Adaptive Control for Jib Crane System with Rope Hoisting and Uncertain Parameters

M2018SC003 Satoshi ISHIKURA
Supervisor : Gan CHEN

1 Introduction

This paper proposes a robust control system which consists of a robust controller and Model Reference Adaptive Control law for an uncertain jib crane system with rope hoisting. In this study, decentralized control comprised of two independent controllers is utilized to control the jib crane. To suppress the influences of system uncertainties e.g., variations of the rope length and nonlinear friction, we design the control system composed of a linear robust controller and an adaptive law for the positioning system of the trolley. We apply the linear robust controller for the variation of system characteristics caused by hoisting the rope. On the other hand, the adaptive law is designed to estimate nonlinear friction. The characteristic of this study is to cope with nonlinear friction by using the robust controller with the adaptive law. The adaptive law is utilized for the estimation and compensation of nonlinear friction. Besides, we approximate nonlinear friction by a nonlinear function in the adaptive law. We show the exponential stability for the system with the proposed method by using Linear Matrix Inequality. Finally, we verify the effectiveness of the adaptive law by contrasting the proposed method with the only robust controller in the simulation of load transferring.

2 Modeling

The crane of this type is composed of a tower, a trolley, a jib, and a load. The crane of this type is utilized to put up the loads and to transfer the loads to target positions. The important things for control of the jib crane are to transfer the loads to target positions fast and to suppress the oscillations of the loads. Fig. 1 shows a simplified schematic of the jib crane. Sensors mounted on the crane can detect the position of the trolley to a horizontal direction $\xi(t)$[m], the angle for the load swing $\gamma(t)$[rad] and the length of the rope $l(t) > 0$[m]. Inputs are an electric current $I_t(t)$[A] to a motor for moving the trolley and an electric current $I_h(t)$[A] to a motor for hoisting the load. An output of the system is the position of the load to a horizontal direction $y(t) = x_p(t) = \xi(t) - l(t)\sin(\gamma(t))$[m].

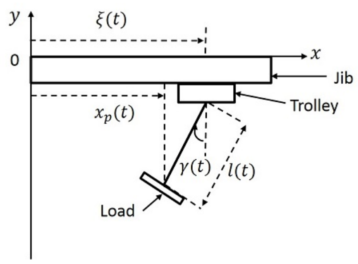

Figure 1 Simplified schematic of the jib crane

Here, the following assumptions are considered, (i) the rope is a rigid rod which doesn't have mass, (ii) the load is a mass point, (iii) the load moves in a horizontal direction and a vertical direction along the jib, (iv) the angle for the load swing γ and its velocity $\dot{\gamma}$ are sufficiently small. Table 1 shows the physical parameters of the crane. Equations of motion for the jib crane are

Table 1 Physical parameters

Parameter	Symbol
Mass of load [kg]	m_p
Mass of trolley [kg]	m_t
Acceleration of gravity [m/s^2]	g
Gear radius for motor of trolley moving [m]	$r_{j,p}$
Reel radius of load [m]	r_r
Gear box efficiency for motor of trolley moving [-]	$\eta_{g,j}$
Gear box efficiency for motor of rope hoisting [-]	$\eta_{g,z}$
Gear ratio for motor of trolley moving [-]	$K_{g,j}$
Gear ratio for motor of rope hoisting [-]	$K_{g,z}$
Efficiency for motor of trolley moving [-]	$\eta_{m,j}$
Efficiency for motor of rope hoisting [-]	$\eta_{m,z}$
Torque constant for motor of trolley moving [Nm/A]	$K_{t,j}$
Torque constant for motor of rope hoisting [Nm/A]	$K_{t,z}$
Moment of inertia for motor of trolley moving [kgm^2]	J_ψ
Moment of inertia for motor of rope hoisting [kgm^2]	J_ϕ

expressed as follows [1]:

$$m_j\ddot{\xi} - m_p l\ddot{\gamma}\cos\gamma - m_p\ddot{l}\sin\gamma$$
$$-2m_p\dot{l}\dot{\gamma}\cos\gamma + m_p l\dot{\gamma}^2\sin\gamma = k_{tj}I_t - F_n, \quad (1)$$
$$-m_p l\ddot{\xi}\cos\gamma + m_p l^2\ddot{\gamma} + 2m_p l\dot{l}\dot{\gamma} + m_p lg\sin\gamma = 0, \quad (2)$$
$$-m_p\ddot{\xi}\sin\gamma + m_l\ddot{l} + m_p l\dot{\gamma}^2 - m_p g\cos\gamma = k_{tl}I_h, \quad (3)$$

