Gain-Scheduled Control of ABS Based on Road Condition Estimation Using Unscented Kalman Filter

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Abstract

In this paper, the real-time road characteristics estimator is proposed for the Anti-lock Braking System (ABS). The key factor of the ABS is the coefficient of friction between car wheel and road surface. However, the parameter cannot be measured directly because there is strong nonlinearity in friction. In order to solve this problem, the LuGre tire friction model is used in this study. The LuGre tire friction model is well known that friction characteristics can be obtained more accurately. For this model, the unknown road characteristics parameter and coefficient of friction are estimated by nonlinear observer, unscented Kalman filter. In the control design, a gain-scheduled controller based on Lyapunov function is proposed. The controller gain can be obtained by solving Linear Matrix Inequality (LMI). Simulations and experiments are performed using the MATLAB/Simulink.

1 Introduction

Automobiles consist of three elements: running, turning, stopping, and the technique of stopping control is particularly important. Currently, most cars have Anti-lock Braking System (ABS). The ABS is a safety device that applies braking force at an appropriate timing so that wheels do not lock, aimed at stopping the vehicle as short as possible. The important thing in the control of the ABS is the various conditions of road surface such as frozen, wet, dry asphalt, etc. However, it is difficult to measure the friction coefficient directly. Recently, some technologies to estimate the state of the road surface by measuring the difference in the rotation of the each tire with attached sensors. However, such sensors are expensive and difficult to be commonly used. So, for improvement of control performance, it's important to estimate the unknown parameters of the tire and the road surface in more practical method. If it becomes possible, it is expected to have various advantages. For example, by sharing road surface information through the Internet, identifying road surface defects and preliminary measures to reduce traffic accidents can be expected. ABS is well known it has a strong nonlinear characteristics, and many of proposed methods have been developed. For example, PID-type control, sliding mode control, fuzzy control and gain-scheduled control [1], etc.

In this study, the LuGre tire friction model is used, and the real-time road characteristics estimator using nonlinear observer is proposed. In the LuGre tire friction model, the unknown friction parameter α have to be estimated, and the Unscented Kalman Filter (UKF) is used in this study. Then, the road friction coefficient $\mu(\alpha)$ can be calculated by obtained α parameter and equations of the LuGre tire friction model. The simulation shows whether this unknown parameter can be estimated properly. In control design, a gain-scheduled controller based on Lyapunov function is designed to guarantee the robust stability for the ABS. The scheduled parameters are the vehicle speed ω_2 and the road friction coefficient $\mu(\alpha)$. The polytopic representation is used to guarantee the robust stability for the uncertain parameters. For polytopic representation, the descriptor form and linear fractional transformation (LFT) are applied in this study. The robust controller can be obtained by solving Linear Matrix Inequality (LMI). Finally, the effectiveness of the proposed method is illustrated by simulation and experiment. The parameters of the LuGre tire friction model are from [2].

2 Controlled Object

Fig.1 is the schematic drawing of ABS experimental device. This is a quarter car model. The upper wheel represents the actual car's wheel, and the lower wheel represents the road surface. There are two identical encoders measuring the rotational angles of two wheels and the deviation angle of the balance lever of the car wheel. The lower wheel has a flat DC motor, and rises to the specified constant rotation speed. Due to the friction between the wheels, the upper wheels also rises to the same speed. Later, the rotational speed is controlled by the disc brake system attached to the rear part of the upper wheel. The parameters are given as Table 1.

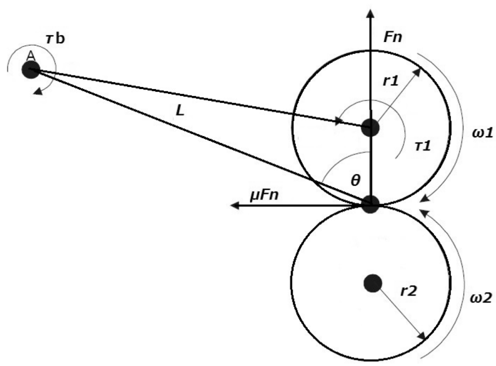

Figure 1 Schematic Drawing of ABS Experimental Device.

