Tracking Control of Control Moment Gyroscope Using SDRE Method

M2016SC011 Reina NAKAGAMI
Supervisor: Isao TAKAMI

Abstract

This research describes tracking control of Control Moment Gyroscope(CMG) based on State Dependent Riccati Equation(SDRE).Motion equations of CMG have nonlinearity such as trigonometric functions.SDRE method is simple ,and it is effective for nonlinear systems such as CMG. However, since trade-off relation between control period and calculation time of Ricatti equation, it is hard to apply it to real systems. In addition, it is difficult to ensure the global stability of SDRE method. To solve these problem, a function approximation and SOS matrix are adapted. Firstly, a function approximation is applied to controller gains of SDRE method. SDRE is solved at each point and obtained controller gains in closed boundary sets of varying parameters. To derive the approximated function, the least squared method is applied to obtained controller gains. Secondly, stability region is discussed. Two-dimensional stability is ensured by Linear Matrix Inequalities(LMI) condition. However, as mentioned above, motion equations of CMG have nonlinearity, so LMI can not be solved directly. Therefore SOS relaxation is applied to LMI conditions, and the stability region of approximated SDRE controller is indicated. Finally, the effectiveness of SDRE controller is illustrated by simulations and experiments.

1 Introduction

Control Moment Gyroscope(CMG) is used as actuators for large space craft, and it can generate larger torque than conventional actuators such as reaction wheels.Nonlinear motion equations of CMG contain many trigonometric functions and squared terms,and the controllability of the system is not ensured by an approximated system at equilibrium point. Therefore it is hard to apply the linear control theory. In previous researches, backstepping approach is applied to CMG[1], approximated model is adapted to control of CMG[2] and nonlinear optimal controller is applied CMG[3]. In generally, CMGs are used in cooperative control, but this study treat CMG as a simple system.

Since the 1990s, State Dependent Riccati Equation (SDRE) method attracted to attentions[5], because it is effective to control nonlinear systems like inverted pendulum[6] and AUV[7]. SDRE method is difficult to apply to real systems,because solving the riccati equation need high-performance calculator and much time. Recently, since various methods are announced to shorten a calculation time of solving riccati equation, application to systems that need fast-response is a minority. Therefore, a function approximation is applied to control gain from SDRE method, but control performance are harmfully influenced by largely error between the function interpolation for control gains and current state. For the reasons stated above, it is hard to apply SDRE method to real systems. Moreover stable guarantee for SDRE method is open problem. Global stability for SDRE method are guaranteed a particular form of closed loop in recently research[4].

Two input and three state system is controlled in this paper, so one of the rotational body in CMG is rocked. Firstly, the motion equations of CMG contain nonlinear terms such as trigonometric function and squared term are convert to state-dependent linear representation. Secondly, SDRE method is applied to CMG, and derived control gains are approximated by function approximation using the least squared method. Thirdly, a stability of the system are ensured by SOS matrix based on Lyapunov function. Lastly, the effectiveness of SDRE method for CMG is illustrated by simulation and experience. Friction disturbances are considered in simulation.

2 Modeling

The schematic model of Model 750 CMG is shown in Fig1. Rotor1 and Gimbal2 are variable speed CMG(VSCMG), and they are driven by DC motors. T_1 is torque of Motor1 which spins Rotor1,and T_2 is torque of Motor2 which tilts Gimbal2. Input torques T_1 and T_2 are restricted as follows:

$$|T_1| < 0.6[\mathrm{Nm}], \tag{1}$$
$$|T_2| < 2.4[\mathrm{Nm}]. \tag{2}$$

Gimbal4 has no driven sources, so it is driven by the gyro torque generated by VSCMG.The control purpose is to follow Gimbal4 the reference without error. Variables q_1, q_2 and q_4 are angles of Rotor1, Gimbal2 and Gimbal4, respectively. The motion range of Gimbal2 is defined $0 < q_2 < \frac{\pi}{2}$ in this study. Variables ω_1, ω_2 and ω_4 are angular velocities of Rotor1, Gimbal2 and Gimbal4 respectively. Gimbal3 is fixed, so the angle and the angular velocity of Gimbal3 are zero. In this study, the centroids of all gimbals are center of Rotor1, and the effectiveness of gravity is ignored. Then, Nonlinear motion equation is indicated as follows:

