Model Reduction via Subsystem Decomposition for Flexible Rotor with Gyro Effect and Application of GS Control

M2017SC009 Takemitsu Takagi
Supervisor : Isao Takami

1 Introduction

Bearing is one of the fundamental mechanical elements. Generally bearings have physical contact with mating parts. It is difficult for traditional bearings to rotate fast because of influence from friction and wear. On the other hand, the magnetic bearings do not have physical contact with mating parts, but instead, are supported by magnets on either end. The magnetic bearing can eliminate difficulties caused by friction and wear. Thus, high-speed rotation of the rotor becomes possible. For example, the magnetic bearings could be applied to turbo-molecular pumps, high speed shafts for machine tools, auxiliary artificial hearts, artificial satellites, and so on [1]. On the other hand, there are disadvantages on the magnetic bearings. The magnetic bearings are unstable, therefore, they need to be controlled at all times. Hence, a more precise model for the magnetic bearing is needed so that they can be controled. Regarding the magnetic bearing, there are two types of rotor models: a rigid rotor model and a flexible rotor model. The former is a rotor which rotates under critical speed. The latter is a rotor that rotates over critical speed. It is necessary to use the flexible rotor in order to increase the angular velocity of the rotor. In the case of using the flexible rotor model, the order of the system increases because the model is generally derived by finite element method. As a result, a controller design becomes difficult. Thus, it is necessary to reduce the order of the model.

Matthew et al. [2] gives more details on the model reduction in rotary systems. In the survey, four general model reduction methods are introduced as follows : Guyan method, Modal analysis method[3], Component mode synthesis method, and Balanced truncation method [4]. There are difficulties of skewness and speed dependency when the order of the rotary system is reduced. These difficulties are caused by gyroscopic effect. In the survey, it has been reported that the methods based on singular value decomposition (SVD) provide the best results when considering gyroscopic effect. Recently, it proposed the model reduction methods resolving the skewness. Nonami et al. proposed the model reduction methods via Cholesky decomposition [5] and singular value decomposition [6]. Seto et al. proposed the model reduction method via an extended reduced order physical model[7].

In this study, decomposing dynamics of the magnetic bearing into horizontal dynamics and vertical dynamics is considered. The horizontal model and the vertical model without gyro terms is derived. Then, the systems and gyro terms is connected by using augmented variables. Hence, The magnetic bearing system can be represented as a closed loop system of the horizontal model without gyro terms, the vertical model without gyro terms and two gyro terms connecting each system. The horizontal and vertical dynamics without gyro terms don't have skewness and speed dependence. Thus, the horizontal and vertical dynamics are linear time-invariant (LTI) systems. The order of the horizontal and vertical dynamics can be reduced by any general reduction methods. Hence, it is not necessary to calculate the reduction procedure every time the angular velocity of the rotor is varied because the proposed reduction procedure doesn't depend on the angular velocity of the rotor. Furthermore, the obtained reduction model is a simple linear parameter-varying system for the angular velocity. It is convenient to design a gain-scheduled (GS) controller. The effectiveness of the proposed model reduction method is illustrated by Bode diagrams. The GS controller is designed with the proposed model. Finally, The effectiveness of the GS controller is illustrated by simulation.

2 Modeling

The flexible rotor model is derived via finite element method. When the rotor is separated in n elements, let h_k and θ_{h_k} be the displacement and the angle in horizontal direction in kth point mass ($k = \{1, 2, \cdots, n\}$), respectively. Similarly, let v_k and θ_{v_k} be the displacement and the angle in vertical direction, respectively. Here, the vectors of horizontal and vertical direction are defined as $q_h = [h_1 \ \theta_{h_1} \ \cdots \ h_n \ \theta_{h_n}]^T$ and $q_v = [\ v_1 \ \theta_{v_1} \ \cdots \ v_n \ \theta_{v_n}]^T$, respectively. The state vector q and input vector u are defined as $q = [q_h \quad q_v]^T$ and $u = [u_1 \quad \cdots \quad u_{n_u}]^T$, respectively. Then a motion equation of magnetic bearing with gyroscopic effect is given as follows [1];

