H_2 Control for Active Suspension Considered as Delayed-Disturbance System to Improve Ride Quality

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Abstract

In this paper, H_2 state feedback controller of active suspension for delayed-disturbance to improve ride quality is proposed. The time-delay included in the disturbance is considered in the proposed controller without approximation. The LMI condition to design the proposed controller is derived by using descriptor representation and Lyapnov function for the time-delay system. Additionally, the ride quality is improved by loop-shaping based on ISO2631-1. The half-car model of the vertical acceleration and the pitch angular acceleration are shaped by reduced-order frequency weight. The effectiveness of the proposed controller is evaluated by simulations and experiments compared with conventional controller ignoring the time-delay. The ride quality is analyzed based on ISO2631-1 and ISO8608 in the simulation and the experiment.

1 INTRODUCTION

A suspension is a shock absorber mainly equipped in between a car body and a wheel. The suspension called passive suspension is composed of a spring and a damper. It is possible to improve ride quality for vehicle by suppressing a disturbance from a road surface. Generally, the ride quality is evaluated by International Organization for Standardization (ISO) 2631-1 [1]. According to ISO2631-1, a frequency weighting curve for human sensibility is defined for a vertical acceleration and a pitch angular acceleration. The vertical acceleration and pitch angular acceleration are evaluated by ratio of 10:4 in ISO2631-1. There is a half-car model as a suspension model. The half-car model can be considered the vertical motion and the pitch angle motion. There is a characteristic that delayed-disturbance in the half-car model. The time-delay is occurred by length of wheelbase between a front wheel and a rear wheel. The vertical acceleration and the pitch angular acceleration are influenced by the time-delay.

An active suspension is to suppress vibration by controlling an electric actuator. It is possible to suppress the vibration by generating an anti-vibration from the electric actuator. The active suspension can improve the ride quality better than the passive suspension to control the vibration. It is mainly used for automobiles and bullet trains to improve ride quality.

Many control theories for the vehicle active suspension have been proposed. Loop-shaping based on ISO2631-1 is proposed to improve ride quality [2]. The time-delay between the front wheel and the rear wheel is approximated by using Padé approximation to improve ride quality in [2]. However, the Padé approximation cannot accurately represent frequency response in high-frequency band [2]. In [3], the delayed-disturbance is carried without approximation to derive necessary conditions for H_2 control using a method of Lagrange multiplier. In addition, the time-delay included in the state-vector and control input is carried without approximation by using descriptor form in [4].

The purpose of this study is designing H_2 state-feedback controller of the active suspension for the delayed-disturbance attenuation without approximation. The ride quality can be improved by loop-shaping based on ISO2631-1. The controller is designed by new LMI conditions. The LMI conditions to design the controller is derived by using descriptor form. The effectiveness of the proposed controller is evaluated by simulations and experiments based on ISO2631-1 and ISO8608[6].

2 MODELING

The half-car model composed of two quarter car models is shown in Figure 1 [2]. The suffixes f and r mean

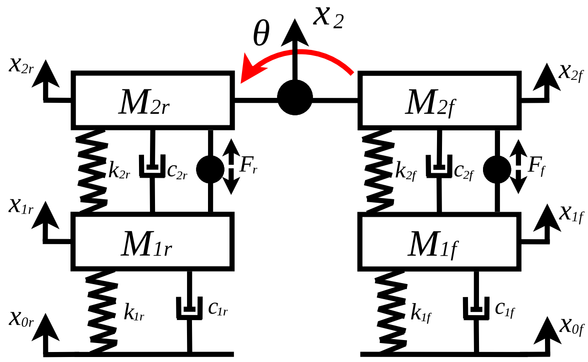

Figure 1 Half Car Model

the front side and the rear side of the vehicle, respectively. In this situation, the rear suspension is not vibrated when the front suspension vibrates. This situation may be occurred when the car body weight is balanced for the suspension mounted position. For example, it is may considered that heavy load such as engine and luggage are on directly above the suspension. For this reason, the same situation with real car body may be occurred in the model. Variables x_2 and θ are displacement of vertical motion and the pitch angle at the center of the car body, respectively. These variables are calculated by following equations.

$$x_2 = \frac{l_r x_{2f} + l_f x_{2r}}{l}, \ \theta = \arcsin(\frac{x_{2f} - x_{2r}}{l}) \qquad (1)$$

Where x_{2i}, x_{1i} and x_{0i} $(i = f, r)$ are the displacement of sprang mass, the displacement of unsprang mass, and the road surface, respectively. Physical constants are shown in Table 1.