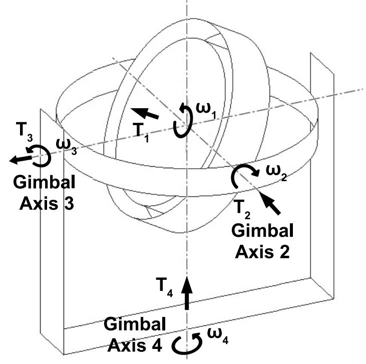

Figure 1 figure of CMG

$$M\begin{bmatrix}\dot{\omega}_1\\ \dot{\omega}_2\\ \dot{\omega}_4\end{bmatrix}+N\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_4\end{bmatrix}=G\begin{bmatrix}T_1 & T_2\end{bmatrix} \tag{3}$$

$$M = \begin{bmatrix} 1 & 0 & \sin(q_2) \\ 0 & 1 & 0 \\ \frac{J_D \sin(q_2)}{a(q_2)} & 0 & 1 \end{bmatrix} \tag{4}$$

$$N = \begin{bmatrix} 0 & \omega_4 \cos(q_2) & 0 \\ \frac{-JD\omega_4 \cos(q_2)}{I_C+I_D} & 0 & \frac{I_1\omega_4 \sin(2q_2)/2}{I_C+I_D} \\ \frac{J_D\omega_2 \cos(q_2)}{a(q_2)} & \frac{I_1\omega_4 \sin(2q_2)}{a(q_2)} & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{1}{J_D} & 0 \\ 0 & \frac{a(q_2)-J_D \sin^2(q_2)}{a(q_2)} \\ \frac{\sin(q_2)}{a(q_2)} & 0 \end{bmatrix}$$

$$a(q_2) = J_2 + J_1 \sin^2(q_2)$$

I_D, J_D : Moment of inertia of Rotor1[kg $\cdot$ m^2]
I_C, J_C, K_C : Moment of inertia of Gimbal2[kg $\cdot$ m^2]
K_B, K_A :Moment of inertia of Gimbal3 and Gimbal4[kg$\cdot$ m^2]
$J_1 = J_C + J_D - K_C - I_D$
$J_2 = K_A + K_B + K_C + I_D$.

2.1 Optimal Servo System Based on SDLR

State Dependent Linear Representation(SDLR)[4] is adapted to apply SDRE method. The motion equations (3) are converted to SDLR[4]. The friction disturbances are in rotational axes of CMG[1]. Therefore to follow the reference without a state error, an optimal servo system is adopted. An optimal servo system is indicated as follows:

$$\begin{cases} \dot{x}_e(t) = A_e(x)x_e(t) + B_e(x)u(t) \\ y_e(t) = C_e x_e(t) \end{cases} \tag{5}$$

$$\begin{aligned} x_e &= [\int (q_2^{\mathrm{ref}} - q_2) \quad \int (q_4^{\mathrm{ref}} - q_4) \\ &\quad q_2^{\mathrm{ref}} - q_2 \quad q_4^{\mathrm{ref}} - q_4 \quad q_2 \quad q_4 \; \omega_1 \quad \omega_2 \quad \omega_4 \;]^{\mathrm{T}}, \\ u &= [T_1 \quad T_2], \\ A_e &= \begin{bmatrix} O_{2\times2} & I_{2\times2} & O_{2\times3} \\ O_{2\times2} & O_{2\times3} & -I_{2\times2} \\ O_{3\times2} & O_{3\times2} & -M^{-1}N \end{bmatrix}, \\ B_e &= \begin{bmatrix} O_{4\times2} \\ M^{-1}G \end{bmatrix}, \\ C_e &= [\; O_{2\times2} \quad I_{2\times2} \quad O_{2\times3} \;]. \end{aligned} \tag{6}$$

Since $A_e(x)$ and $B_e(x)$ contain state variables q_2, ω_2 and ω_4, state dependent matrices $A_e(x)$ and $B_e(x)$ depend on state variables. State dependent matrices are nonunique, and the control performance depends on the choice of them. State dependent matrices can be chosen freely, but the system that contains them must be controllable.