Table 1 Parameters

Parameter	Symbol	Units
Radius of the upper wheel	r_1	[m]
Radius of the lower wheel	r_2	[m]
Angular velocity of the upper wheel	ω_1	[rad/s]
Angular velocity of the lower wheel	ω_2	[rad/s]
Moment of inertia of the upper wheel	J_1	[kgm^2]
Moment of inertia of the lower wheel	J_2	[kgm^2]
Braking torque	τ_1	[Nm]
Torques acting on the balance lever (A)	τ_b	[Nm]
Normal force	F_n	[N]
Road friction coefficient	μ	[-]
Slip ratio	λ	[-]

3 Modeling

3.1 LuGre Tire Friction Model

In this paper, the LuGre tire friction model is used in order to more accurately estimate the road friction characteristics. The equations of this model can be written as follows:

$$\dot{z} = v_r - \alpha \frac{\sigma_0 |v_r|}{g(v_r)} z, \tag{1}$$

$$F = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) F_n, \tag{2}$$

$$g(v_r) = \mu_c + (\mu_s - \mu_c) e^{-|\frac{v_r}{v_s}|^{\beta}}. \tag{3}$$

The parameter z is the friction internal state, $v_r = r_1\omega_1 - v = r_1\omega_1 - r_2\omega_2$ is the relative velocity. The unknown parameter α is introduced to capture the changes in the road characteristics [3]. σ_0 is the rubber longitudinal lumped stiffness, σ_1 is the rubber longitudinal lumped damping, σ_2 is the viscous relative damping. F_n is the normal force which is function of the weight of the vehicle or the component of the weight acting in a vertical plane relative to the road surface. F is the longitudinal force. The friction coefficient is defined as $\mu = F/F_n$. μ_c is the normalized Coulomb friction, The μ_s is the normalized static friction, v_s is the Stribeck relative velocity. The parameter β is a curve parameter that further tunes the Stribeck effect [3].

The reason for picking up this model is to estimate unknown parameter $\mu(\alpha)$, and set the scheduling parameter of gain-scheduled controller. The relationship between the road characteristics parameter α and road conditions is set as shown in Fig.2 and Table 2 from Eq.(4) [3],

$$\begin{aligned}\mu(\lambda, v_r) = &\frac{1}{\alpha}\mathrm{sgn}(v_r) g(v_r) \\ &(1 + \frac{g(v_r)|1+\lambda|}{\sigma_0 \alpha L |\lambda|}(e^{-\frac{\sigma_0 L|\lambda|}{g(v_r)|1+\lambda|}} - 1)) + \sigma_2 v \lambda,\end{aligned} \tag{4}$$

where L is the length of the contact patch. The setting of the wet asphalt is decided assuming the experimental device.

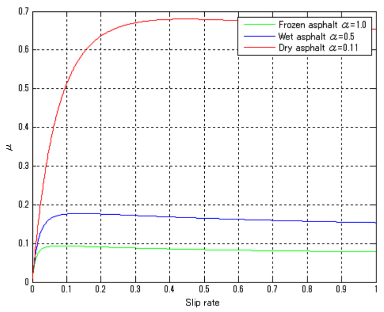

Figure 2 $\mu - \lambda$ relation under $v = 50$[km/h].

Table 2 The relationship between α and road conditions

Road condition	α	$\mu(\lambda = 0.2)$
Dry asphalt	0.11	0.63
Wet asphalt	0.5	0.17
Frozen asphalt	1.0	0.09

3.2 Quarter Car Model

The motion equations of the upper wheel and lower wheel can be written as Eq.(5) and Eq.(6).

$$J_1 \dot{\omega}_1 = F_n r_1 \mu - \tau_1, \tag{5}$$

$$J_2 \dot{\omega}_2 = -F_n r_2 \mu. \tag{6}$$

The slip ratio is defined as follows.

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2}. \tag{7}$$

From the sum of torques corresponding to the point A, the normal force F_n can be written as follows.

$$F_n L(\sin\theta - \mu\cos\theta) = \tau_b + \tau_1. \tag{8}$$

The following equation can be obtained from Eq.(7).

$$\dot{\lambda} = -\frac{r_1}{r_2\omega_2}\dot{\omega}_1 + \frac{r_1\omega_1}{r_2\omega_2^2}\dot{\omega}_2. \tag{9}$$

Putting formulas Eq.(5)-Eq.(8) to formula (9) we obtain

$$\dot{\lambda} = \frac{1}{\omega_2} f(\lambda) + \frac{1}{\omega_2} g(\lambda)\tau_1, \ \omega_2 \neq 0. \tag{10}$$