2.1 Motion equation

Motion equations for a displacement of sprung mass and a displacement of unsprung mass at equilibrium point are derived as Eq. (2) - Eq. (5) by using Newto-

Table 1 Physical Constants

M_{1f}	Front unsprung mass	[kg]
M_{1r}	Rear unsprung mass	[kg]
k_{1f}	Front wheel stiffness	[N/m]
k_{1r}	Rear wheel stiffness	[N/m]
c_{1f}	Front wheel damping	[Ns/m]
c_{1r}	Rear wheel damping	[Ns/m]
M_{2f}	Front sprung mass	[kg]
M_{2r}	Rear sprung mass	[kg]
k_{2f}	Front suspension stiffness	[N/m]
k_{2r}	Rear suspension stiffness	[N/m]
c_{2f}	Front suspension damping	[Ns/m]
c_{2r}	Rear suspension damping	[Ns/m]
l	Length of car body	[m]
l_f	Length from center of gravity to front axis	[m]
l_r	Length from center of gravity to rear axis	[m]

nian Equation of motion.

$$\begin{aligned} M_{1f}\ddot{x}_{1f} &= k_{2f}(x_{2f}-x_{1f})+c_{2f}(\dot{x}_{2f}-\dot{x}_{1f}) \\ &\quad -k_{1f}(x_{1f}-x_{0f})-c_{1f}(\dot{x}_{1f}-\dot{x}_{0f})-F_f \quad (2) \\ M_{1r}\ddot{x}_{1r} &= k_{2r}(x_{2r}-x_{1r})+c_{2r}(\dot{x}_{2r}-\dot{x}_{1r}) \\ &\quad -k_{1r}(x_{1r}-x_{0r})-c_{1r}(\dot{x}_{1r}-\dot{x}_{0r})-F_r \quad (3) \\ M_{2f}\ddot{x}_{2f} &= -k_{2f}(x_{2f}-x_{1f})-c_{2f}(\dot{x}_{2f}-\dot{x}_{1f})+F_f (4) \\ M_{2r}\ddot{x}_{2r} &= -k_{2r}(x_{2r}-x_{1r})-c_{2r}(\dot{x}_{2r}-\dot{x}_{1r})+F_r \quad (5) \end{aligned}$$

2.2 State-space representation

The state-space representation is obtained from motion equations. The state-vector $x(t)$, the control input $u(t)$, and the output $y(t)$ are defined as follows.

$$\begin{aligned} x(t) = [\ & x_{2f}-x_{1f} \quad x_{2r}-x_{1r} \quad x_{1f}-x_{0f} \quad x_{1r}-x_{0r} \\ & \dot{x}_{2f} \quad \dot{x}_{2r} \quad \dot{x}_{1f} \quad \dot{x}_{1r}\ ]^T \end{aligned} \tag{6}$$

$$y(t) = [\ \ddot{x}_2 \quad \ddot{\theta}\ ]^T,\ u(t) = [\ F_f \quad F_r\ ]^T \tag{7}$$

The disturbance vector is defined in Eq. (8).

$$w(t) = [\ w_f(t) \quad w_r(t)\ ]^T = [\ \dot{x}_{0f} \quad \dot{x}_{0r}\ ]^T \tag{8}$$

Here $w_f(t)$ and $w_r(t)$ are the front disturbance and the rear disturbance respectively. There is the time-delay $\tau(t)$ between the front disturbance and the rear disturbance generated by length of between the front wheel and the rear wheel. The time-delay is calculated by Eq. (9).

$$\tau(t) = \frac{l}{V(t)\cdot 1000/3600} \tag{9}$$

Here $V(t)$[km/h] is a car velocity. Accordingly, the state-space representation is derived as Eq. (10) by assuming $w_r(t) = w_f(t-\tau(t))$.

$$\left\{ \begin{array}{ll} \dot{x}(t) & = Ax(t)+B_u u(t) \\ & +B_{wf}w_f(t)+B_{wr}w_f(t-\tau(t)) \\ y(t) & = Cx(t)+Du(t) \end{array} \right. \tag{10}$$

