Sliding Mode Control of Anti-lock Braking System Using Unscented Kalman Filter

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1 Abstract

In this study, a method is proposed to sliding mode control for Anti-lock Braking System(ABS) using Unscented Kalman Filter(UKF). The main objective of the ABS is to prevent of wheel-lock while braking. The ABS dynamics has strong nonlinearity with system uncertainties. In addition, the car speed and road surface friction to continue changing. The proposed sliding mode controller guarantees a highly robust performance for the system parameters and disturbance. The road surface friction estimate using a UKF. The optimal reference slip rate is determined from estimated road surface friction. Finally, effectiveness of the proposed method is verified by simulations and experiments.

2 Introduction

The main objective of the ABS is to prevent of wheel-lock while braking. The wheel-lock cause vehicle cannot make a turn. It is necessary to control slip rate. The slip rate depends on the car speed and wheel speed. The ABS to make a following slip rate a reference slip rate. Namely, the ABS is to ensure the braking force, car body posture, and maneuverability. However, the braking dynamics are highly nonlinear and there exist uncertainties. These are changed by car speed, road surface friction and slip rate. Therefore, the many kinds of study approach to ABS. For example, PID-type control[1], fuzzy control[2] and Gain scheduling control[3]. In this study, sliding mode controller is designed[4]. The sliding mode controller guarantees a highly robust performance. Although the reference slip rate is set at 0.2[5], the slip rate depends on the friction coefficient. In this study, the road surface friction is estimated using the UKF. The optimal reference slip rate is determined from estimated road surface friction.

3 Modeling

The control target is the quarter model of the real car. The upper wheel represents the car wheel and the lower wheel represents the road. The upper wheel has brake and the lower wheel has DC motor. There are two identical encoders measuring the rotational angles of two wheels. The model of the simplified ABS experimental device used in this study is shown in Fig.1. The physical parameter used in this study shown in Table.1.

Table 1 The physical parameter

radius of the wheel	r_1, r_2	[m]
moment of inertia of the wheel	J_1, J_2	[kgm^2]
total force generated	F_n	[Nm]
braking torque	τ_1	[Nm]
angular velocity of the wheel	ω_1, ω_2	[rad/s]
slip rate	λ	[-]
friction coefficientl	μ	[-]

The dynamical equations of the rotational motion of the upper and lower wheel are shown by Eq.(1) and Eq.(2).

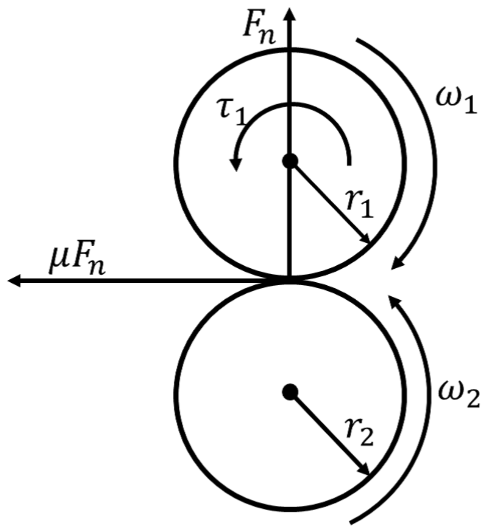

Figure 1 The simplified ABS experimental device

$$J_1\dot{\omega}_1 = F_n r_1 \mu - \tau_1 \tag{1}$$

$$J_2\dot{\omega}_2 = -F_n r_2 \mu \tag{2}$$

The slip rate is defined by Eq.(3) as the function of the car velocity and the wheel velocity.

