Real-time Compensation of Varying Friction using the Square Root Unscented Kalman Filter

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Abstract

This paper presents a method to design the contour control of the ball screw system. The friction is one of the main factors to deteriorate the positioning performance. There exists the nonlinearity in the friction such as the Stribeck effect and the hysteresis property. The friction has the position dependent and asymmetric characteristics which change the friction size depending on the position and the direction of object. In addition, the friction varies depending on the temperature and the wear. From the above, it is necessary to successively estimate the friction. In these conditions, we propose the method to estimate and compensate the friction in real time. The LuGre model is applied to describe the dynamics of friction, which is able to represent the characteristics by a few parameters. The square root unscented Kalman filter is used to estimate the friction which is applicable to nonlinear systems. The estimated variables are used to compensate the friction in real time. The effectiveness is illustrated by simulations and experiments.

1 Introduction

In recent years, the high accuracy positioning and the fast response are required for machine tools and semiconductor manufacturing equipment. The ball screw system is applied to the positioning control. It is used in robot arms, medical equipment and cars. The ball screw is rotated by the motor, and the rotation translates the linear motion which is attached to the ball screw system. The ball screw system has the characteristic that the friction is reduced to change the surface contact to the point contact by balls between the screw and the nut. However, the friction influence cannot be ignored in the precision positioning control.

The friction has nonlinear characteristics such as the Stribeck effect and the hysteresis property. The stick slip phenomenon and the limit cycle are caused by them. Many friction models are proposed to express these characteristics. The LuGre model presents these friction characteristics such that the friction is approximated by the spring and the dumper effects of bristles in the contact surface [1] [2]. In addition, the Tustin model, the generalized Maxwell-slip model(GMS) [3] and the Leuven model [4] are proposed. The methods which compensate the friction using these friction models are proposed [5]. There are various types of the observer such as the inverse plant based on observer and the Kalman filter. Among the Kalman filters, the extended Kalman filter (EKF) and the unscented Kalman filter (UKF) [6] are proposed. They are able to estimate the state variables in the nonlinear system. The EKF estimates the state variables by linearizing the nonlinear system at every sampling time. The UKF approximates the statistic model of the nonlinear dynamics using the probability distribution function at a few sampling points called sigma points which are determined by the covariance matrix. It has the high accurate estimation while it has a large amount of calculation. In case, the square root matrix of the covariance matrix can not be calculated in the UKF algorithm. The reason is that the Cholesky decomposition is applied when the covariance matrix is not positive definite by the cancellation of significant digits. The square root unscented Kalman filter (SRUKF) is proposed to solve this problem [7]. The SRUKF is derived the square root matrix of the covariance matrix by using the QR decomposition. The square root matrix can be derived when the covariance matrix is not positive definite.

The contribution of this paper is to show the method which estimates and compensates the nonlinear and asymmetric friction in real time for positioning control system. The nonlinear friction is estimated by using the LuGre model which is able to express the friction characteristics using a few parameters. The friction has the strong nonlinearity such as the Stribeck effect and the hysteresis property, therefore the SRUKF is applied to treat the nonlinearity. As the number of estimated variables increases, the calculation amount of estimation increases. The method is implemented in real time by combining the SRUKF and the LuGre model which needs a few parameters to reduce the calculation cost. The friction has the asymmetric characteristic and the position dependency. In this study, the parameter in the LuGre model is identified by using the SRUKF in real time. The time varying friction influence is suppressed by the friction compensator based on the LuGre model using identified parameters. The effectiveness of the proposed method is verified by simulations and experiments.

2 Models

In this section, the mathematical models are shown to describe the dynamics of the ball screw system and the friction.

2.1 Model of the ball screw system

The two axes ball screw system used in the experiment is shown in Figure 1. The ball screw system moves the table which is connected to the ball screw through the coupling by transforming the motor rotation to the linear motion. The rotational angle of the motor is θ, the