3 CONTROLLER SYNTHESIS

3.1 Loop-shaping

The ride quality is improved by loop-shaping based on ISO2631-1. In ISO2631-1, the frequency weighting curves of the vertical acceleration and the pitch angular acceleration are defined. They have peaks at 4−8[Hz] and 0.63−0.8[Hz]. In this study, a second-order frequency weight like the defined frequency weight in ISO2631-1 is designed to simplify the controller. State-space representation of the frequency weight is defined as Eq. (11).

$$\left\{ \begin{array}{lll} \dot{x}_w(t) & = & A_w x_w(t)+B_w y(t) \\ y_w(t) & = & C_w x_w(t)+D_w y(t) \end{array} \right. \tag{11}$$

Bode diagrams of the frequency weight for the loop-shaping and ISO2631-1 are shown in Figure 2, respectively.

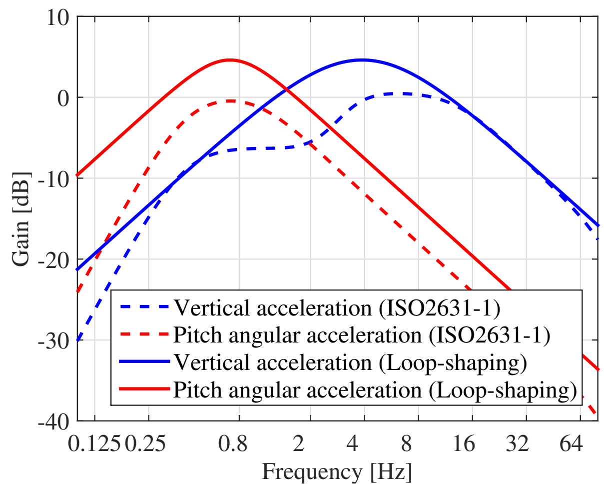

Figure 2 Frequency weight for ISO2631-1 and loop-shaping

3.2 Augmented system

A new state vector is defined as Eq. (12).

$$\hat{x}(t) = [\ x^T(t) \quad x_w^T(t)\ ]^T \tag{12}$$

Accordingly, the state-space representation of the augmented system is derived as Eq. (13).

$$\left\{ \begin{array}{ll} \dot{\hat{x}}(t) & = \hat{A}\hat{x}(t)+\hat{B}_u u(t) \\ & +\hat{B}_{wf}w_f(t)+\hat{B}_{wr}w_f(t-\tau(t)) \\ \hat{y}(t) & = \hat{C}\hat{x}(t)+\hat{D}_u u(t) \end{array} \right. \tag{13}$$

$$\hat{A} = \begin{bmatrix} A & O \\ B_w C & A_w \end{bmatrix},\ \hat{B}_{wf} = \begin{bmatrix} B_{wf} \\ O \end{bmatrix},\ \hat{B}_{wr} = \begin{bmatrix} B_{wr} \\ O \end{bmatrix},$$
$$\hat{B}_u = \begin{bmatrix} B_u \\ B_w D_u \end{bmatrix},\ \hat{C} = [\ D_w C \quad C_w\ ]\ \hat{D}_u = D_u$$

Furthermore, the evaluated output $z(t)$ is defined by the weighted vertical acceleration, the weighted pitch angular acceleration, and the control inputs as Eq. (14).

$$\begin{aligned} z(t) &= W_z [\ y_w^T(t) \quad F_f \quad F_r\ ]^T \\ &= W_z C_z \hat{x}(t) + W_z D_{uz} u(t) \\ W_z &= \begin{bmatrix} W_1 & O \\ O & W_2 \end{bmatrix},\ W_1 = \begin{bmatrix} w_1 & 0 \\ 0 & 0.4\cdot w_1 \end{bmatrix} \end{aligned} \tag{14}$$

Here W_z is the weight for the evaluate output. W_1 and W_2 are weight for the vertical and the pitch angular accelerations and weight for the control inputs. The W_1 is chosen as ratio of 1:0.4 for the vertical acceleration and the pitch angular acceleration by multiplying factor based on ISO2631-1.