$$M\ddot{q} + (C + G(p))\dot{q} + Kq = Fu, \tag{1}$$
$$M = \begin{bmatrix} M_h & O \\ O & M_v \end{bmatrix}, C = \begin{bmatrix} C_h & O \\ O & C_v \end{bmatrix}, K = \begin{bmatrix} K_h & O \\ O & K_v \end{bmatrix},$$
$$G(p) = \begin{bmatrix} O & -J_p(p) \\ J_p(p) & O \end{bmatrix}, F = \begin{bmatrix} F_h & O \\ O & F_v \end{bmatrix},$$

where M, C, K, F, G and p are the mas matrix including inertia force, the damping matrix, the stiffness matrix, the locations of electromagnets, the gyro matrix and the angular velocity of the rotor, respectively. It is known that M, C, K and $G(p)$ are a positive definite matrix, a semi-positive definite matrix, a symmetric indefinite matrix (not positive definite matrix) and a skew-symmetric matrix [1]. Note that $M, C, K, G(p)$ and F have a certain structure as above. A model reduction method by using the certain structure is proposed in a later section.

It is known that the skew-symmetric matrix causes difficulties when the ordinary reduction method is applied. It has been reported that a model reduction method based on Cholesky decomposition is effective when K is positive definite. In this study, the model reduction method based on Cholesky decomposition can not used because K is not positive definite. The state variable x is defined as $x = [q_h \quad q_v \quad \dot{q}_h \quad \dot{q}_v]^T$. The state equation is obtained as follows;

$$P(s)\left\{\begin{array}{l}\dot{x}=A(p)x+Bu\\ y=Cx\end{array}\right., \tag{2}$$

$$A(p)=\begin{bmatrix}O & I\\ -M^{-1}K & -M^{-1}(G(p)+C)\end{bmatrix},$$

$$B=\begin{bmatrix}O\\ M^{-1}F\end{bmatrix}, C=\begin{bmatrix}F^T & O\end{bmatrix},$$

where $A(p)\in\mathbb{R}^{8n\times 8n}, B\in\mathbb{R}^{8n\times n_u}, C\in\mathbb{R}^{n_y\times 8n}, x\in\mathbb{R}^{8n\times 1}, u\in\mathbb{R}^{n_u\times 1}$ and $y\in\mathbb{R}^{n_y\times 1}$. n_u and n_y are the number of input and output, respectively. This state equation (2) is called Full Model in the following sections.

3 Model Reduction Method

In this section, the following method is proposed to resolve the above difficulty. The motion equation (1) is decomposed into the horizontal motion and the vertical motion. The motion equation (1) can be represented as follows;

$$M_h\ddot{q}_h+C_h\dot{q}_h+K_hq_h=F_hu_h-J_p(p)\dot{q}_v, \tag{3}$$
$$M_v\ddot{q}_v+C_v\dot{q}_v+K_vq_v=F_vu_v+J_p(p)\dot{q}_h. \tag{4}$$

Let augmented variables z_h, z_v, w_h and w_v be as follows;

$$z_h=\dot{q}_h, \tag{5}$$
$$z_v=\dot{q}_v, \tag{6}$$
$$w_h=-J_p(p)z_v, \tag{7}$$
$$w_v=J_p(p)z_h. \tag{8}$$

The motion equations are represented as follows by augmented variables;

$$M_h\ddot{q}_h+C_h\dot{q}_h+K_hq_h=F_hu_h+Iw_v, \tag{9}$$
$$M_v\ddot{q}_v+C_v\dot{q}_v+K_vq_v=F_vu_v+Iw_h. \tag{10}$$

The each model without gyro terms is derived. The motions (3), (4) and gyro terms (7), (8) by augmented variables are connected. Hence, the magnetic bearing system can be represented as the closed loop system of the horizontal model without gyro terms, the vertical model without gyro terms and two gyro terms connecting each system. The closed loop system of the motions and gyro terms is shown in Figure 1. The both systems (9) (10) are linear time-invariant (LTI) which do not depend on the angular velocity. The orders of both systems can be reduced by any general reduction methods.

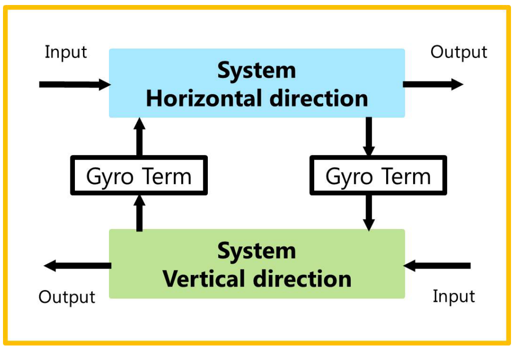

Figure 1 Block diagram of magnetic bearing system

In this study, the reduced model is obtained by using the modal analysis method[3].