where $m_j = m_p + m_t + J_\psi\frac{K_{g,j}^2}{r_{j,p}^2}$, $k_{tj} = \frac{\eta_{g,j}K_{g,j}\eta_{m,j}K_{t,j}}{r_{j,p}}$, $m_l = m_p + \frac{J_\phi}{r_r^2}$ and $k_{tl} = \frac{\eta_{g,z}K_{g,z}\eta_{m,z}K_{t,z}}{r_r}$. The nonlinear friction between the jib and the trolley F_n is shown as follows:

$$F_n = \begin{cases} \mathrm{sgn}(F_{total})\min(|F_{total}|, f_s) & (\dot{\xi} = 0) \\ f_c\mathrm{sgn}(\dot{\xi}) + f_v\dot{\xi} & (\dot{\xi} \neq 0) \end{cases}, \quad (4)$$

where f_s, f_c, f_v, and $\mathrm{sgn}(\cdot)$ are maximum static friction, Coulomb friction, the viscous friction coefficient, and signum function, respectively. Besides, $F_{total} = k_{tj}I_t - m_p g\sin\gamma\cos\gamma - m_p l\dot{\gamma}^2\sin\gamma + m_p\ddot{l}\sin\gamma$. These friction parameters are unknown in controller design. For (1)–(3), let $\sin\gamma \simeq \gamma$, $\cos\gamma \simeq 1$, $\dot{\gamma}^2 \simeq 0$ because the angle for the load swing γ and its velocity $\dot{\gamma}$ are sufficiently small on the assumption (iv). Then, (1)–(3) are expressed as follows:

$$m_j\ddot{\xi} - m_p l\ddot{\gamma} - m_p\ddot{l}\gamma - 2m_p\dot{l}\dot{\gamma} = k_{tj}I_t - F_n, \quad (5)$$

$$-m_p\ddot{\xi} + m_p l\ddot{\gamma} + 2m_p\dot{l}\dot{\gamma} + m_p g\gamma = 0, \quad (6)$$

$$-m_p\ddot{\xi}\gamma + m_l\ddot{l} - m_p g = k_{tl}I_h. \quad (7)$$

In this study, we employ the decentralized control composed of two independent controllers because this control method is effective for control of cranes. We control the positioning system of the trolley by the controller proposed in this paper. On the other hand, we control the system of rope hoisting by a typical servo controller. Hence, we focus on the equations of motion for the movement of the trolley. In this study, we suppose that the velocity and the acceleration of the rope variation are sufficiently small. It is not always efficient for the crane control to consider the velocity and the acceleration of the rope change because these parameters don't affect the stability of the crane system. Therefore, we omit the terms that contain the velocity and the acceleration of the rope variation in (5) and (6) to design the controller efficiently. For (5) and (6), let $q = [\xi\ \gamma]^{\mathrm{T}}$ be generalized coordinate. An approximate model for the system of the trolley positioning can be derived as follows:

$$E_m(l)\ddot{q} + F_m q = G_m I_t - H_m F_n, \quad (8)$$

$$E_m(l) = \begin{bmatrix} m_j & -m_p l \\ -m_p & m_p l \end{bmatrix}, F_m = \begin{bmatrix} 0 & 0 \\ 0 & m_p g \end{bmatrix},$$

$$G_m = \begin{bmatrix} k_{tj} \\ 0 \end{bmatrix}, H_m = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Let $x = [q\ \dot{q}]^{\mathrm{T}}$ and $u = I_t$ be the state vector and input, respectively. Then, the model for the trolley positioning is described as follows:

$$\begin{cases} \dot{x} &= A_p(l)x + B_p(l)u - B_f F_n \\ y &= C_p(l)x \end{cases}, \quad (9)$$

$$A_p(l) = \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ -E_m^{-1}F_m & 0_{2\times2} \end{bmatrix}, B_p(l) = \begin{bmatrix} 0_{2\times1} \\ E_m^{-1}G_m \end{bmatrix},$$

$$B_f = \begin{bmatrix} 0_{2\times1} \\ H_m \end{bmatrix}, C_p(l) = \begin{bmatrix} 1 & -l & 0_{1\times2} \end{bmatrix}.$$

In this study, we use the linearized model for the system of the trolley positioning in (9) to design a robust H_∞ controller for the change of the rope length l. Then, we design the control system composed of a robust positioning controller and an adaptive law in the following section.