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} \tag{3}$$

The following equation is obtained from Eq.(3).

$$\dot{\lambda} = -\frac{r_1}{r_2\omega_2}\dot{\omega}_1 + \frac{r_1\omega_1}{r_2\omega_2^2}\dot{\omega}_2 \tag{4}$$

The car speed V and wheel speed V_w defined by $V = r_2\omega_2$ and $V_w = r_1\omega_1$. In addition, the following equation is obtained from Eq.(3).

$$V_w = (1-\lambda)V \tag{5}$$

The reference wheel speed is defined by Eq.(6) is derived from Eq.(5).

$$V_w^* = (1-\lambda^*)V \tag{6}$$

4 State Space Representation

The state space representation is derived from Eq.(6). In order to track output of the system to optimal value without error, an integrator added to the state variable. The state variable $x(t)$ and input $u(t)$ are defined as follows

$$\begin{aligned} x(t) &= [x_1(t)\ x_2(t)]^{\mathrm{T}} \\ &= [\int (V_w - V_w^*)dt\ V_w - V_w^*]^{\mathrm{T}} \end{aligned} \tag{7}$$

$$u(t) = \tau_1. \tag{8}$$

Then the state space representation is obtained as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + Bh(t). \tag{9}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -\frac{r_1}{J_1} \end{bmatrix}$$

In order to design sliding mode controller, it is necessary to satisfy matching requirements. Therefore, $h(t)$ defined by Eq.(10). $Bh(t)$ is satisfy matching requirement in this system.

$$h(t) = -\mu F_n(r_1 + \frac{r_2^2 J_1}{r_1 J_2}(1 - \lambda^*)) \tag{10}$$

5 Sliding Mode Controller

The siding mode controller has the following characteristics. The controller guarantees highly robust performance namely resistant to modeling error and disturbances. The robust performance and high tracking can be made compatible, thus the sliding mode controller used in design servo system. Many sliding mode control methods exist. However, tracking wheel speed controller is designed from Eq.(9) in this study. The input for sliding mode control is composed of two input, a equivalent control input and nonlinear control input. The equivalent control input is tied down to a switching hyperplane. The nonlinear control input is made to a switching hyperplane. The input for sliding mode controller is defined as follows

$$u = u_{eq} + u_{nl}. \tag{11}$$

The switching hyperplane is designed to use in sliding mode control. Many method for designing switching hyperplane exist, i.e. characteristic vector method and pole assignment method and so on. The method for design optimum switching hyperplane is used in this study. The state behavior is defined as follows

$$\begin{aligned} \sigma &= Sx \\ &= [A_{12}^{\mathrm{T}} P + Q_{12}^{\mathrm{T}},\ Q_{22}]x. \end{aligned} \tag{12}$$

The cost function J is defined as follows

$$J = \int_{t_s}^{t} (x_1^{\mathrm{T}} Q_{11} x_1 + v^{\mathrm{T}} Q_{22} v) dt. \tag{13}$$

The minimum solution for cost function J from the Riccati equation.

$$PA_{11}^* + A_{11}^{*\mathrm{T}} P - PA_{12} Q_{22}^{-1} A_{12}^{\mathrm{T}} P + Q_{11}^* = 0 \tag{14}$$

The switching hyperplane is defined by Eq.(16).

$$\begin{aligned} \sigma &= Sx \\ &= [A_{12}^{\mathrm{T}} P + Q_{12}^{\mathrm{T}},\ Q_{22}]x \end{aligned} \tag{15}$$

The following equation is derived from $\dot{\sigma} = 0$ and Eq.(10).

$$\begin{aligned} \dot{\sigma} &= S\dot{x} \\ &= S\{Ax + Bu\} \\ &= 0 \end{aligned} \tag{16}$$

The equivalent control input is defined by Eq.(17) from Eq.(17).

$$u_{eq} = -(SB)^{-1}(SAx) \tag{17}$$

The control input defined by Eq.(18).

$$u = -K \frac{\sigma}{\|\sigma\| + \delta} \tag{18}$$

The control input derived from Lyapunov function. In order to σ guarantees $\sigma \to 0$ the Lyapunov function is defined $V_s = \frac{\sigma^{\mathrm{T}}\sigma}{2}$.

$$\begin{aligned} \dot{V}_s &= \sigma^{\mathrm{T}} \dot{\sigma} \\ &= \sigma^{\mathrm{T}} SAx + \sigma^{\mathrm{T}} SBu \end{aligned} \tag{19}$$

The following equation is derived from Eq.(18).

$$\begin{aligned} \dot{V}_s &= \sigma^{\mathrm{T}} \dot{\sigma} \\ &= \sigma^{\mathrm{T}} SAx + \sigma^{\mathrm{T}} - KSB \frac{\sigma}{\|\sigma\| + \delta} \end{aligned} \tag{20}$$

$$\leq -\|\sigma\|[K\|SB\| - \|SA\|\|x\|] < 0 \tag{21}$$

K is defined by Eq.(22). The Lyapunov function is negative constant function for σ then $x \neq 0$. Namely σ guarantees $\sigma \to 0$.

