Gain-Scheduling Observer for Twin Rotor Helicopter considering Unmeasurable Slung Load Angle

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1 Abstract

In this paper, the Robust controller and Gain-Scheduling Observer are designed for Twin Rotor Helicopter including unmeasurable slung load angle. Designing the Robust state feedback controller, Descriptor representation and Linear Functional Transformation (LFT) are applied. Designing Gain-Scheduling Observer, Descriptor, LFT and pole placement condition are applied on the separation theorem. The effectiveness of proposed method is verified by simulation comparing the nominal controller which control only airframe. The proposed controller's problems is confirmed by experiment. At last, the problem of proposed controller and the points of improvement is mentioned.

2 Introduction

The helicopter can land without a runway, and stay in the air (hovering). For this convenience, the helicopter is used by the load transport, military activity, rescue operation and so on. Also, the helicopter can not only carry humans and a load but also hang a load or roll up and down by wire. On the other hand, the helicopter flight slung a load is more dangerous than the general flight. Actually, the influence of strong wind, a slinging load swings largely, an accident may occur. Accordingly, several papers have been reported for flight stabilization considering slung load swing. Bisgaard proposed that estimation of slung load swing angle by image processing[1]. Sonobe. M is applied delay feedback of helicopter's output not using slung load estimated value[2]. However, in order to estimate by image processing is needed a additional sensor, and delay feedback has a disadvantages that convergence time is long. There, we consider the output feedback using estimated value of slung load from helicopter's input and output. Advantage of this method is not needed a additional sensor and it can be expected to respond quickly to the load swing.

The purpose of this study is to design a controller suppressing a slung load swing while being affected by the wind disturbance. In this study, variation of rope length by rolling up and down is also verified during helicopter flight. As a control target, Twin Rotor Helicopter experimental unit produced by Quanser is used. The state of the slung load can not be measured directly the same as actual helicopter, so we consider stabilizing closed loop system by output feedback control theory using Robust state feedback and Gain-scheduling Observer[3][4]. First, the state feedback gain that the varying parameter is rope length is designed. Designing Robust state feedback controller, Descriptor representation and Linear Functional Transformation (LFT) are applied. Optimal regulator theoretical problem is reduced to LMIs, and stability is guaranteed using polytope representation. Next, the Gain Scheduling Observer gain in which the scheduling parameter is rope length is designed on the separation theorem. For designing Gain-Scheduling Observer, Descriptor and LFT are applied. The pole placement condition is added to the LMI condition to design to set the real part of the eigenvalue of the estimated error system rather than the real part of the eigenvalue of the merged system.

3 Modeling

The schematic diagram of Twin Totor Helicopter is shown in Figure 1.

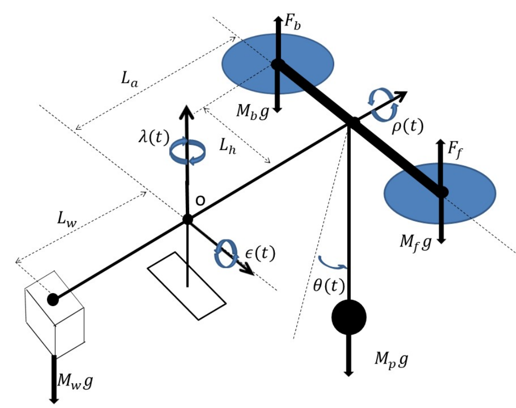

Figure 1 Schematic diagram of Twin Totor Helicopter

Controlled plant has two rotors in front and behind and it can maintain an airframe attitude to gain a proper input.ϵ[rad] is the angle in vertical plane named elevation angle and λ[rad] is the angle in horizontal plane named traveling angle and ρ[rad] is the pitch angle. In addition, θ[rad] is the alung load swing angle and let M_p[kg] is a load mass slung by a rope from the airframe. Then, l[m] is the rope length which is the variation parameter. A state variable of the plant is defined as $x(t)=[\,\epsilon(t)\,\rho(t)\,\lambda(t)\,\theta(t)\,\dot\epsilon(t)\,\dot\rho(t)\,\dot\lambda(t)\,\dot\theta(t)\,]^T$. An input is defined as $u(t)=[\,u_f(t)\,u_b(t)\,]^T$. The state space representation is given as follows.