3.3 LMI condition for delayed-disturbance system

In this section, H_2 state-feedback controller for the delayed-disturbance is designed. The closed-loop system is derived as Eq. (15) by the control input $u(t) = K\hat{x}(t)$.

$$\begin{cases} \dot{\hat{x}}(t) = A_{cl}\hat{x}(t) + \hat{B}_{wf}w_f(t) + \hat{B}_{wr}w_f(t-\tau(t)) \\ z(t) = C_{cl}\hat{x}(t) \end{cases} \quad (15)$$

$$(A_{cl} = \hat{A} + \hat{B}_u K,\ C_{cl} = W_z C_z + W_z D_{uz} K)$$

The term of $w_f(t-\tau(t))$ in Eq. (15) is transformed as Eq. (16).

$$w_f(t-\tau(t)) = w_f(t) - \int_{t-\tau(t)}^{t} \dot{w}_f(s)ds \quad (16)$$

The $\dot{w}_f(t)$ is assumed as output of Eq. (17) using descriptor form[5]

$$\begin{cases} E_{dif}\dot{x}_{dif}(t) &= A_{dif}x_{dif}(t) + B_{dif}w_f(t) \\ \dot{w}_f(t) &= C_{dif}x_{dif}(t) \end{cases} \quad (17)$$

$$x_{dif}(t) = \begin{bmatrix} w_f(t) \\ \dot{w}_f(t) \end{bmatrix},\ E_{dif} = \begin{bmatrix} I & O \\ O & O \end{bmatrix},$$

$$A_{dif} = \begin{bmatrix} O & I \\ I & \epsilon \end{bmatrix},\ B_{dif} = \begin{bmatrix} O \\ -I \end{bmatrix},\ C_{dif} = \begin{bmatrix} O & I \end{bmatrix}$$

Here ϵ is considered as a resistance value of the differentiator circuit to maintain the controllability of the descriptor system. The descriptor variable is defined as Eq. (18).

$$\tilde{x}(t) = \begin{bmatrix} \hat{x}^T(t) & w_f^T(t) & \dot{w}_f^T(t) \end{bmatrix}^T \quad (18)$$

Accordingly, the descriptor system is derived by the descriptor variable $\tilde{x}(t)$.

$$\begin{aligned} E\dot{\tilde{x}}(t) &= (\bar{A} + \bar{B}_u\bar{K})\tilde{x}(t) \\ &- \begin{bmatrix} \hat{B}_{wr} \\ O \\ O \end{bmatrix} \int_{t-\tau}^{t} \begin{bmatrix} O & O & I \end{bmatrix} \tilde{x}(s)ds \\ &+ \begin{bmatrix} O \\ I \\ -I \end{bmatrix} w_f(t) \end{aligned} \quad (19a)$$

$$\tilde{z}(t) = (\bar{C} + W_z D_{uz}\bar{K})\tilde{x}(t) \quad (19b)$$

$$E = \begin{bmatrix} I & O & O \\ O & I & O \\ O & O & O \end{bmatrix},\ \bar{A} = \begin{bmatrix} \hat{A} & \hat{B}_{wf} + \hat{B}_{wr} & O \\ O & -I & I \\ O & I & \epsilon \end{bmatrix},$$

$$\bar{B}_u = \begin{bmatrix} \hat{B}_u \\ O \\ O \end{bmatrix},\quad \begin{aligned} \bar{K} &= \begin{bmatrix} K & O & O \end{bmatrix} \\ \bar{C} &= \begin{bmatrix} W_z C_z & O & O \end{bmatrix}, \end{aligned}$$

$$\tilde{x}(0+) = \begin{bmatrix} O \\ I \\ -I \end{bmatrix},\ w_f(t-\tau(t)) = O\ (t \in [0,\ \tau(t))),$$

$$w_f(t) = O\ (t \in [-\tau(t),\ 0)),\ \tilde{x}(t) = O\ (t \in [-\tau(t),\ 0))$$

The $2,2$ element in the $\bar{A}$ and the $2,1$ element in the $\bar{B}_w$ are identity matrix respectively to consider the disturbance in LMI condition. Lyapunov function candidate is chosen as follows [4].

$$V(\tilde{x}(t)) = V_1(\tilde{x}(t)) + V_2(\tilde{x}(t)) \quad (20)$$

$$V_1(\tilde{x}(t)) = \tilde{x}^T(t)EP\tilde{x}(t) \quad (21)$$

$$V_2(\tilde{x}(t)) = \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \tilde{x}^T(s)\bar{B}_w^T Q^{-1}\bar{B}_w\tilde{x}(s)dsd\theta \quad (22)$$

$$\bar{B}_w = \begin{bmatrix} O & O & \hat{B}_{wr} \\ O & O & O \\ O & O & O \end{bmatrix},\ P = \begin{bmatrix} P_{11} & O & O \\ O & P_{22} & O \\ P_{31} & P_{32} & P_{33} \end{bmatrix},$$

$$P_{11} = P_{11}^T > 0,\ P_{22} = P_{22}^T > 0,\ Q = Q^T > 0$$

Using completing square and integrate by substitution, the upper bound of the derivative of the Lyapunov function is derived as Eq. (23).