Here, $f(\lambda)$ and $g(\lambda)$ are given as follows:

$$f(\lambda) = -\frac{r_1^2 S(\mu)\tau_b}{J_1 r_2} - \frac{r_2(1-\lambda)S(\mu)\tau_b}{J_2}, \tag{11}$$

$$g(\lambda) = -\frac{r_1^2 S(\mu)}{J_1 r_2} - \frac{r_2(1-\lambda)S(\mu)}{J_2} + \frac{r_1}{J_1 r_2}. \tag{12}$$

In both equations we have the common factor

$$S(\mu) = \frac{\mu}{L(\sin\theta - \mu\cos\theta)}. \tag{13}$$

Eq.(9) is nonlinear, so it have to be linearized around an equilibrium point (λ^*, τ_1^*). Here, λ^* is the reference slip ratio, and τ_1^* is the equilibrium braking torque to keep the slip ratio. Eq.(14) can be obtained from Eq.(10).

$$0 = \frac{1}{\omega_2} f(\lambda^*) + \frac{1}{\omega_2} g(\lambda^*)\tau_1^*. \tag{14}$$

The τ_1^* is derived from Eq.(14).

$$\begin{aligned}\tau_1^* &= -\frac{f(\lambda^*)}{g(\lambda^*)} \\ &= \frac{J_2 r_1^2 S(\mu)\tau_b - J_1 r_2^2 (1-\lambda^*) S(\mu)\tau_b}{J_2 r_1 - J_2 r_1^2 S(\mu) - J_1 r_2^2 (1-\lambda^*) S(\mu)}.\end{aligned} \tag{15}$$

Using Taylor expansion around the equilibrium point (λ^*, τ_1^*), the nonlinear model Eq.(10) can be linearized as follows [4].

$$\dot{\lambda} \simeq \dot{\lambda}(\lambda^*, \tau_1^*) + \frac{\partial \dot{\lambda}}{\partial \lambda}|_{\lambda=\lambda^*}(\lambda - \lambda^*) + \frac{\partial \dot{\lambda}}{\partial \tau_1}|_{\tau=\tau_1^*}(\tau_1 - \tau_1^*). \tag{16}$$

The linearized equation is derived from Eq.(15)-(16).

$$\begin{aligned}\dot{\lambda} = &\left(\frac{C_1 S(\mu) + C_2}{\omega_2 (C_3 S(\mu) + C_4)}\right)(\lambda - \lambda^*) \\ &+ \frac{1}{\omega_2}(C_5 S(\mu) + C_6)(\tau_1 - \tau_1^*).\end{aligned} \tag{17}$$

Here, $C_1, C_2, \ldots, C_8$ are constants.

3.3 State Space Representation

In this study, the polytopic representation is used to guarantee the robust stability for the uncertain parameters, vehicle speed ω_2 and road friction coefficient μ. First, consider the state space representation of this system. In order to track the output of the system to the target value without error, an integrator is applied to the state variable. The state variable $\mathbf{x}(t)$ and control input $u(t)$ are defined as follows:

$$\begin{aligned}\mathbf{x}(t) &= [x_1(t)\ x_2(t)]^T \\ &= [\int (\lambda - \lambda^*)dt \quad \lambda - \lambda^*]^T, \end{aligned} \tag{18}$$

$$u(t) = \tau_1 - \tau_1^*. \tag{19}$$

Then, the state space representation can be obtained as follows.

$$E\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bu(t), \tag{20}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & \omega_2(C_3 S(\mu) + C_4) \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 \\ 0 & C_1 S(\mu) + C_2 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & C_3 C_5 S(\mu)^2 + (C_3 C_6 + C_4 C_5) S(\mu) + C_4 C_6 \end{bmatrix}^T.$$

In the gain-scheduled control, there is a structure depending on scheduling parameter. Therefore, if this structure is expressed in an easy to handle form, the design of this controlled system becomes easy to use polytopic representation. In this study, a LPV system applied to descriptor form and linear fractional transformation (LFT) is designed.

4 Nonlinear Observer Design

In this study, a nonlinear observer based on unscented Kalman filter (UKF) theory is designed. This nonlinear observer provides the estimated value of unobserved parameters. In this study, the purpose is to estimate road friction coefficient $\mu(\alpha)$. The estimated value of road friction coefficient is applied to the Gain-scheduled controller as a scheduling parameter. To apply the UKF algorithm, discretization of the continuous time model is needed. For the discretization, the forward Euler method is used in this study.