$$\begin{cases} E_0(l,l^2)\dot x(t) = A_0(l)x(t) + B_0u(t) \\ \quad y(t) = Cx(t) \end{cases} \tag{1}$$

$$E_0 = \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix}, \quad A_0 = \begin{bmatrix} 0 & I \\ 0 & A \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & \sqrt{L_a^2+L_b^2}K_f & K_fL_h & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{L_a^2+L_b^2}K_f & -K_fL_h & 0 & 0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} e_1 & 0 & 0 & 0 \\ 0 & e_2 & 0 & 0 \\ 0 & 0 & e_3 & e_4 \\ 0 & 0 & e_4 & e_5 \end{bmatrix}, \ A = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & a_3 \end{bmatrix}$$

$$e_1 = (L_a^2 + L_b^2)(M_f + M_b + M_p) + (L_w^2 + L_b^2)M_w$$
$$e_2 = L_h^2(M_f + M_b)$$
$$e_3 = (L_a^2 + L_h^2)(M_f + M_b) + L_w^2 M_w + L_a^2 M_p$$
$$e_4 = L_a l M \quad , \quad e_5 = l^2 M_p$$
$$a_1 = -(M_f + M_b + M_w + M_p) g L_b$$
$$a_2 = -(u_{f0} + u_{b0})\sqrt{L_a^2 + L_b^2} K_f \quad , \quad a_3 = -M_p g L_a - C_{f2}$$

4 Control system design

In this study, the robust controller and Gain-Scheduling Observer are designed. First, the robust controller design is explained.

4.1 Servo system

In order to follow the output $\epsilon(t)$ and $\lambda(t)$ to the reference without error, servo system is designed. The error between the observed output $y(t)$ and the reference $r(t)$ is $e(t)$. Integrated value of $e(t)$ between 0 to t is $\int e(t)dt$. A new state variable of the plant is defined as $x_e(t) = [\int e_\epsilon(t)dt \ \int e_\lambda(t)dt \ x(t)]^T$, a new system including servo system is as follow.

$$\begin{cases} E_e(l, l^2)\dot{x}_e(t) = A_e(l)x_e(t) + B_e u_e(t) \\ \quad e(t) = C_e x_e(t) \end{cases} \tag{2}$$

$$E_e = \begin{bmatrix} I & O \\ O & E_0(l, l^2) \end{bmatrix}, A_e = \begin{bmatrix} O & -C \\ O & A_0(l) \end{bmatrix}$$
$$B_e = \begin{bmatrix} O \\ B_0 \end{bmatrix}, C_e = [\ O \quad -C \ ]$$

4.2 Descriptor representation

Eq.(3) has a time varying parameter at matrix E_e. For system with such characteristic, we can derive calculable LMI conditions by introducing redundant descriptor variables. Descriptor variables is defined as $x_d = [\ x_e(t) \ \ddot{\epsilon}(t) \ \ddot{\rho}(t) \ \ddot{\lambda}(t) \ \ddot{\theta}(t) \ ]^T$, a new system is as follow.

$$E_d\dot{x}_d(t) = A_d(l, l^2)x_d(t) + B_d u(t) \tag{3}$$
$$E_d = diag(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0)$$
$$A_d = \begin{bmatrix} 0 & -C_e & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & A_0(l) & 0 & -E_0(l, l^2) \end{bmatrix} B_d = \begin{bmatrix} 0 \\ B_e \end{bmatrix}$$

4.3 Linear Functional Transformation (LFT)

Matrix A_d has a l^2 which is a second team of the variation parameter. In this case, matrix A_d is transformed by introducing Linear Functional Transformation for deriving calculable LMI conditions. Where, let scheduling parameter is $\Delta = l$, Matrix $A_d(l)$ which is equivalent to Matrix A_d is as follow.

$$A_d(l) = A_n + B_\delta \Delta C_\delta \tag{4}$$

Then, the system which is equivalent to Eq.(3) is as follow.

$$\begin{cases} E_d\dot{x}_d(t) = A_n x_d(t) + B_\delta \omega_\delta(t) + B_d u(t) \\ \quad z_\delta(t) = C_\delta x_d(t) \\ \quad \omega_\delta(t) = \Delta z_\delta(t) \end{cases} \tag{5}$$
$$B_\delta = [\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$$
$$C_\delta = [\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \ -M_p l]$$

Here, new state variable of the plant is defined as $x_l(t) = [x_d(t) \ \ z_\delta(t)]^T$, we can derive calculable LMI conditions. After the transformed system is as follow.