3 Controller Design

The variation of the rope length and the nonlinear friction between the jib and the trolley make it difficult to control the positioning system of the trolley safely. To deal with these difficulties, we design the control system composed of the robust H_∞ controller and Model Reference Adaptive Control (MRAC) law. We design the robust H_∞ controller to deal with the negative impact of the variation of rope length. In the design of this controller, we omit the terms that contain the velocity and the acceleration of the rope variation because these parameters don't affect the stability of the crane system. On the other hand, MRAC law handles friction. Besides, H_∞ control is utilized to cope with the friction in the case when MRAC law doesn't behave well.

3.1 Robust H_∞ Controller

To deal with the negative impact of the variation of the rope length, we design the robust H_∞ controller. Note that we don't deal with the effect of the rope velocity and the acceleration. Here, the servo control system is constructed to get rid of the stationary error between the target position and the actual position of the load. Let r, $e = r - y$ and $x_e = \int e\ dt$ be the reference trajectory, the error of the system and its integral value, respectively. Here, $x_s = [x^{\mathrm{T}}\ x_e]^{\mathrm{T}}$ and u_{rc} denote the state vector and the controller input of the servo control system, respectively. The servo control system is obtained as follows:

$$\begin{cases} \dot{x}_s &= A_s(l)x_s + B_s(l)u_{rc} - B_{ef}F_n \\ y_s &= C_s(l)x_s \end{cases}, \quad (10)$$

$$A_s(l) = \begin{bmatrix} A_p(l) & 0_{4\times1} \\ -C_p(l) & 0 \end{bmatrix}, B_s(l) = \begin{bmatrix} B_p^{\mathrm{T}}(l) & 0 \end{bmatrix}^{\mathrm{T}},$$

$$B_{ef} = \begin{bmatrix} B_f^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}}, C_s(l) = \begin{bmatrix} C_p(l) & 0 \end{bmatrix}.$$

Subsequently, the polytope (11) is given by upper and lower bounds of a time-varying parameter, a rope length l.

$$\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1], \theta_1 = l \quad (11)$$

In this study, we utilize the polytope in (11) to avoid solving the infinite set of LMI conditions. By solving LMI at only two vertices of θ_1 instead of all $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1]$, we can guarantee H_∞ norm condition for all $\theta \in [\underline{\theta}_1, \overline{\theta}_1]$. Here, let $w = -F_n$ as the disturbance to the servo control system. From (10), the generalized plant is derived as follows:

$$\begin{cases} \dot{x}_s &= A_s(\theta_1)x_s + B_s(\theta_1)u_{rc} + B_{ef}w \\ z &= C_z x_s + D_z u_{rc} \end{cases}, \quad (12)$$

where C_z and D_z are weighting matrixes for the state variables of the servo control system x_s and those for the input to the motor for the movement of the trolley u_{rc}, respectively. Here, we consider minimizing H_∞ norm $\|G_{zw}\|_\infty$ from disturbance w to the evaluated output z. H_∞ norm $\|G_{zw}\|_\infty$ is defined as follows:

$$\|G_{zw}\|_\infty = \sup_{w \neq 0 \in L_2} \frac{\|z\|_2}{\|w\|_2} < \gamma_\infty, \quad (13)$$

where $\|w\|_2$, $\|z\|_2$ and γ_∞ are L_2 norm of the disturbance w, that of the evaluated output z and upper value of H_∞ norm $\|G_{zw}\|_\infty$, respectively. The LMI condition to employ the robust H_∞ controller which stabilizes the servo control system (10) are derived as follows:

Theorem 1 If there exist X_s and Y_s meeting the following LMI conditions, the system is stable by the state feedback $u_{rc} = Y_s X_s^{-1} x_s := K x_s$, where $K = Y_s X_s^{-1}$ is the state-feedback gain. Additionally, $\|G_{zw}\|_\infty < \gamma_\infty$ is guaranteed.

$$
\begin{aligned}
&\text{minimize} : \gamma_\infty^2 \\
&\text{subject to} : X_s > 0, \qquad (14)\\
&\begin{bmatrix} \mathrm{He}\{A_s(\theta_1)X_s + B_s(\theta_1)Y_s\} & B_{ef} & M_s^{\mathrm{T}} \\ B_{ef}^{\mathrm{T}} & -I & 0 \\ M_s & 0 & -\gamma_\infty^2 I \end{bmatrix} < 0, \qquad (15)\\
&M_s := C_z X_s + D_z Y_s, \qquad (16)
\end{aligned}
$$

where the notation $\mathrm{He}\{M\}$ means $M^{\mathrm{T}} + M$.