$$K = \alpha\|x\|,\ \alpha \geq \|SB\|^{-1}\|SA\| \tag{22}$$

The nonlinear control input is defined by Eq.(23).

$$u_{nl} = -K \frac{\sigma}{\|\sigma\| + \delta} \tag{23}$$

6 Existence Condition of Sliding Mode

The input for sliding mode control is defined by Eq.(24) from Eq.(11), Eq.(17) and Eq.(23).

$$u = -(SB)^{-1}(SAx) - K \frac{\sigma}{\|\sigma\| + \delta} \tag{24}$$

The existence condition of sliding mode is defined by Eq.(26) from Eq.(20) and Eq.(24).

$$\begin{aligned} \dot{V}_s &= \sigma^{\mathrm{T}}[SAx + SB\{-(SB)^{-1}SAx - K\frac{\sigma}{\|\sigma\| + \delta}\}] \\ &= -KSB\frac{\sigma^2}{\|\sigma\| + \delta} < 0 \end{aligned} \tag{25}$$

$SB > 0$ and $K > 0$, or $SB < 0$ and $K < 0$ are derived from Eq.(4.15). Thus, the stable sliding mode control can be realized. In this study, it is clear that SB is $SB < 0$. Therefore, K is also $K < 0$. Thereby, the existence condition of sliding mode is satisfied.

7 Unscented Kalman Filter

In this study, the changed road surface friction is estimated using UKF. The optimal reference slip rate is determined by estimated road surface friction. The best of estimate method because Kalman Filter can estimate state[6]. Additionally, UKF is adequate for the nonlinear system. The state variable $x_f(t)$ is defined by Eq.(26).

$$x_f(t) = [x_{f1}(t)\ x_{f2}(t)\ x_{f3}(t)]^{\mathrm{T}} = [\lambda\ \omega_2\ \mu]^{\mathrm{T}} \tag{26}$$

The continuous time nonlinear state equation is defined by Eq.(27).

$$\begin{bmatrix} \dot{x}_{f1} \\ \dot{x}_{f2} \\ \dot{x}_{f3} \end{bmatrix} = \begin{bmatrix} \frac{r_1\tau_1 - F_n r_1^2 x_{f3}(t)}{r_2 J_1 x_{f2}(t)} - \frac{(1-x_{f1}(t))F_n r_2 x_{f3}(t)}{J_2 x_{f2}(t)} \\ -\frac{F_n r_2 x_{f3}(t)}{J_2} \\ 0 \end{bmatrix} \tag{27}$$

The discretize Eq.(27) by Euler method. The discrete time nonlinear state equation is defined by Eq.(28). (dT:sampling time)

$$\begin{bmatrix} x_{f1}(k+1) \\ x_{f2}(k+1) \\ x_{f3}(k+1) \end{bmatrix} =$$

$$\begin{bmatrix} x_{f1}(k) + \{\frac{r_1\tau_1 - F_n r_1^2 x_{f3}(k)}{r_2 J_1 x_{f2}(k)} - \frac{(1-x_{f1}(k))F_n r_2 x_{f3}(k)}{J_2 x_{f2}(k)}\}\mathrm{dT} \\ x_{f2}(k) - \frac{F_n r_2 x_{f3}(k)}{J_2}\mathrm{dT} \\ x_{f3}(k) \end{bmatrix} \tag{28}$$

$$y(k) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_f(k) \tag{29}$$

The discrete time state space representation is defined by (30) from Eq.(28) and Eq.(29). f, h_f is nonlinear function. Additionally, the initial state x_{f0} and initial covariance P_0 are derived from the beginning.

$$x(k+1) = f(x_f(k), u(k)) + bv_f(k)$$

$$y(k) = h_f(x_f(k)) + w_f(k) \tag{30}$$

The Kalman gain is defined by Eq.(31). $P_{xy}^-(k)$ is the covariance of advance state error and advance output error. $P_{yy}^-(k)$ is the advance output error covariance.