3 Control Synthesis

3.1 Tracking Controller Design Based on SDRE Method

Control gains of SDRE method are obtained by calculating Riccati equation freezing state variables every control period. Shortcoming of SDRE method is to need enough calculating time for solving Riccati equation. On the other hand, a system is controlled by the optimal input at all times, because CPU of system can get the latest information of plant states in real time. Therefore nonlinearities of a system can be simply treated. SDRE method is designed by SDLR and weight matrices. A cost function is defined as follows:

$$J = \int_0^{\infty} (x_e(t)^{\mathrm{T}} Q_e x_e(t) + u(t)^{\mathrm{T}} R_e u(t))dt \tag{7}$$

Positive definite matrix $P(x)$ is obtained by solving Riccati equation(8).

$$\begin{aligned} A(x)^{\mathrm{T}} P(x) + P(x)A(x) + Q_e \\ - P(x)B(x)R_e^{-1}B(x)^{\mathrm{T}}P(x) = O \end{aligned} \tag{8}$$

An optimal input $u(x)$ that minimize the cost function(7) is obtained from Riccati solution $P(x)$. The optimal input $u(x)$ is indicated as follows:

$$u(x) = -R(x)^{-1}B(x)^{\mathrm{T}}P(x)x(t). \tag{9}$$

where weight matrices Q_e, R_e are defined as constant. They are decided by try and error.

$$Q_e = \mathrm{diag}[3 \; 30 \; 1 \; 25 \; 0.01 \; 0.1 \; 0.1] \quad R_e = \mathrm{diag}[10.2 \; 0.3] \tag{10}$$

State feedback controller are defined as follows:

$$u(t) = K_p x(t) + K_i \int (y_e^{\mathrm{ref}} - y_e). \tag{11}$$

where $K_p$and K_i are controller gains given by SDRE method. The positive definite matrix $P(x)$ are obtained by applying the method of Arimoto$\cdot$Potter to Hamilton matrix as follows:

$$H(t) = \begin{bmatrix} A_e(x) & -B_e(x)R_e^{-1}B_e(x)^{\mathrm{T}} \\ -Q_e & -A_e^{\mathrm{T}}(x) \end{bmatrix}. \tag{12}$$

Algorithm of SDRE method to obtain the controller gains is indicated as follows.

1. Current states are measured in a control period.
2. State dependent matrices $A_e(x), B_e(x)$ are derived from current states.
3. Riccati solution $P(x)$ is given by calculation of SDRE.
4. The optimal input $u(t)$ is obtained.
5. The input torque $u(t)$ is continued to outputting to a system during a control period.
6. Back to 1.

3.2 Function Approximation

As mentioned in section1, since a calculation cost of SDRE method is large, it is hard to apply to real systems. In this study,a function approximation is adopted for overcoming the problem. Varying parameters of SDLR model are q_2, ω_2 and ω_4, but ω_2 is small enough to ignore, so it is ignored in the function approximation. The controller gains are approximated two-dimensional function on basis q_2, ω_4. Firstly, the controller gains are obtained by solving SDRE at each point that enough small interval in closed boundary sets. Next, the least squares method is applied to the controller gains obtained by SDRE method,and controller gains approximated two-dimensional function.

4 Stability Analysis

A stability guarantee is an open problem of SDRE method. The stability of CMG controlled by SDRE method is discussed. The stability is ensured by the approximated controller gains and LMI condition relaxed by SOS. Firstly,a function approximation is adopted to control gains obtained by SDRE method. Secondly, LMI condition is defined, and relaxed to SOS condition. Lastly,two-dimensional stability is confirmed in closed boundary set.