The robust H_∞ controller to deal with the influence of the variation of the rope length is derived by solving (14)–(16).

3.2 Adaptive Law with σ-modification

We design a compensator to cancel the influence of the nonlinear friction because the friction causes the performance deterioration of the robust positioning controller and the residual oscillations of the load. In this study, the adaptive controller with σ-modification is utilized as the real-time compensator for the friction.

Controller design of the adaptive controller is proposed by [2]. Let x_s, u and y_s be the state vector of the system which is considered as the actual plant in MRAC design, input, and the output, respectively. Then, an approximate model of the actual plant is expressed by (17).

$$
\begin{cases} \dot{x}_s = A_s(l)x_s + B_s(l)\{u + W^{\mathrm{T}}\phi(x_s)\} \\ y_s = C_s(l)x_s \end{cases}, \qquad (17)
$$

where W and $\phi(x_s)$ are an uncertain parameter vector and a known basis function, respectively. Besides, we try to represent the nonlinear friction $-F_n$ by nonlinear uncertainty $k_{tj}\{W^{\mathrm{T}}\phi(x_s)\}$. Here, the controller input of the system is given as follows.

$$
u = Kx_s - u_{ad}, \qquad (18)
$$

where u_{ad} denotes an adaptive signal to suppress the effect of $W^{\mathrm{T}}\phi(x_s)$. Here, the adaptive signal is as follows.

$$
u_{ad} = \hat{W}^{\mathrm{T}}\phi(x_s), \qquad (19)
$$

where $\hat{W}$ is the estimated value of the uncertain parameter vector W. In this study, Coulomb friction, static friction and viscous friction coefficient are uncertain parameters in controller design. In the case when $\dot{\xi} = 0$, the nonlinear friction involves the product of absolute value and signum function of a resultant force F_{total}, the product of absolute value of x_e and signum function of $k_{tj}x_e$ from (4). We consider approximating $\mathrm{sgn}(F_{total})$ by $\tanh\{f_{cv}(k_{tj}x_e)\}$ to avoid chattering by the signum function in controller input, where f_{cv} is a related coefficient of the basis function. Therefore, we employ $-|x_e|\tanh\{f_{cv}(k_{tj}x_e)\}$ to the basis function as the replacement for $-\mathrm{sgn}(F_{total})\min(|F_{total}|, f_s)$. On the other hand, in the case when $\dot{\xi} \neq 0$, the nonlinear friction involves the trolley velocity $\dot{\xi}$ and its signum function $\mathrm{sgn}(\dot{\xi})$. We employ $-\tanh(f_{cv}\dot{\xi})$ to the basis function as the replacement for $-\mathrm{sgn}(\dot{\xi})$ because the controller input including the signum function causes chattering. For the above reasons, the basis function is determined by considering $-|x_e|\tanh\{f_{cv}(k_{tj}x_e)\}$, the trolley velocity $-\dot{\xi}$ and its arctangent function $-\tanh(f_{cv}\dot{\xi})$. The basis function $\phi(x_s)$ is as follows:

$$
\phi(x_s) = [\phi_1(x_e), \phi_2(\dot{\xi}), \phi_3(\dot{\xi})]^{\mathrm{T}}, \qquad (20)
$$

where $\phi_1(x_e) = -|x_e|\tanh\{f_{cv}(k_{tj}x_e)\}$, $\phi_2(\dot{\xi}) = -\tanh(f_{cv}\dot{\xi})$ and $\phi_3(\dot{\xi}) = -\dot{\xi}$. Let us consider the reference model for an ideal behavior of the system. The reference model is described as follows:

$$
\begin{cases} \dot{x}_r = A_{rm}(l)x_r + B_{rm}r \\ y_r = C_{rm}(l)x_r \end{cases}, \qquad (21)
$$

$$
A_{rm}(l) = A_s(l) + B_s(l)K,\ B_{rm} = \begin{bmatrix} 0_{1\times 4} & 1 \end{bmatrix}^{\mathrm{T}},\ C_{rm}(l) = C_s(l).
$$