The following equation is obtained by the normalized mode matrix Φ_h;

$$M_h\Phi_h=\Lambda_hK_h\Phi_h, \quad \Lambda_h=diag\{\lambda_{h_1},\lambda_{h_2},\cdots,\lambda_{h_{2n}}\}. \tag{11}$$

λ_{h_i} is the eigenvalue of $K^{-1}M$. The vector q_h is defined as $q_h=\Phi_h\xi_h$, where let ξ_h be the mode variable ($\xi_h=[\xi_{h_1} \ \cdots \ \xi_{h_{2n}}]$). The equation is obtained as follows from (3);

$$M_h\Phi_h\ddot{\xi}_h+C_h\Phi_h\dot{\xi}_h+K_h\Phi_h\xi_h=F_hu_h+Iw_v, \tag{12}$$
$$\ddot{\xi}_h+\Delta_h\dot{\xi}_h+\Omega_h\xi_h=\Phi_h^TF_hu_h+\Phi_h^Tw_v, \tag{13}$$
$$I=\Phi_h^TM_h\Phi_h, \Delta_h=\Phi_h^TC_h\Phi, \Omega_h=\Phi_h^TK_h\Phi_h,$$

where Δ_h is approximated as $\Delta_h=diag\{2\zeta_{h_1}\omega_{h_1},\cdots,2\zeta_{h_{2n}}\omega_{h_{2n}}\}$ because the damping matrix C_h is small. ζ_{h_k} and ω_{h_k} $(k=\{1,\cdots,2n\})$ are the damping rate and the natural frequency, respectively. Here, the state variable x_{fh} is defined as $x_{fh}=\begin{bmatrix}\xi_h & \dot{\xi}_h\end{bmatrix}^T$. The following system is obtained;

$$\left\{\begin{array}{l}\dot{x}_{fh}=A_{fh}x_{fh}+B_{fh}w_h+B_{fuh}u_h\\ z_h=C_{fh}x_{fh}\\ y_h=C_{fyh}x_{fh}\end{array}\right., \tag{14}$$

$$A_{fh}=\begin{bmatrix}O & I\\ -\Omega_h & -\Delta_h\end{bmatrix}, B_{fh}=\begin{bmatrix}O\\ \Phi_h^T\end{bmatrix}, B_{fuh}=\begin{bmatrix}O\\ \Phi_h^TF_h\end{bmatrix},$$

$$C_{fh}=\begin{bmatrix}O & \Phi_h\end{bmatrix}, C_{fyh}=C_{yh}\begin{bmatrix}\Phi_h & O\\ O & \Phi_h\end{bmatrix}.$$

The state variable x_{sh} is defined as $x_{sh}=\begin{bmatrix}\xi_{h_1} & \cdots & \xi_{h_r} & \dot{\xi}_{h_1} & \cdots & \dot{\xi}_{h_r}\end{bmatrix}^T$. The reduced model is obtained as follows;

$$\left\{\begin{array}{l}\dot{x}_{sh}=A_{sh}x_{sh}+B_{sh}w_h+B_{suh}u_h\\ z_h=C_hx_{sh}\\ y_h=C_{syh}x_{sh}\end{array}\right.. \tag{15}$$

The reduced model for the vertical dynamics is obtained as follows as well as the model reduction procedure for the horizontal dynamics;

$$\left\{\begin{array}{l}\dot{x}_{sv}=A_{sv}x_{sv}+B_{sv}w_v+B_{suv}u_v\\ z_v=C_vx_{sv}\\ y_v=C_{syv}x_{sv}\end{array}\right.. \tag{16}$$

The closed loop systems are represented as follows by the augmented variables (7), (8) and the reduced model (15), (16);

$$\dot{x}_{sh}=A_{sh}x_{sh}-B_{sh}J_p(p)C_{sv}x_{sv}+B_{suh}u_h, \tag{17}$$
$$\dot{x}_{sv}=A_{sv}x_{sv}+B_{sv}J_p(p)C_{sh}x_{sh}+B_{suv}u_v. \tag{18}$$