$$G(k) = \frac{P_{xy}^-(k)}{P_{yy}^-(k)} \tag{31}$$

The state estimate is defined by Eq.(32). $\hat{x}^-(k)$ is the advance state estimate. $\hat{y}^-(k)$ is the advance output estimate.

$$\hat{x}_f(k) = \hat{x}_f^-(k) + G(k)\{y(k) - \hat{y}^-(k)\} \tag{32}$$

8 Simulation

In this section, the effectiveness of the designed controller is verified by simulations. The hypothesis of road condition are frozen road, wet road and dry road. The initial car speed is 50[km/h]. The reference slip rate is shown in Table.2 [7].

Table 2 The reference slip rate

Road condition	Friction coefficient	Reference slip rate
Frozen road	0.2	0.05
Wet road	0.4	0.12
Dry road	0.8	0.2

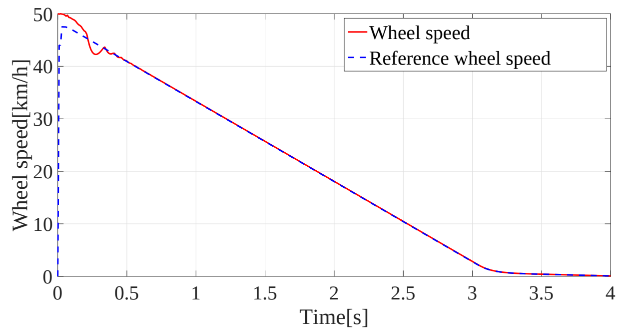

Figure 2 The wheel speed simulation at the frozen road

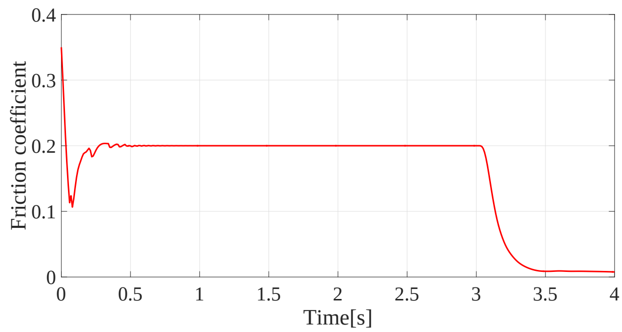

Figure 3 The estimate friction coefficient simulation at the frozen road

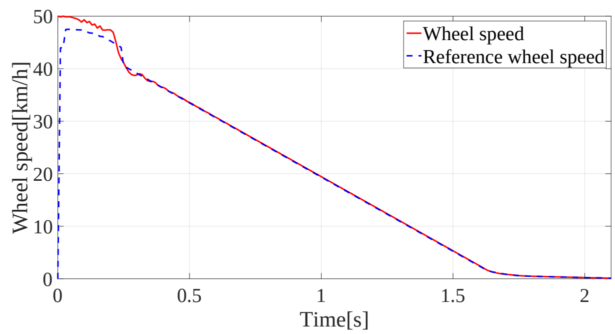

Figure 4 The wheel speed simulation at the wet road

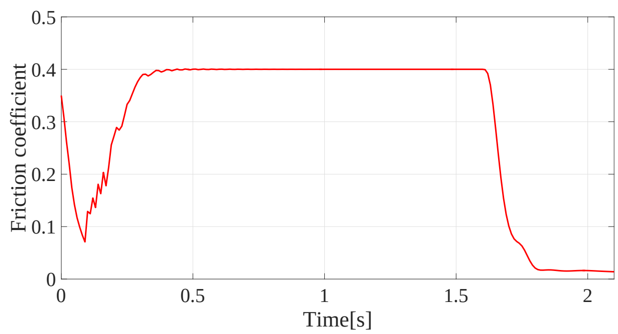

Figure 5 The estimate friction coefficient simulation at the wet road

The simulation result of wheel speed at any road condition are shown in Fig.2, Fig.4 and Fig.6. The simu-

