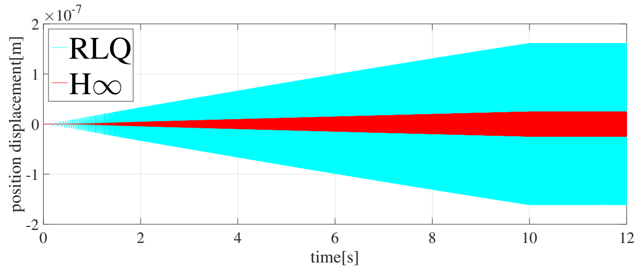

Figure 2 The displacement of the rotor on the vertical direction of the left side

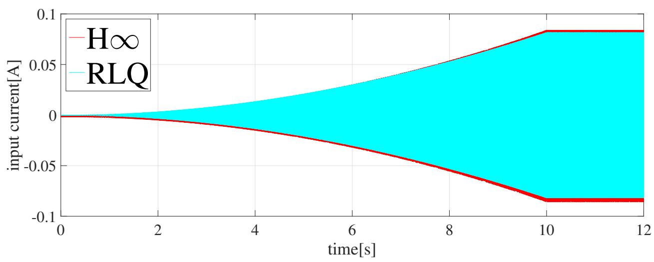

Figure 3 The input current on the vertical direction of the left side

RLQ are approximately equal. However, from Figure 2, the vibration is suppressed by H_∞ control than RLQ. The effectiveness of proposed H_∞ control is illustrated.

5 Conclusion

This paper proposes design of a robust H_∞ controller for AMB system whose rotor has both static and dynamic imbalance. The proposed controller is designed to suppress the vibration caused by imbalance and gyroscopic effect. Furthermore the effectiveness of the proposed controller is illustrated by simulations comparing with RLQ.

References

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