Let the tracking error e_t between the state of the reference model x_r and that of the approximate model for the actual plant x_s be $e_t = x_r - x_s$. The estimated uncertain parameter vector W is renewed by (22).

$$
\dot{\hat{W}} = -\gamma_s\phi(x_s)e_t^{\mathrm{T}}PB_s(l) - \sigma\hat{W}, \qquad (22)
$$

where $\gamma_s = \mathrm{diag}\{\gamma_{s1}, \gamma_{s2}, \gamma_{s3}\} \in \mathbb{R}_{3\times 3}$ $(\gamma_{s1}, \gamma_{s2}, \gamma_{s3} > 0)$ and $\sigma > 0$ are the adaptive gain and σ-modification gain, respectively. From (12), the uncertain parameter $\theta_1 = l, \theta_1 \in \{\underline{\theta}_1, \overline{\theta}_1\}$. We get the matrix $P = P^{\mathrm{T}} > 0$ by solving the following equation at the both vertex matrixes $A_{rm}(\underline{\theta}_1)$ and $A_{rm}(\overline{\theta}_1)$.

$$
\mathrm{He}\{PA_{rm}(\theta_1)\} < 0 \qquad (23)
$$

Here, the tracking error dynamics and renewal rule of the weight estimation error are shown as follows [2]:

$$
\dot{e}_t = A_{rm}(\theta_1)e_t + B_s(\theta_1)\tilde{W}^{\mathrm{T}}\phi(x_s), \qquad (24)
$$

$$
\dot{\tilde{W}} = -\gamma_s\phi(x_s)e_t^{\mathrm{T}}PB_s(\theta_1) - \sigma\tilde{W} - \sigma W. \qquad (25)
$$

Let $\zeta = [e_t^{\mathrm{T}}\ \tilde{W}^{\mathrm{T}}]^{\mathrm{T}}$. We express the dynamics that consist of the tracking error and the weight estimation error by (26).

$$
\dot{\zeta} = \bar{A}(\theta_1, \phi(x_s))\zeta + \bar{B}\sigma W, \qquad (26)
$$

$$
\bar{A} = \begin{bmatrix} A_{rm}(\theta_1) & B_s(\theta_1)\phi^{\mathrm{T}} \\ -\gamma_s\phi B_s(\theta_1)^{\mathrm{T}}P & -\sigma I \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ -I \end{bmatrix}
$$

Basis function $\phi(x_s) = [\phi_1(x_e), \phi_2(\dot{\xi}), \phi_3(\dot{\xi})]^{\mathrm{T}}$ is given as $\phi \in \Phi$, where the parameter box $\Phi = \{\phi = [\phi_1, \phi_2, \phi_3]^{\mathrm{T}} : \phi_j \in \{\underline{\phi}_j, \overline{\phi}_j\}\}$. Therefore, we can calculate the lower bounds and the upper bounds of $\phi_j \in [\underline{\phi}_j, \overline{\phi}_j]$. If we can see the system in (26) as an exponentially stable system with the constant disturbance σW, the stability of the entire control system is guaranteed. Besides, uncertain parameter vector W is a bounded input. Hence, we consider the exponential stability analysis for the system in (27) which doesn't have the constant disturbance σW.

$$
\dot{\zeta} = \bar{A}(\theta_1, \Phi)\zeta \qquad (27)
$$

The exponential stability of the system in (27) is inspected by solving the following LMI [2].

Theorem 2 If there exists $X = X^{\mathrm{T}} > 0$ meeting the following inequality, the system in (27) is exponentially stable for variations of θ_1 and ϕ_j.

$$\mathrm{He}\{X\bar{A}(\theta_1, \Phi)\} < 0 \tag{28}$$

By solving the matrix inequality (28), the exponential stability of the system in (27) is guaranteed theoretically.