Here, the state variable x_s is defined as $x_s=\begin{bmatrix}x_{sh} & x_{sv}\end{bmatrix}^T$. Finally, the state equation is obtained as follows from the closed loop systems (17) and (18);

$$P_{ma}(s)\left\{\begin{array}{l}\dot{x}_s=A_s(p)x_s+B_su\\ y=C_sx_s\end{array}\right., \tag{19}$$

$$A_s(p)=\begin{bmatrix}A_{sh} & -B_{sh}J_p(p)C_{sv}\\ B_{sv}J_p(p)C_{sh} & A_{sv}\end{bmatrix},$$

$$B_s=\begin{bmatrix}B_{suh} & O\\ O & B_{suv}\end{bmatrix}, C_s=\begin{bmatrix}C_{syh} & O\\ O & C_{syv}\end{bmatrix},$$

where $A_s(p) \in \mathbb{R}^{2r\times 2r}, B_s \in \mathbb{R}^{2r\times n_u}, C_s \in \mathbb{R}^{n_y\times 2r}, x_s \in \mathbb{R}^{2r\times 1}, u \in \mathbb{R}^{n_u\times 1}$ and $y \in \mathbb{R}^{n_y\times 1}$.

The conventional method based on singular value decomposition(SVD) [6] requires to recalculate the reduction procedure every time the angular velocity of the rotor is varied. On the other hand, $A_s(p)$ of (19) only has to be recalculated when the angular velocity of the rotor is varied because the proposed reduction procedure doesn't depend on the angular velocity of the rotor. Hence, its calculation cost of the proposed method is much less than that of the conventional method based on SVD. Note that $P_{ma}(s)$ is a simple linear parameter-varying system for the angular velocity of the rotor. It is convenient to design a gain-scheduled (GS) controller.

4 Controller design

The gain scheduled (GS) controller [8] is designed with the proposed model. The scheduling parameter of the designed controller is the angular velocity p. The state feedback controller which minimizes the following cost function is derived;

$$J = \int_0^\infty (x_s^T Q x_s + u^T R u) dx, \tag{20}$$

where $Q = Q_h^T Q_h \geq 0, R > 0$. If there exist $X(p)$ and $Y(p)$ satisfying the following LMI conditions, the system is stabilized by the state feedback controller with gain $K_{gs}(p) = Y(p)X(p)^{-1}$. The LMI conditions are as follows;

$$\begin{bmatrix} -He\{N(p)\} + \dot{X}(p) & (Q_h X(p))^T & (RY(p))^T \\ Q_h X(p) & I & O \\ RY(p) & O & R^{-1} \end{bmatrix} \succ 0,$$

$$\begin{bmatrix} Z & I \\ I & X(p) \end{bmatrix} \succ 0,$$

$$Trace(Z) < \gamma,$$

$$N(p) = A_s(p)X(p) + B_s Y(p), X(p) = X_0 + pX_P,$$

$$X_P = \begin{bmatrix} X_{P1} & X_{P1} \\ X_{P1} & X_{P1} \end{bmatrix}, X_{P1} = \begin{bmatrix} X_{P11} & O \\ O & O \end{bmatrix}.$$

Then, the cost function J is lower than γ.

The magnetic bearing have sensors only at either end, therefore, the other state variable can not observed. In this study, the full order observer is used. Estimated error is defined as $e = x_s - \tilde{x}_s$, where let $\tilde{x}_s$ be estimate value. Therefore, output feedback controller designed by the full order observer is as follows;

$$\begin{cases} \dot{\tilde{x}}_s = A_s \tilde{x}_s + B_s u - L(y - C_s \tilde{x}_s) \\ u = K_{gs} \tilde{x}_s \end{cases}. \tag{21}$$

The observer gain L is designed by optimal regulator from separation theorem.