$$E_l\dot{x}_l(t) = A_l(l)x_l(t) + B_l u(t) \tag{6}$$
$$A_l(l) = A_{l0} + lA_{l1} = \begin{bmatrix} A_n & B_\delta\Delta \\ C_\delta & -I \end{bmatrix}$$
$$E_l = \begin{bmatrix} E_d & 0 \\ 0 & 0 \end{bmatrix}, B_l = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$$

4.4 Polytopic representation

Let Upper and lower bounds of the varying parameter l be l_m and l_M. The parameter is defined as follows.

$$l = [\underline{l}\ ,\ \bar{l}] = [l_m\ ,\ l_M] \tag{7}$$

From Eq.(7), polytopic representation of matrix $A_l(l)$ can be described as follows.

$$A_l(l) = \lambda A_l(l_m) + (1-\lambda)A_l(l_M)\lambda \in [0\ , 1] \tag{8}$$

By using polytopic representation Eq.(8), designing problem of the control system can be formulated as a finite set of LMI conditions.

4.5 Robust controller design

From, Eq.(6), the system including Robust controller K_l is as follows.

$$\begin{cases} E_l\dot{x}_l(t) = A_l(l)x_l(t) + B_l u(t) \\ \quad u(t) = K_l x_l(t) \end{cases} \tag{9}$$

To guarantee the quadratic stability for the system, the following Lyapunov function $V(t)$ is dened as $V(x_l(t))= x_l^T(t)E_l P x_l(t)$, so stability condition for the system can be described as follows.

$$\dot{V}(x_l(t)) = \mathrm{He}\{P^T(A_l(l) + B_l K_l)\} \prec 0 \tag{10}$$

If there exist P and K_l such that Eq.(10) holds, then the closed loop system Eq.(10) is stabilized by the state feedback u(t) = $K_l x(t)$. In order to derive the stabilizing state feedback, the following quadratic stability condition is considered.

$$\begin{aligned} \dot{V}(x_l(t)) &= -x_l^T(t)(Q + K_l^T R K_l)x_l(t) \prec 0 \\ &= \mathrm{He}\{P^T(A_l(l) + B_l K_l)\} + (Q + K_l^T R K_l) \prec 0 \end{aligned} \tag{11}$$

Where, $Q = Q^T \succeq 0$ and $R = R^T \succ 0$ are weight matrices. The inequality Eq. (11) is equivalent to Riccati's differential inequality. Therefore, this inequality can be regarded as problem of optimal regulator that minimizes the following cost function J.

$$J = \int_0^\infty (x_l(t)^T Q x_l(t) + u^T(t) R u(t))dt \tag{12}$$

If there matrix $X_l = P^{-1}$ such that the inequality Eq.(11) holds, a upper bound on the cost function is represented as follows.

$$J \prec x_l^T(t)Px_l(t) = x_l^T(t)X_l^{-1}x_l(t) \tag{13}$$

For stabilizing sysytem Eq.(9), LMI condition's which derive LQ controller minimize the cost function J can be obtained as follows by Eq.(11) - Eq.(13) using schur complement. If there exist matrix $E_lX_l = (E_lX_l)^T$ and $Y_l = K_lX_l$ is satisfying the LMI conditions Eq.(14) - Eq.(17), state feedback $u(t) = Kx(t)$ stabilizes the system Eq.(9).

$$\begin{bmatrix} \mathrm{He}\{A_l(l)X_l + B_lY_l\} & (QX_l)^T & Y_l^T \\ QX_l & -I & O \\ Y_l & O & -R^{-1} \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} W & I \\ I & X_{11} \end{bmatrix} > 0 \tag{15}$$

$$Trace(W) < \gamma^2 \tag{16}$$

Then, Considering structure of the matrix $E_lX_l = (E_lX_l)^T$, candidates of lyapunov matrix X_l and variable matrix Y_l are restricted as Eq.(17).

$$X_l = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}, Y_l = \begin{bmatrix} Y_{11} & 0 \end{bmatrix} \tag{17}$$

Robust LQ controller is as follow.

$$K = Y_{11}{X_{11}}^{-1} \tag{18}$$

4.6 Full order obsever

In the previous section, the state feedback controller is designed under the condition that all the state variables can be observed. However, the actual experimental unit can acquire only the value of $\epsilon(t)$, $\rho(t)$, $\lambda(t)$ by using sensors. Therefore, full order observer is used to estimate the others state variables. A block diagram of output feedback using the full order observer is shown in Figure 2.

