Correction to the Paper “Formulas in modal logic S4”

Katsumi Sasaki
Faculty of Information Sciences and Engineering, Nanzan University,
27 Seirei-Cho, Seto 489-0863, Japan
e-mail: sasaki@nanzan-u.ac.jp.
2011.8.16.

In our paper [1], there are two points, which should be modified. First, in [1], “an exact model for $\mathbf{F}^n$” is treated as a model, which verify all the provable formulas in **S4** in **F**. However, this did not described. Second, in page 605 in [1], we describe that there are three differences between Fine’s normal forms and members of our $\mathbf{ED}^n$. However, there is one more difference between them. In other words, the set of normal forms corresponding to Fine in our sequent style is not exactly $\mathbf{Fine}(n)$.

Specifically, we have to correct the paper as follows. The list below also includes other typos.

Page 602, at the end of subsection 1.3 We add the following.

“Among the exact models for $\mathbf{F}^n$, we mainly consider the exact models for $\mathbf{F}^n$ satisfying

$$M \models A \text{ for any } A \in \{A' \in \mathbf{F} \mid A' \in \mathbf{S4}\}.$$

We refer to this model as an exact S4-model for $\mathbf{F}^n$.

Lemma 1.5 Let $M = \langle W, R, P\rangle$ be an exact S4-model for $\mathbf{F}^n$. Then M is reflexive and transitive.

Proof. We show that M is reflexive. Suppose that $\alpha \not R \alpha$ for some $\alpha \in W$. Then by the second condition of an exact model, there exists a formula $A \in \mathbf{F}^n$ such that $P(A) = \{\alpha\}$. Therefore, we have

$$(M, \alpha) \not\models \Box(A \supset \bot) \supset (A \supset \bot),$$

which is in contradiction with $\Box(A \supset \bot) \supset (A \supset \bot) \in \{A' \in \mathbf{F} \mid A' \in \mathbf{S4}\}$. Similarly, by $\Box(A \supset \bot) \supset \Box\Box(A \supset \bot) \in \{A' \in \mathbf{F} \mid A' \in \mathbf{S4}\}$. M is transitive, similarly. ⊣

We use the above lemma without specific justification in the rest of the paper.”

Page 603, lines 1↑–2↑ “$\cdots$ to list all exact S4-models for $\mathbf{F}^n$ (in section 4, we will give 14 exact S4-models for $\mathbf{F}^1$),”

Page 604, lines 13–14 “$\cdots$ In section 4, we will give a finite method to list all exact S4-models for $\mathbf{F}^n$, and for each exact S4-model $M = \langle W, R, P\rangle$, $\cdots$”

Page 604, lines 22–23 “$\cdots$ Here, in order to construct exact S4-models for $\mathbf{F}^n$, we also use some members of $\mathbf{ED}^n$, but not all. We also use members of $\mathbf{ED}^{n+i}$ $(i = 1, 2, \cdots)$ for worlds of exact S4-models for $\mathbf{F}^n$.”

Page 604, line 24 “Also, by the description to list all exact S4-models for $\mathbf{F}^n$, $\cdots$”

Page 605, line 5 “ $\cdots$ Comparing Fine’s normal form with ours, there are four differences.

Page 605, lines 15–16 We replace these lines with

“ $\mathbf{Fine}(0) = \{(\mathbf{V} - V_1 \to V_1) \mid V_1 \subseteq \mathbf{V}\}$,

$\mathbf{Fine}(n+1) = \{(\Box\Gamma, \mathbf{V} - V_1 \to V_1, \Box\Delta) \mid \Gamma \cup \Delta = \mathbf{for}(\mathbf{Fine}(n)), \Gamma \cap \Delta = \emptyset, V_1 \subseteq \mathbf{V}\}$.

Second, we use members of $\displaystyle\bigcup_{i=0}^{n} \mathbf{ED}^i$ to construct a member of $\mathbf{ED}^{n+1}$. Specifically, we modify $\mathbf{Fine}(n)$ as follows.

$\mathbf{Fine}'(0) = \{(\mathbf{V} - V_1 \to V_1) \mid V_1 \subseteq \mathbf{V}\}$,

$\displaystyle\mathbf{Fine}'(n+1) = \bigcup_{X \in \mathbf{Fine}'(n)} \mathbf{next}^+(X)$,”

Page 605, lines 17–20 " where

$$\textbf{next}^+(X) = \{(\Box\Gamma, \textbf{ant}(X) \to \textbf{suc}(X), \Box\Delta) \mid \Gamma \cup \Delta = \textbf{for}(\textbf{Fine}'(n)), \Gamma \cap \Delta = \emptyset\}. \qquad (*)$$

A sequent in $\textbf{Fine}'(n+1)$, the next step of $\textbf{Fine}'(n)$, is constructed from a sequent $X \in \textbf{Fine}'(n)$ by adding each member of $\Box\textbf{for}(\textbf{Fine}'(n))$ to either the antecedent or the succedent"

Page 605, line 21 We add the following sentence: "We note that there may be a sequent in $\{X \in \textbf{Fine}'(n) \mid X \in \textbf{K}\}$. "

Page 605, line 22 "Third, we remove the sequents provable in **S4**. $\cdots$"

Page 605, line 27 "Fourth, we distinguish the sequents with a maximal antecedent.$\cdots$"

Page 617, line 7 "$\cdots$ to list exact S4-models for $\mathbf{F}^1$."

Page 618, lines 2–3 "exact S4-models for $\mathbf{F}^n$; as a result, we obtain methods for (II) in subsection 1.4. For example, in the case that $m = 1$, there are just four exact S4-models for $\mathbf{F}^0$ and just 14 exact S4-models"

Page 618, line 29 "$\cdots$ is an exact S4-model for $\mathbf{F}^n$."

Page 618, line 30 "(2) For any exact S4-model M for $\mathbf{F}^n$, $\cdots$"

Page 619, line 4 "exact S4-models for $\mathbf{F}^n$. By (4), for each exact S4-model for $\mathbf{F}^n$, $\cdots$"

Page 620, line 2 "be exact S4-model for $\mathbf{F}^n$.

Page 622, line 21 "From the above lemma, Lemma 1.2, and Lemma 4.7, we obtain $\cdots$"

Page 622, the last line "$\cdots$ be an exact S4-model for $\mathbf{F}^n$,"

Page 623, line 10 We remove "**for**($\in$".

Page 624, line 6 "$\cdots$ exact S4-models M_1 and M_2 for $\mathbf{F}^1$:"

References

[1] K. Sasaki, *Formulas in modal logic S4*, The Review of Symbolic Logic, 3, 2010, pp. 600–627.