5 Numerical Example

The proposed method is evaluated by Bode diagram. In this study, an active Magnetic Bearing MBC500 of Launch Point is taken for example [8]. The magnetic bearing has pairs of four electromagnets and two hall effect sensors at the each end of the rotor, i.e., $n_u = 4$ and $n_y = 4$. The rotor model is shown in Figure 2. In this study, the rotor is separated into three elements by finite element method. The number is $n = 3$. Let i_j be input current. The levitation force of the electromagnet

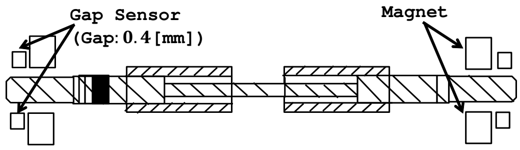

Figure 2 Rotor schematic view

Table 1 Natural Frequency

ω_{h_1}	0	[rad/s]
ω_{h_2}	0	[rad/s]
ω_{h_3}	1042	[rad/s]
ω_{h_4}	2306	[rad/s]
ω_{h_5}	3896	[rad/s]
ω_{h_6}	4817	[rad/s]

is approximated by Taylor expansion around the equilibrium point. The approximated equation is as follows;

$$f_j = k_{sf}\frac{4bI_j}{G_0^2} + K_{hvj}q_j + K_c i_j, \tag{22}$$

$$K_{hvj} = k_{sf}\frac{4(b^2 + I_j)}{G_0^3}, K_c = \frac{4b}{G_0^2},$$

where $j = \{h_1, h_3, v_1, v_3\}$. G_0, k_{sf}, I_j and b are the nominal gap, the suction force constant, the steady-state current and the bias current, respectively. Let control input u be $u = [i_{h_1}\ i_{h_3}\ i_{v_1}\ i_{v_3}]^T$. The following equation is obtained by the levitation force of the electromagnet (22);

$$M\ddot{q} + (C + G(p))\dot{q} + Kq = FK_c u, \tag{23}$$

$$K = K_0 - K_{hv}.$$

K_0 and K_{hv} are the stiffness matrix and the matrix including levitation force of the electromagnet, respectively. K is not positive definite. The details of the matrices is omitted for the sake of space. Here, the model of horizontal in (23) is used for eigenvalue analysis. The natural frequency is shown in Table 1. Here, the proposed method and the conventional method based on SVD [6] is compared. The first resonance frequency is 1042[rad/s] by eigenvalue analysis. the orders of the systems by considering the rigid mode and the first flexible mode is reduced because maximum angular velocity of the rotor is 1571[rad/s], thus , the orders of the systems based on the conventional model $P_{svd}(s)$ and the proposed model $P_{ma}(s)$ are both 12.

The effectiveness of the proposed method is verified by comparing full model and two reduced model. The input and the output are defined as input current and sensor of left horizontal direction. Bode diagrams of the systems are shown in Figure 3. As the angular velocity of the rotor increase, it can be seen that each flexible mode is splitted into backward flexible mode and forward mode by gyroscopic effect [6]. From this figure, it is noted that the reduced model based on the proposed method is almost as approximate accuracy as the reduced model based on SVD.

Next, the effectiveness of the proposed controller (GS controller) is evaluated by simulations. The proposed controller is verified by comparing with the conventional controller. The conventional controller means

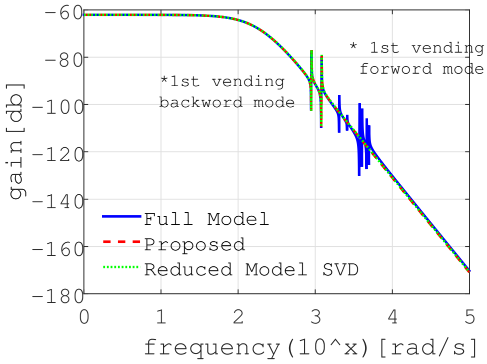

Figure 3 Bode diagram($p = 628$[rad/s])

the LQ controller in an angular velocity of the rotor (p = 1042[rad/s]). The angular velocity and the angular acceleration of the rotor are set up as 0 [rad/s] and 26.1[rad/s^2], respectively. The result of simulation is shown in Figure 4. Figure 4 illustrates the displacement of horizontal in the left side of the rotor when the whirling vibrations around the first resonance point is generated. The proposed controller can reduce vibration more than the conventional controller.

However, the difference of the proposed controller and the conventional controller is small. Here, the mass and the radius in second element is tripled. The effectiveness of the proposed controller are evaluated by simulations as with the above simulations. Displacement in left side of the rotor around the first resonance point are shown in Figure 5. The result shows the effectiveness of the proposed controller.