4.1 SOS Formulation

A robust stability is guaranteed by Linear Matrix Inequality (LMI). LMI condition is indicated as follows:

$$P(x) > 0, \tag{13}$$

$$P(x)A_{cl}(x)^{\mathrm{T}} + A_{cl}(x)P(x) < 0, \tag{14}$$

$$A_{cl}(x) = A(x) + B(x)K(x). \tag{15}$$

If positive definite matrix $P(x)$ exist ,and it is satisfied above LMI condition, the closed loop system $A_{cl}(x)$ is a robust stable. However, to obtain the solution of the LMI condition, infinite number of LMI conditions must be solved. In this study, to avoid the problem, above conditions are relaxed based on the result [8]. The LMI conditions are formulated to SOS conditions as follows:

$$P(x) - g_1(x)S_{a1}(x) - g_2(x)S_{a2}(x) \\ -g_3(x)S_{a3} - \epsilon I > 0 \tag{16}$$

$$-P(x)A_{cl}(x)^{\mathrm{T}} - A_{cl}(x)P(x) \\ -g_1(x)S_{b1}(x) - g_2(x)S_{b2}(x) - g_3(x)S_{b3}(x) - \epsilon I > 0 \tag{17}$$

where $S_{ai}(x), S_{bi}(x)(i = 1,2,3)$ are SOS matrices. $g_i(x)(i = 1,2,3)$ are closed boundary sets of varying parameters q_2, ω_2 and ω_4 respectively. SOS condition is ensured in the closed boundary sets. The closed boundary sets are defined as follows:

$$g_1(x) = (q_2 + \overline{q_2})(\underline{q_2} - q_2) \tag{18}$$

$$g_2(x) = (\omega_2 + \overline{\omega_2})(\underline{\omega_2} - \omega_2) \tag{19}$$

$$g_3(x) = (\omega_4 + \overline{\omega_4})(\underline{\omega_4} - \omega_4). \tag{20}$$

SOS relaxation can be applied to LMI condition when the mathematical model is represented only polynomials. Since motion equations of CMG contain trigonometric functions, trigonometric functions must be represented to polynomial.

4.2 Transforming to Polynomial

To apply SOS relaxation for LMI condition,trigonometric functions are express polynomial. Firstly, sine is defined as follows:

$$\sin(q_2) = \alpha(t) \tag{21}$$

where $\alpha(t)$ is a varying parameter in $-1 < \alpha(t) < 1$. Then,cosine is expressed as follows:

$$\cos(q_2) = \sqrt{1 - \alpha(t)^2}. \tag{22}$$

Since square root is not polynomial, the above equation is applied Taylor series expansion to apply SOS relaxation. Then, cosine can be expressed as follows:

$$\cos(q_2) \cong 1 - \frac{\alpha(t)^2}{2}. \tag{23}$$

Sine and cosine applied polynomial approximation are used in SOS condition.

4.3 Stability Region

In this section, derived stability regions is indicated. sine and cosine are represented $\sin q_2 = \alpha, \cos q_2 = \sqrt{1-\alpha^2} \cong 1 - \alpha(t)^2/2$ in motion equations of CMG, and semi definite problems (16)(17) are solved by SOS matrix. The result is indicated as follows. The point indicates that SOS condition (16)(17) is satisfied. As can be seen Fig. 2, when CMG is controlled in the stability region, the robust stability is ensured.

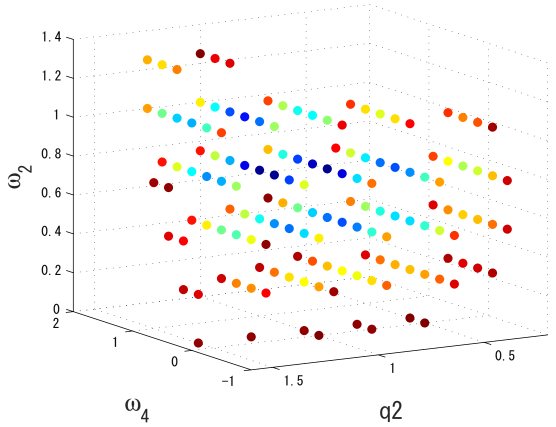

Figure 2 stability region of CMG ($\sin q_2 = \alpha(t), \cos q_2 = 1 - \alpha(t)^2/2$)