Define $\mathbf{x}$, y, and u as follows:

$$\mathbf{x} = \begin{bmatrix} z & \omega_1 & \omega_2 & \alpha \end{bmatrix}^T, \tag{21}$$
$$y = \omega_1, \quad u = \tau_1. \tag{22}$$

where α is the road characteristics parameter. The first derivatives of $\mathbf{x}$ can be calculated from Eq.(1)-(3), Eq.(5)-(6).

$$\dot{z}(t) = v_r - \alpha \frac{\sigma_0 |v_r|}{\mu_c + (\mu_s - \mu_c) e^{-|\frac{v_r}{v_s}|^\beta}} z, \tag{23}$$

$$\dot{\omega}_1(t) = \frac{1}{J_1}(F_n r_1(\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) - \tau_b), \tag{24}$$

$$\dot{\omega}_2(t) = \frac{1}{J_2}(-F_n r_2(\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r)). \tag{25}$$

The parameter α is assumed to be constant.

$$\dot{\alpha} = 0. \tag{26}$$

Therefore, the discrete-time nonlinear system of ABS based on LuGre tire friction model is given by

$$\begin{bmatrix} z(k+1) \\ \omega_1(k+1) \\ \omega_2(k+1) \\ \alpha(k+1) \end{bmatrix} = \begin{bmatrix} z(k) + T\dot{z}(k) \\ \omega_1(k) + T\dot{\omega}_1(k) \\ \omega_2(k) + T\dot{\omega}_2(k) \\ \alpha(k) \end{bmatrix} + \mathbf{b}v(k), \tag{27}$$

where T is a sampling interval. Fig.3 is the simulation test of estimating road characteristics parameter α. The blue line represents the estimated value. The initial state of the estimated value is set to 0.5.

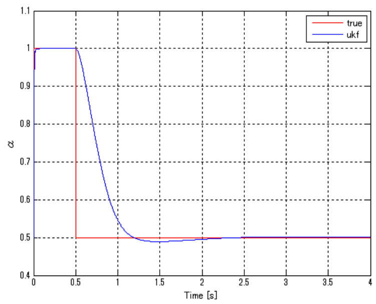

Figure 3 Road characteristics parameter α and $\hat{\alpha}$.

5 Controller synthesis

In this study, a gain-scheduled controller based on Lyapunov function is proposed. The scheduled parameters of vehicle speed ω_2 and road friction coefficient μ are guaranteed by polytopic representation. Here, the parameter μ is unobserved parameter, so estimated value by nonlinear observer is used. As for the reference slip ratio, the constant value 0.2 is set. The control method adopted linear-quadratic optimal control.

5.1 Polytopic Representation

The range of the vehicle speed is supposed from 10[km/h] to 50[km/h].

$$\mu \in [0.05,\ 0.7], \tag{28}$$
$$\omega_2 \in [20.0584,\ 140.2918]. \tag{29}$$

Let scheduling parameter $\theta_1,\ \theta_2,\ \ldots,\ \theta_6$ be $\theta_1 = \omega_2$, $\theta_2 = S(\mu)$, $\theta_3 = \omega_2 S(\mu)$ and their differential. The parameter box Θ is defined as follows.

$$\Theta = \{[\theta_1 \quad \theta_2 \quad \ldots \quad \theta_6] : \theta_i \in \{\underline{\theta}_i, \overline{\theta}_i\}\},\ (i = 1, 2, ..., 6)$$
$$\Theta_1 = (\underline{\theta}_1, \underline{\theta}_2, \underline{\theta}_3, \underline{\theta}_4, \underline{\theta}_5, \underline{\theta}_6), \cdots, \Theta_{64} = (\overline{\theta}_1, \overline{\theta}_2, \overline{\theta}_3, \overline{\theta}_4, \overline{\theta}_5, \overline{\theta}_6).$$

5.2 LMI Conditions

If there exists $\tilde{X}_d(\Theta)$, $\tilde{Y}_d(\Theta)$ satisfying the following LMI, the control system is stable.

minimize : γ, subject to:

$$X(\Theta_p) > 0, \tag{30}$$

$$\begin{bmatrix} \tilde{M}(\Theta_p) - \tilde{E}_d \dot{\tilde{X}}_d & \tilde{X}_d^T (Q^{\frac{1}{2}})^T & \tilde{Y}_d^T (R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}} \tilde{X}_d & -I_{3\times3} & O_{3\times3} \\ R^{\frac{1}{2}} \tilde{Y}_d & O_{3\times3} & -I_{3\times3} \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} W_{2\times2} & I_{2\times2} \\ I_{2\times2} & X(\Theta_p) \end{bmatrix} > 0, \tag{32}$$

$$\mathrm{trace}(W) < \gamma, \tag{33}$$

$$(p = 1, ..., 64)$$

where $\tilde{M}(\Theta_p) = He[\tilde{A}_d(\Theta_p)\tilde{X}_d(\Theta_p) + \tilde{B}_d \tilde{Y}_d(\Theta_p)]$.