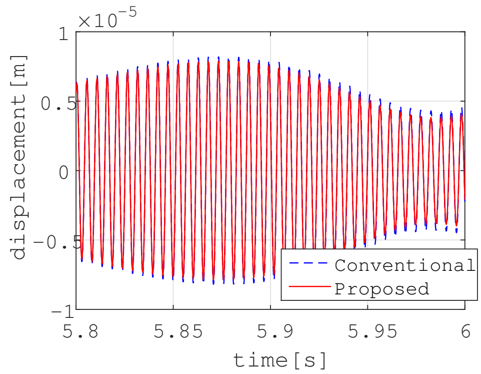

Figure 4 Displacement in left side of the rotor

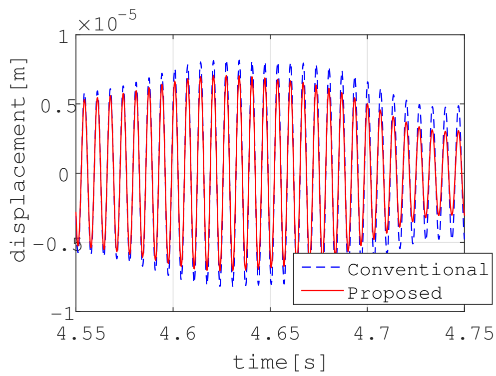

Figure 5 Displacement in left side of the rotor

6 Conclusion

This paper proposed the model reduction method using subsystem decomposition. The magnetic bearing system is represented as a closed loop system of the horizontal model without gyro terms, the vertical model without gyro terms and two gyro terms connecting each system. The order of the horizontal dynamics and vertical dynamics are reduced by modal analysis method and fractional balanced reduction method. The effectiveness of the proposed reduction method is verified by the Bode diagrams. The reduced model based on the proposed method has almost as approximate accuracy as the reduced model based on SVD. However, the proposed method is superior in three ways. First, any general model reduction can be chosen. Second, its calculation cost of the proposed method is mush less than the conventional method based on SVD. Third, it is easy to design the GS controller. Therefore, controllers need not to be switched such as conventional method[6]. The GS controller is designed with the proposed model. Finally, The effectiveness of the GS controller is illustrated by simulation.

References

[1] Gerhard Schweitzer, Eric H. Maslen, "Magnetic Bearings", Springer, 2009.

[2] Matthew B. Wagner, Amir Younan, Paul Allaire and Randy Cogill, "Model reduction methods for rotor dynamic analysis: a survey and review", International Journal of Rotating Machinery, Vol. 2010, Article ID 273716, 17 pages, 2010.

[3] Stimac Goranka, Braut Sanjin, Zigulic Roberto, "Vibration Suppression of a Flexible Rotor Using Active Magnetic Bearings (AMB)", Transactions of FAMENA, , Vol. 35, No. 3, p. 27-38, 2011.

[4] Mushi, Simon E., Zongli Lin and Paul E. Allaire, "Design, construction and modeling of a flexible rotor active magnetic bearing test rig", proc. ASME Turbo Expo 2010: Power for Land, Sea, and Air. American Society of Mechanical Engineers, 2010.

[5] Yajun Zhang, Kenzo Nonami and Hiromasa Higata, "Sliding Mode Control of 10MWh Class Energy Storage Flywheel System Using Superconducting Magnetic Bering with Gyroscopic Effect", Transactions of the Japan Society of Mechanical Engineers, Vol. 67, No. 662, 2001. (Written in Japanese)

[6] Ming Ren and Kenzo Nonami, "Modeling of Flexible Rotor Magnetic Bering Flywheel System with Gyroscopic Effect Using Singular Value Decomposition (1st Report: Precise modeling and control system design)", Journal of the Japan Society Applied Electromagnetic and Mechanics, Vol. 14, No. 4, 2006. (Written in Japanese)

[7] Naoyuki Tanaka, Naoki Uchiyama, Toru Watanabe and Kazuto Seto, "Levitation and vibration control of a flexible rotor by using active magnetic bearing", Journal of System Design and Dynamics Vol. 3, No. 4, p. 551-562, 2009.

[8] Sanbayashi, Akio, Masanori Narita, Gan Chen and Isao Takami, "Gain scheduled control for active magnetic bearing system considering gyroscopic effect", proc. 7th International Conference on Information Technology and Electrical Engineering, pp. 553-558, 2015.