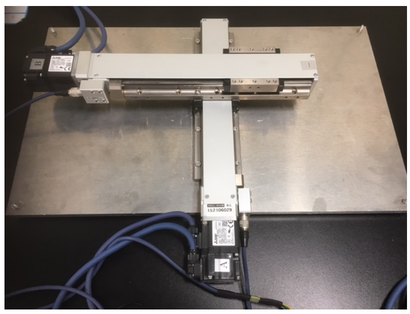

Figure 1 Ball Screw System

displacement of the table is x_p, the torque of the motor is T, the nonlinear friction generated in the ball screw system is F_n, the mass of the table is M_t, the spring constant is K, the motor inertia is J_m, the viscous friction coefficient is σ_2 and the the ball screw coefficient is R. The motion equations of the motor and the table are given as follows;

$$J_m\ddot{\theta} = T - RK(R\theta - x_p) \quad (1)$$

$$M_t\ddot{x}_p = K(R\theta - x_p) - F_n - \sigma_2\dot{x}_p. \quad (2)$$

Assuming the ball screw does not have the delay caused by the torsion, the ball screw system has the relationship $x_p = R\theta$. By using Eq. (1) and (2), the motion equation of the ball screw system is given as follows.

$$(J_m + R^2M_t)\ddot{\theta} = T - R^2\sigma_2\dot{\theta} - RF_n. \quad (3)$$

Here, the input is $u = T$, the state variable vector is $x_m = [\theta \ \ \dot{\theta}]^T$. The state space representation of the ball screw system is given as follows;

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m u - RB_m F_n \\ y &= C_m x_m \end{aligned} \quad (4)$$

$$A_m = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{R^2\sigma_2}{J_m+R^2M_t} \end{bmatrix}, B_m = \begin{bmatrix} 0 \\ \frac{1}{J+R^2M_t} \end{bmatrix}, C_m = [1 \quad 0].$$

2.2 the LuGre model

The friction occurs on the contact surface. The LuGre model is applied to present the nonlinear characteristics of the friction in the ball screw system. In the LuGre model, the contact surface is considered by the aggregation of the bristles. The friction is derived from the elasticity and viscosity of the bristles. The LuGre model has some advantages. First, the friction of the stick mode and the slip mode are uniformed by using the LuGre model. Second, the static characteristics of the friction such as the Stribeck effect and the hysteresis property are represented with a few parameters. The mathematical expression of LuGre model is given as follows [1];

$$F_r = \sigma_0 z + \sigma_1\dot{z} + R\sigma_2\dot{\theta} \quad (5)$$

$$\dot{z} = R\dot{\theta} - \sigma_0\frac{R|\dot{\theta}|}{g(\dot{\theta})}z \quad (6)$$

$$g(\dot{\theta}) = sgn(\dot{\theta})(F_c + (F_s - F_c)e^{-\frac{R|\dot{\theta}|}{v_s}}). \quad (7)$$

Here, F_s is the static friction, F_c is the Coulomb friction, v_s is the Stribeck velocity, σ_0 is the spring coefficient of the bristles, σ_1 is the damper coefficient of the bristles. Eq. (5), (6) and (7) show the total of the friction, the dynamics of the bristles and the static characteristics of the friction.

2.3 Friction Identification

In this section, the identification methods of the parameters in the LuGre model are described. As we have mentioned before, six parameters are set in the LuGre model. Each parameter is identified by changing the migration distance of the table and the input. When the static friction and the Coulomb friction are identified, the sinusoidal waves are input to the ball screw. The static friction is corresponding to the input when the table of ball screw system starts to move. Similarly, the Coulomb friction is corresponding input when the table stops. Eq. (7) is expanded to Eq. (8) to simulate the asymmetricity of the friction. F_{s+}, F_{s-}, F_{c+}, F_{c-} show the difference of friction force by the moving direction of the table.

$$g(\dot{\theta}) = \begin{cases} F_{c+} + (F_{s+} - F_{c+})e^{-R|\dot{\theta}|/v_s} & (\dot{\theta} > 0) \\ F_{c-} + (F_{s-} - F_{c-})e^{-R|\dot{\theta}|/v_s} & (\dot{\theta} < 0) \\ (F_{s+} + F_{s-})/2 & (\dot{\theta} = 0). \end{cases} \quad (8)$$

Next, the Stribeck velocity is identified. The Stribeck effect is the characteristic which depends on the velocity of the object. The velocity when the friction is minimum is called as the Stribeck velocity. To identify the Stribeck velocity, the table is controlled by the uniform motion in several velocities using the PI controller. The Stribeck velocity is the one when the friction is minimum. Next, the parameter σ_0 and σ_1 are identified which are the bristles parameters. The hysteresis curve appears in experiments when the sinusoidal wave of the small amplitude is inputted to the motor. The parameter σ_0 is identified by the slope from the top-right corner to the lower-left corner of the orbit. Also, σ_1 is identified by consisting the output result of simulations and experiments based on the obtained parameters F_s, F_c, v_s, σ_0 and σ_2.