4 Simulation

In this section, we express the effectivity of the robust H_∞ controller with the MRAC law (the proposed method) by simulation. We validate the performance of the proposed method by contrasting it with the only robust H_∞ controller (the conventional method). In other words, we show the effectiveness of the MRAC law. Besides, we use the robust H_∞ controller which is adjusted moderately to show the validity of the MRAC law. Note that this robust controller is not adjusted strictly. In the simulation, the initial position for the trolley $\xi(0)$ and the final position for the load r are set as $\xi(0) = 0$[m] and $r = 0.6$[m], respectively. To ensure the soft start of the trolley, we use the following equation for reference y_{ref}.

$$y_{ref} = 0.6\{1 - \exp(-8.33t^3)\}[\mathrm{m}] \tag{29}$$

The upper and the lower bounds of θ_1 are set as $\theta_1 \in [0.1, 0.7]$. The maximum value of static friction f_s, the Coulomb friction f_c and the viscous friction coefficient f_v are set as $f_s = 2.3$, $f_c = 2.2$ and $f_v = 6.2$, respectively. Note that these friction-related parameters are unknown in controller design. A related coefficient of the basis function $f_{cv} = 1.0 \times 10^4$ is used in this study. To see the robustness to θ_1 of the robust controller, we control the rope lengths by another controller as illustrated in Fig. 2. Note that we consider the time responses of rope length as the uncertainty of the system for the trolley positioning.

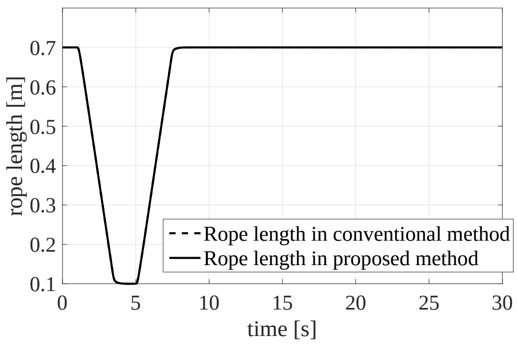

Figure 2 Rope length l (dashed line: rope length in the conventional method, solid line: rope length in the proposed method)

The simulation results are shown in Figs. 3 and 4. Fig. 3 shows the time responses of the horizontal positions of the load. As you can see in Fig. 3, the load reaches the target position r swiftly by utilizing the proposed method. On the other hand, the conventional method causes the error between the target position and the load position, and overshoot under the influence of friction. Fig. 4 shows the time responses of the swing angle. From Fig. 4, the proposed method suppresses the vibrations of the load. On the other hand, the conventional method causes the residual oscillation of the load. From these results, we can find that the MRAC law suppresses the influence of nonlinear friction.

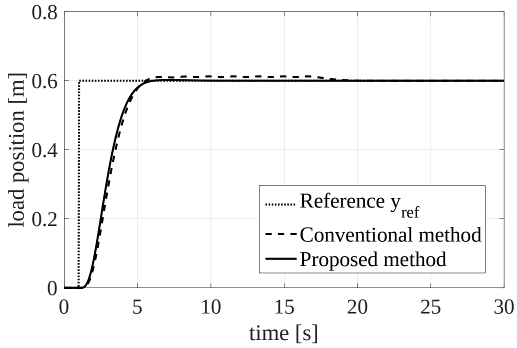

Figure 3 Load position x_p (dotted line: reference, dashed line: the conventional method, solid line: the proposed method)

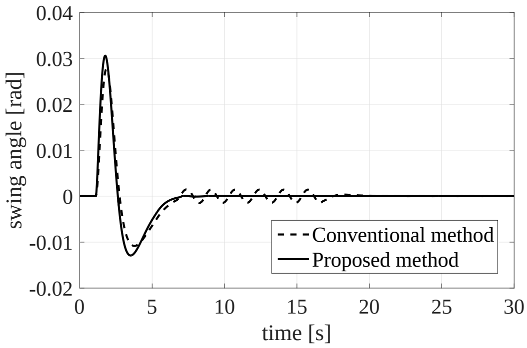

Figure 4 The angle for load swing γ (dashed line: the conventional method, solid line: the proposed method)

5 Conclusions

For the jib crane with rope hoisting, we have presented the robust H_∞ control system with the MRAC law. The main object of this paper is to cope with the effects which uncertain parameters such as rope length and nonlinear friction cause. In this study, we have handled an uncertainty of the rope length l by the robust H_∞ controller. Additionally, the nonlinear uncertainties for friction are dealt with the MRAC law with σ-modification. We have shown the performance of the proposed method by contrasting the proposed method with the only robust H_∞ controller in the simulation. As you can see in the simulation, the proposed method can decrease the influence of nonlinear friction by the MRAC law.

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