$$\begin{aligned} \frac{d}{dt}V(\tilde{x}(t)) &\le \tilde{x}(t)^T(He[P(\bar{A} + \bar{B}_u\bar{K})])\tilde{x}(t) \\ &+ \tau(t)\tilde{x}^T(t)P^TQP\tilde{x}(t) + \tau(t)\tilde{x}^T(t)\bar{B}_w^TQ^{-1}\bar{B}_w\tilde{x}(t) \end{aligned} \quad (23)$$

The term $w_f(t)$ is ignored at Eq. (23) to consider the H_2 norm is calculated from initial value response of the descriptor system as Eq. (19a). The following inequality is assumed.

$$\frac{d}{dt}V(\tilde{x}(t)) + \tilde{z}^T(t)\tilde{z}(t) < 0 \quad (24)$$

Eq. (24) is transformed as the following equations by assuming $V_2(\tilde{x}(0)) = 0$ ($\tilde{x}(t) = O\ (t \in [-\tau,\ 0))$).

$$\int_0^\infty \frac{d}{dt}V(\tilde{x}(t))dt + \int_0^\infty z(t)^Tz(t)dt < 0 \quad (25)$$

$$\Leftrightarrow V(\tilde{x}(\infty)) - V(\tilde{x}(0)) + \|z(t)\|_2^2 < 0 \quad (26)$$

$$\Leftrightarrow \|z(t)\|_2^2 < V(\tilde{x}(0)) = \begin{bmatrix} O \\ I \\ -I \end{bmatrix}^T EP \begin{bmatrix} O \\ I \\ -I \end{bmatrix}$$

$$= \begin{bmatrix} O \\ I \end{bmatrix}^T \begin{bmatrix} P_{11} & O \\ O & P_{22} \end{bmatrix} \begin{bmatrix} O \\ I \end{bmatrix} < Z$$

$$\Leftrightarrow trace(Z) < \gamma^2 \quad (27)$$

For this result, the upper bound of H_2 norm is less than γ by Eq. (27). The following equation is derived by Eq. (24), pre-multiplying X and post-multiplying X^T ($X = P^{-1}$) respectively, and the change-of-variables by $\bar{Y} = \bar{K}X$ ($M = \bar{C}X + W_zD_{ut}\bar{Y}$).

$$\begin{aligned} &He[\bar{A}X + \bar{B}_u\bar{Y}] + \tau(t)Q + M^TM \\ &\quad + \tau(t)X^T\bar{B}_w^TQ^{-1}\bar{B}_wX < 0 \end{aligned} \quad (28)$$

Therefore, the following LMI conditions are derived by using Schur complement for Eq. (27) and Eq. (28).

Theorem 3.1 If there exist X, Y, Q, and Z satisfying LMI conditions Eq. (29) and Eq. (30), the upper bound of H_2 norm is less than γ and the stabilizing state feedback gain is given as $K = YX_{11}^{-1}$.

$$minimize\ \gamma^2\ such\ that$$

$$\begin{bmatrix} He[\bar{A}X + \bar{B}_u\bar{Y}] + \tau(t)Q & \tau(t)(\bar{B}_wX)^T & M^T \\ \tau(t)\bar{B}_wX & -\tau(t)Q & O \\ M & O & -I \end{bmatrix} < 0 \quad (29)$$

$$X_{11} > 0,\ \begin{bmatrix} Z & I \\ I & X_{22} \end{bmatrix} > 0,\ trace(Z) < \gamma^2 \quad (30)$$

$$\bar{B}_w = \begin{bmatrix} O & O & \hat{B}_{wr} \\ O & O & O \\ O & O & O \end{bmatrix},\ X = \begin{bmatrix} X_{11} & O & O \\ O & X_{22} & O \\ X_{31} & X_{32} & X_{33} \end{bmatrix},$$

$$\bar{Y} = \begin{bmatrix} Y & O & O \end{bmatrix},$$

4 SIMULATION & EXPERIMENT

The proposed controller is evaluated by simulations and experiments. The effectiveness of the proposed controller designed by using $\epsilon = 0.005$ is compared with the passive suspension and the conventional H_2 controller ignoring the time-delay.