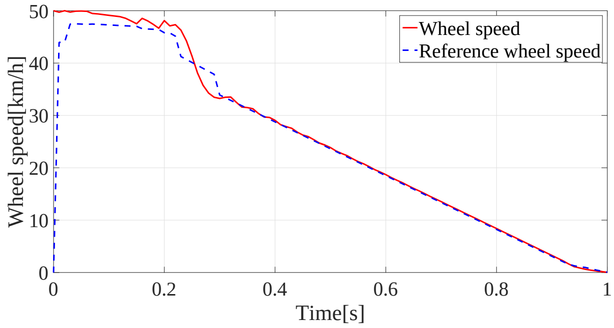

Figure 6 The wheel speed simulation at the dry road

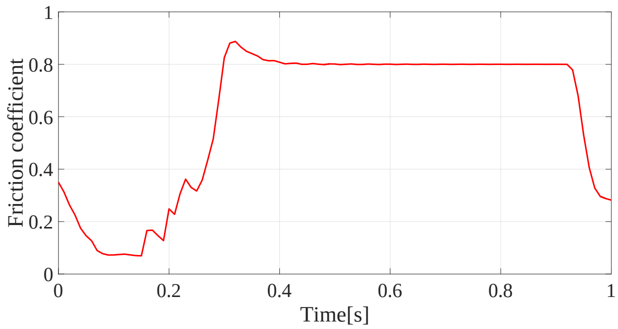

Figure 7 The wheel speed simulation at the dry road

lation result of estimate friction coefficient at any road condition are shown in Fig.3, Fig.5 and Fig.7.

As can been seen in Fig.2, Fig.4 and Fig.6 the wheel speed converge the reference wheel speed and dose not oscillate. As can been seen in Fig.3, Fig.5 and Fig.7 the estimate friction coefficient converge around road condition friction.

9 Experiment

In this section, the effectiveness of the proposed method is illustrated by experiments. The experiment result of wheel speed is shown in Fig.8. The experiment result of estimate friction coefficient using UKF is shown in Fig.9.

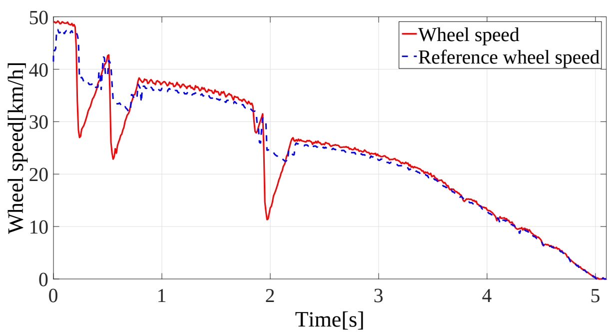

Figure 8 The wheel speed of experiment result

As can been seen in Fig.8, overshoot is occur. However, wheel speed follow a reference wheel speed. It is possible that overshoot results from the unexpected delay. The purpose of sliding mode is to restrain switching hyperplane as soon as possible. Thereby, sliding mode controller tends to be high gain controller. High gain controller tends to be weak unexpected delay. As can been seen in Fig.9, the estimation friction coefficient converge around 0.2. However, the estimation accuracy is low. It is possible that estimation accuracy is low results from wheel speed overshooting and the adjusting parameter.

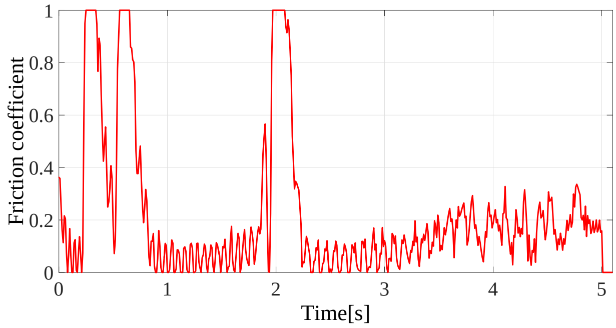

Figure 9 The estimate friction coefficient of experiment result

10 Conclusion

In this paper, a friction coefficient between a road surface and the wheels is estimated using Unscented Kalman Filter(UKF). In addition, the optimal reference slip rate is determined from estimated road surface friction. Sliding mode controller designed so that the method to guarantee the robust stability for uncertain parameters such as road surface friction and the car speed. Furthermore, the effectiveness of the proposed controller is illustrated by simulation and experiments.

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