6 Simulation

Fig.4-5 depicts the simulation results for an anti-lock braking on wet asphalt.

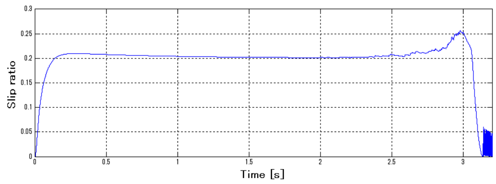

Figure 4 Slip ratio ($\alpha = 0.5$).

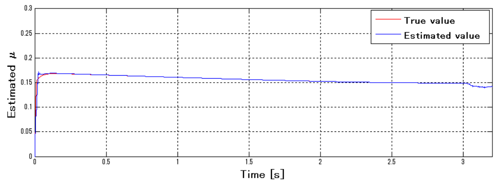

Figure 5 Road friction coefficient ($\alpha = 0.5$).

Fig.6-7 depicts the robustness of the controller with negative μ-jump from high-μ to low-μ. The μ-jump ($\alpha = 0.5 \rightarrow \alpha = 1.0$) is happened at t=0.5[s].

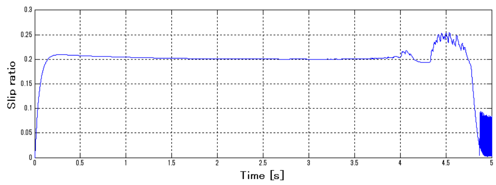

Figure 6 Slip ratio (High-$\mu\rightarrow$ Low-μ).

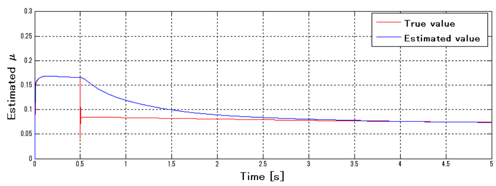

Figure 7 Road friction coefficient (High-$\mu\rightarrow$ Low-μ).

In both situations, the slip ratio λ reaches the target slip ratio 0.2, and the road frciton coefficient $\hat{\mu}(\alpha)$ (blue line) can be estimated properly.

7 Experiment

In this section, the effectiveness of proposed method is shown by experiment data. Fig.8 shows the slip ratio and Fig.9 shows the wheel velocity and wheel speed.

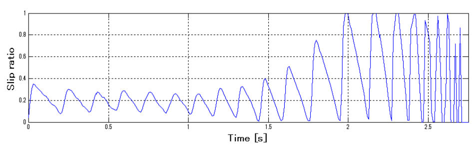

Figure 8 Slip ratio.

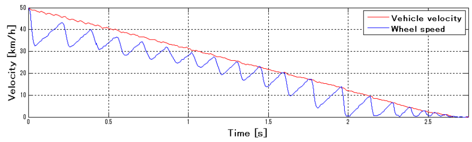

Figure 9 Vehicle velocity(red) and Wheel speed(blue).

The slip ratio is followed target value 0.2 in 50-20[km/h]. It is conducted that ABS works well at high speed and the designed gain-scheduled controller is practical. Next, Fig.10 shows the estimated road friction coefficient.

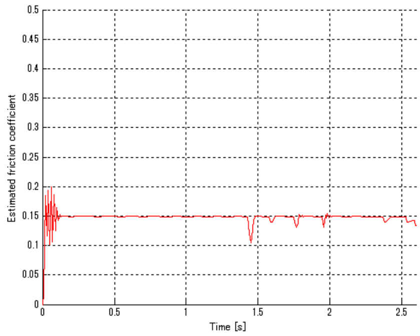

Figure 10 Estimated road friction coefficient data.

According to the previous research and consistency with simulation, the value of the friction coefficient of the experimental device is around 0.17. So, it is said that the proposed road characteristics estimator is useful.

8 Conclusion

In this study, the LuGre tire friction model is used, and the real-time road characteristics estimator is proposed. In the LuGre tire friction model, the unknown friction parameter α have to be estimated, and the unscented Kalman filter is used. It is shown in simulations and experiments that the unknown parameter $\mu(\alpha)$ can be estimated, and the slip ratio is followed target value 0.2. Therefore, the effectiveness of the proposed method is verified.

References

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