5 Simulation and Experiment

5.1 Simulation Setting

In this section,the effectiveness of SDRE method for CMG is illustrated by simulations. SDRE controller are compared with linear quadratic servo controller by simulations. Tracking orbits are defined as follows:

$$q_2^{\mathrm{ref}}\mathrm{rad} = \begin{cases} \frac{\pi}{18} \ (t < 4) \\ -\frac{1}{2}(\sin(\frac{\pi}{4}t - \frac{\pi}{2}) - 1) \quad (4 \le t \le 8) + \frac{\pi}{18} \\ 1 + \frac{\pi}{18} \ (8 < t) \end{cases} \tag{24}$$

$$q_4^{\mathrm{ref}}\mathrm{rad} = \begin{cases} 0 \quad (t < 4) \\ -\frac{1}{2}(\sin(\frac{\pi}{4}t - \frac{\pi}{2}) - 1) \quad (4 \le t \le 8) \\ 1 \quad (8 < t) \end{cases} \tag{25}$$

Initial values are defined as $q_0 = [q_1 \ q_2 \ q_4] = [0 \ \frac{\pi}{18} \ 0]$. The effectiveness of friction can not be ignored for CMG[1]. Therefore friction is considered in simulations. F_s, F_c and F_v are coefficients of Static friction, Coulomb friction and Viscous friction respectively. Friction disturbance varies by the angular velocity as follows:

$$F_i = \begin{cases} F_{is} \quad (\omega_i = 0) \\ F_{ic}\mathrm{sgn}\omega_i + F_{iv}\omega_i(\omega_i \ne 0)(i = 1,2,4). \end{cases} \tag{26}$$

Table 1 Parameter of friction coefficient

	F_{is}[Nm]	F_{ic}[Nm]	F_{iv}[Nm · sec/rad]
Rotor1	0.045600	0.045465	0.0002983
Gimbal2	0.050800	0.050556	0.0543675
Gimbal4	0.030819	0.031900	0.0009627

5.2 Simulation Result

Simulations of SDRE controller are shown in this section. SDRE controller is compared with a nonlinear controller designed by backstepping approach by the simulations. In this section, the simulation results of approximated SDRE method are shown in Fig. 3-6. The solid line, the dashed line and the dotted line show the simulation of SDRE controller, the nonlinear controller and the reference, respectively. As can be seen Fig. 3 and 4, the response of SDRE controller tracks the reference without error. As can be seen Fig. 4, SDRE controller and nonlinear controller are tracking to the reference almost at the same time at angle of Gimbal4 that is the main control target. Since SDRE method is similar to Linear Quadratic Regulator control, the input torques of SDRE controller are smaller than nonlinear controller in Fig. 5 and 6.