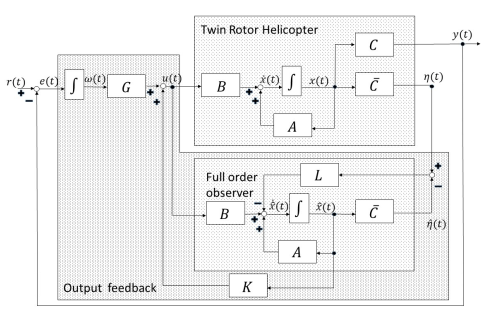

Figure 2 output feedback using the full order observer

From Figure 2, the system of the output feedback is as follows.

$$\begin{cases} E_0(l, l^2)\dot{\hat{x}}(t) = A_0(l)\hat{x}(t) + B_0u(t) - L(l)(\eta(t) - \bar{C}\hat{x}(t)) \\ u(t) = K\hat{x}(t) + G\omega(t) \end{cases} \tag{19}$$

Using the separation theorem, the state feedback gain K and the servo gain G and the observer gain $L(l)$ can be designed independently. In order to stabilize the estimated error system is defined as follows. Here, the state variable ξ_s is $[\ x(t)\ \ddot{\epsilon}(t)\ \ddot{\rho}(t)\ \ddot{\lambda}(t)\ \ddot{\theta}(t)\ z_\delta(t)]^T$.

$$E_s\dot{\xi}_s(t) = (A_s(l) + L(l)\bar{C}_s)\xi_s(t) \tag{20}$$

To guarantee the quadratic stability for the above system, the following Lyapunov function $V_s(t)$ is considered as $V_s(\xi_s(t)) = \xi_s^T(t)E_sP_s\xi_s(t)$. If $E_sP_s = (E_sP_s)^T \succeq 0$ exists, $\dot{V}_s(x_s(t)) \prec 0$ becomes asymptotically stable at the equilibrium point. In addition, Letting $X_s = P_s^T$ and $L(l) = {X_s}^{-1}H(l)$. $E_sP_s = (E_sP_s)^T$ and $\dot{V}_s(\xi_s(t))$ are rewritten as follows.

$$E_sX_s = (E_sX_s)^T \succeq 0 \tag{21}$$

$$\mathrm{He}\{X_s(A_s(l) + H(l)\bar{C}_s)\} \prec 0 \tag{22}$$

In this time, in order to design the real part of the eigenvalue of the estimated error system to be more negative than -5, pole placement condition is added at LMI condition Eq.(22).

$$\mathrm{He}\{X_s^T(A_s(l) + H(l)\bar{C}_s)\} + 2 * 5 * E_sX_s \prec 0 \tag{23}$$

Then, Considering structure of the matrix $E_sX_s = (E_sX_s)^T$, candidates of lyapunov matrix X_s is restricted.

$$X_s = \begin{bmatrix} X_{s11} & 0 \\ X_{s21} & X_{s22} \end{bmatrix} \tag{24}$$

Variable matrix $H(l)$ is defined as follows.

$$H(l) = \begin{bmatrix} H_{11} \\ 0 \end{bmatrix} + l \begin{bmatrix} H_{21} \\ 0 \end{bmatrix} \tag{25}$$

The Gain-Scheduling Observer is as follow.

$$L(l) = X_{s11}^{-1}(H_{11} + lH_{21}) \tag{26}$$

From above theories, the poles of state feed back system and the estimated error system are derived as Figure 3.

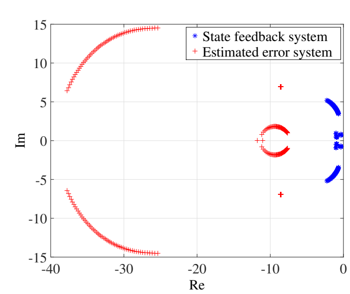

Figure 3 Eigenvalue of merging system

5 Simulations

In this section, the verification of the proposed method is illustrated by comparing nominal controller.