4.1 H_2 norm analysis (Simulation)

The effectiveness of the proposed controller is analyzed by relationship between the time-delay and H_2 norm. The evaluate output for the H_2 norm analysis is defined as Eq. (31).

$$z_e(t) = \begin{bmatrix} \ddot{\tilde{x}}_2 \\ 0.4 \cdot \ddot{\tilde{\theta}} \end{bmatrix} \tag{31}$$

These signals $\ddot{\tilde{x}}_2$, $\ddot{\tilde{\theta}}$ are the vertical acceleration and the pitch angular acceleration weighted by the frequency weight in ISO2631-1 respectively. A relationship between the H_2 norm and the time-delay $\tau(t)$ is shown in Figure 3.

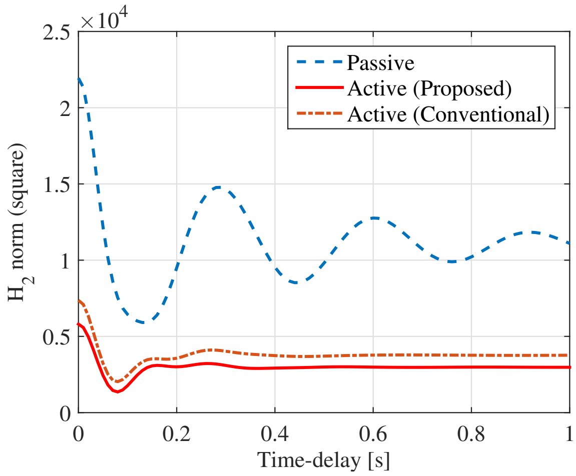

Figure 3 Relationship between H_2 norm and time-delay

According to this graph, the H_2 norm is reduced about 21.7% than the conventional controller at the time-delay is 0.27[s] (The vehicle speed is 40[km/h]). From these results, the ride quality is improved by the proposed controller.

4.2 Time-domain analysis based on ISO8608 (Experiment)

The ride quality is evaluated at the velocity by the classified road roughness based on ISO8608[6]. In this experiment, the road surface Class:A of the ISO8608. The road surface deflection used at experiment is shown in Figure 4. The RMS value weighted by ISO2631-1 of the time-domain response of the vertical acceleration and the pitch angular acceleration at 40[km/h] are calculated by Table. 2.

Table 2 RMS value of the weighted signals at 40[km/h]

	Vertical acceleration	Pitch angular acceleration	Total
Passive	0.808	0.127	0.809
Proposed	0.315	0.102	0.317
Conventional	0.368	0.100	0.370

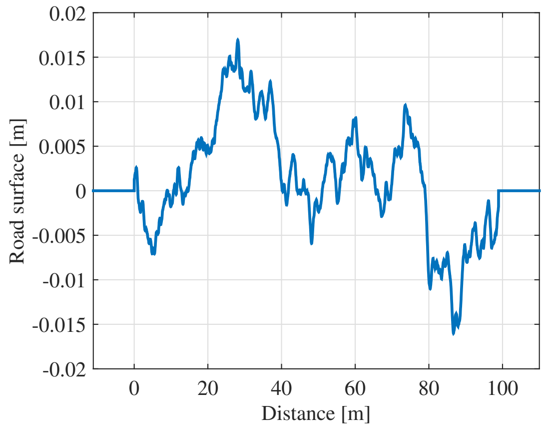

Figure 4 Road surface for ISO8608 Class:A

Here total is sum of square the RMS value of the weighted vertical acceleration and the weighted pitch angular acceleration under the ratio of 1:0.4. As you can see from this table, the total RMS value is reduced about 14.3% than the conventional controller. Therefore, the ride quality is improved by the proposed controller.

5 CONCLUSION

In this study, the H_2 controller for the active suspension on half-car model is designed by considered as the delayed-disturbance system. The time-delay in between the front disturbance and the rear disturbance is carried without approximation by using the descriptor form and incomplete differentiator circuit to maintain the controllability. The loop-shaping based on reduced-order frequency weight of the weighting curve in ISO2631-1 is used to improve ride quality. The effectiveness of the proposed controller is evaluated by simulations and experiments. As a result, the ride quality is improved by the proposed controller more than the conventional controller ignoring the time-delay.

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