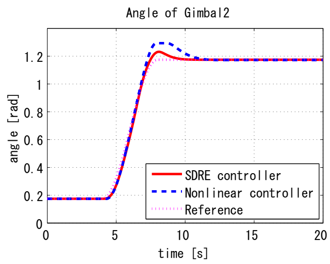

Figure 3 Simulation of angle of Gimbal2

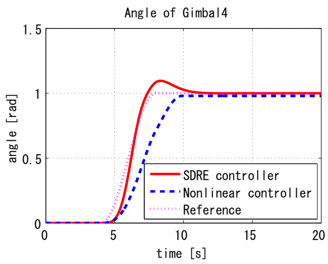

Figure 4 Simulation of angle of Gimbal4

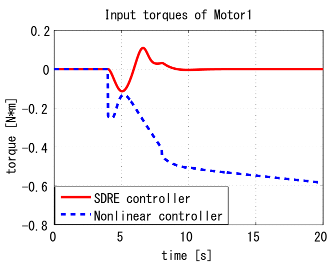

Figure 5 Simulation of torque of Motor1

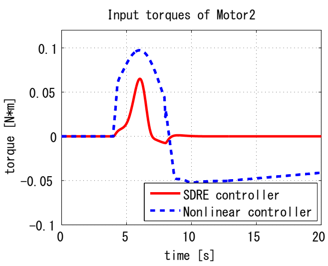

Figure 6 Simulation of torque of Motor2

5.3 Experiment Result

The experiment results are shown in Fig. 7-10. The solid line and the dashed line show the experiment result and the simulation result respectively. As can be seen Fig.7 and 8, the response of approximated SDRE controller tracks the reference without error. As can be seen Fig. 9 and 10, input torques are in the restricted ranges.

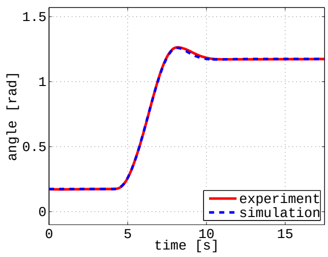

Figure 7 Experiment of angle of Gimbal2

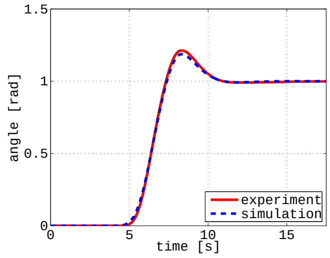

Figure 8 Experiment of angle of Gimbal4

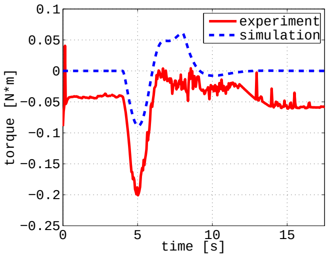

Figure 9 Experiment of torque of Motor1

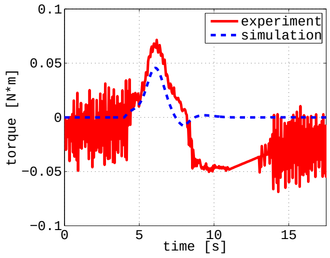

Figure 10 Experiment of torque of Motor2

6 Conclusion

In this study, tracking controller of CMG is designed by SDRE method. The nonlinear model of CMG is adopted , and SDRE method is approximated the two-dimensional function. The robust stability region is confirmed by LMI condition. The LMI condition are relaxed by SOS matrices, and trigonometric functions in the mathematical model of CMG are approximated polynomial to apply SOS relaxation. Lastly, the effectiveness of SDRE controller is illustrated by simulations and experiments.

References

[1] C. Murai, S. Washizu, I.Takami and G. Chen: "Nonliner Control for First-Order Nonholonomic System with Hardware Restriction and Disturbance", 10th Asian Control Conference (ASCC), ,2015

[2] H.S.Abbas,S.M.Hashemi,A.Ali and H.Werner:"LPV Gain-Scheduled Control of a Control Moment Gyroscope.", American Control Conference,2013

[3] K.Ishikawa and N.Sakamoto:"Optimal control for control moment gyros-center-stable manifold approach",The 53rd IEEE Conference on Decision and Control,2014

[4] T.Cimen:"State-Dependent Riccati Equation (SDRE) Control: A Survey",The 17th IFAC World Congress, 2008

[5] J. R. Cloutier, C. N. D'Souza, and C. P. Mracek:"Nonlinear Regulation and Nonlinear H∞ Control via the State Dependent Riccati Equation Technique: Part1,Theory",International Conference on Nonlinear Problems in Aviation and Aerospace,1996

[6] M.Izutu and K.Furuta: "Design of a Model Following Stabilizer for Furuta Pendulum", The 50th Japan Joint Automatic Control Conference , 2007

[7] Mugdha S. Naik and Sahjendra N. Singh: "State-dependent Riccati equation-based robust dive plane control of AUV with control constraints", Ocean Engineering, Vol. 34, No.1112, pp.1711-1723, 2007

[8] H.Ichihara:"Control System Analysis and Synthesis Based on Sum of Squares",Systems, control and information, 2011