5.1 Rolled up a rope

In simulation, the rope is rolled up between 2.0[s] to 12.0[s]. Upper and lower bounds of the varying parameters are assigned as $l \in [0.45, 1.0]$. This is a simulation that wind disturbance is added for a Twin Rotor heliconter during rolled up a rope. Here, Nominal written in the simulation grahp is only body control controller. Proposed is controller that we design without using Gain-Scheduling Observer. Figure 4 and Figure 5, are slung load swing and pitch angle graph. From Figure 4, slung load swing is suppressed than Nominal controller by moving pitch angle according to slung load swing. But, Using Gain-Scheduling Observer, airframe

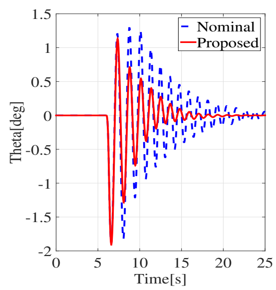

Figure 4 Slung load swing angle

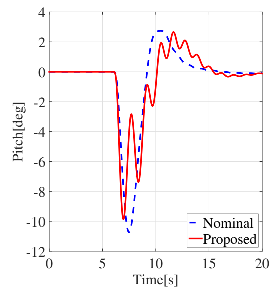

Figure 5 Pitch angle

is moved largely. This result is assumed that slung load angle of the estimation accuracy is low or the feedback gain response is excessive for slung load angle.

6 Experiment

From the simulation results, proposed controller has some points of uncertainty. First, we confirmed estimation accuracy of Gain-Scheduling Observer by using nominal controller for experimental unit equipped with a slung load. The experimental results are shown Figure 6 and Figure 7. Here, Actual value is measured by a angle sensor.

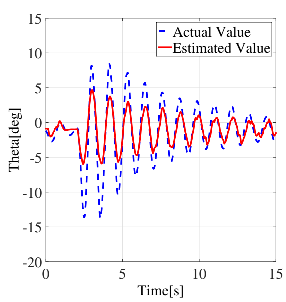

Figure 6 Estimated slung load swing angle (l=0.45)

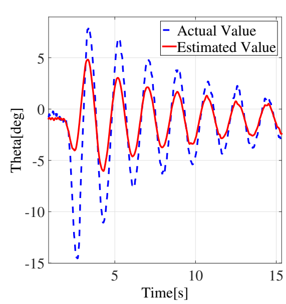

Figure 7 Estimated slung load swing angle (l=1.0)

From Figure 6 and Figure 7, it can be seen that the swing can be estimated although the slung load angle is small. This result is assumed that model error or low estimation accuracy is occurring. Next, we verified the proposed controller, however helicopter is not stabilized. Then, in order to verify that the feedback gain response is excessive for slung load angle, estimated values of θ and $\dot{\theta}$ is reduced by division.The result is from Figure 8 to Figure 11. In this experiment, the situation rolling up a rope could not prepare, so we experimented in upper bound an upper bound of parameter l. Figure 8 and Figure 9 are slung load swing angle graph, pitch angle graph in the lower bounds $l = 0.45$. Figure 10 and Figure 11 are slung load swing angle graph, pitch angle graph in the lower bounds $l = 1.0$.

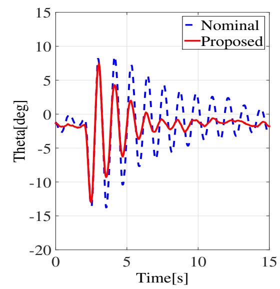

Figure 8 Slung load swing angle(l=0.45)

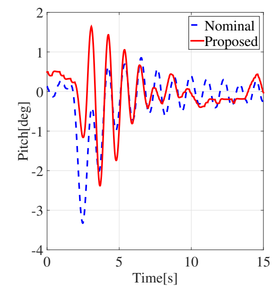

Figure 9 Pitch angle

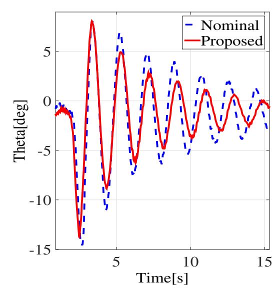

Figure 10 Slung load swing angle(l=1.0)

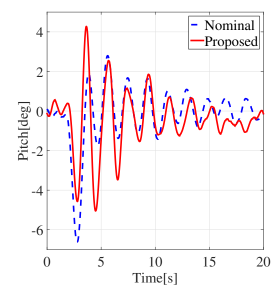

Figure 11 Pitch angle

7 Conclusion

In this paper, the Robust controller and Gain-Scheduling Observer are designed for Twin Rotor Helicopter including unmeasurable a slung load angle. The effectiveness of proposed method is confirmed in experiment by gain turning. However proposed controller is not stabilized without gain turning. The reason is that the feedback gain response is excessive for slung load angle. For the points of improvement, eliminating model error and designing a controller that response is not too excessive for slung load angle.

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