3 Controller Design

The control system consists of the friction compensator and the tracking controller. The nonlinear friction which deteriorates the positioning performance is estimated and compensated by the friction compensator based on the SRUKF in real time. There exist asymmetric and position dependent characteristics which change the friction size depending on the position and the direction of movement of the object. The static friction and the Coulomb friction have the asymmetricity. In this paper, the friction in Eq. (5) is estimated and compensated accurately by estimating $g(\dot{\theta})$ in Eq. (8) as the unknown parameter. F_s and F_c identified in the previous section are compared with the estimated friction by the SRUKF. After the friction is compensated, the tracking controller based on the model predictive controller (MPC) is applied to control the table position of the ball screw system.

3.1 Friction Compensator based on the LuGre model

In this section, the friction compensator based on the LuGre model is designed to compensate the nonlinear friction by extending the state space representation of the ball screw system. At first, Eq. (5) is discretized. The static friction and the Coulomb friction are treated as unknown parameters. These parameters are estimated by using the SRUKF in real time. The accurate estimation is deteriorated when these two unknown parameters are estimated independently. In the LuGre model, $g(\dot{\theta})$ in Eq. (8) is defined as the function of $\dot{\theta}$, F_s, F_c and v_s. In this paper, $g(\dot{\theta})$ is regarded as an unknown parameter g_p, which is estimated using the SRUKF. The friction dynamics is given by Eq. (5) and (6) using the estimated parameter g_p. The parameter g_p is assumed as time invariant, i. e. $\frac{d}{dt}g_p = 0$. The nonlinear equation of control system including the LuGre model is obtained;

$$\begin{aligned} x_{Ld}(k+1) &= f(x_{Ld}(k), u_d(k)) + Bv_d(k) \\ y_{Ld}(k) &= h(x_{Ld}(k)) + w_d(k) \end{aligned} \quad (9)$$

$$f(x_{Ld}(k), u_d(k)) = \begin{bmatrix} F_{11}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + F_{12}(x_{Ld}(k))x_3(k) + B_{md}u_d(k) \\ f_{21}(x_{Ld}(k))x_2(k) + f_{22}(x_{Ld}(k))x_3(k) \\ x_4(k) \end{bmatrix}$$

$$h(x_{Ld}(k)) = [1 \quad 0 \quad 0 \quad 0]\, x_{Ld}(k)$$

$$F_{11} = A_{md} + [O \quad -B_{md1}\sigma_1]$$

$$F_{12}(x_{Ld}(k)) = -B_{md2}(\sigma_0 - \sigma_0\sigma_1\alpha(x_2(k)))$$

$$f_{21}(x_{Ld}(k)) = \frac{e^{(-\sigma_0\alpha(x_2(k)))T_s} - 1}{\sigma_0\alpha(x_2(k))}R$$

$$f_{22}(x_{Ld}(k)) = e^{-\sigma_0\alpha(x_2(k))T_s}, \alpha(x_2(k)) = \frac{|Rx_2(k)|}{x_4(k)}$$

$$B_{md1} = \int_0^{T_s} e^{A_m\tau}B_{m1}d\tau, B_{m1} = \begin{bmatrix} 0 \\ R/(J+M_tR^2) \end{bmatrix}$$

$$B_{md2} = \int_0^{T_s} e^{A_m\tau}B_{m2}d\tau, B_{m2} = \begin{bmatrix} 0 \\ R^2/(J+M_tR^2) \end{bmatrix}.$$

Here, $x_{Ld}(k) = [x_1(k) \quad x_2(k) \quad x_3(k) \quad x_4(k)]^T = [\theta(k) \quad \dot{\theta}(k) \quad z(k) \quad g_p(k)]^T$ is the state variable vector, $v_d(k)$ is the system noise, $w_d(k)$ is the observation noise and T_s is the sampling time. When four state variables are estimated, the nonlinear friction is given by applying the SRUKF algorithm to Eq. (9). The estimated nonlinear friction is obtained as follows;

$$\hat{F}_n(k) = \sigma_1\hat{x}_2(k) - \sigma_0\sigma_1\hat{x}_3(k)\hat{x}_4(k). \quad (10)$$

The influence of nonlinear friction is canceled by adding the estimated $\hat{F}_n(k)$ to the input of the ball screw system.

3.2 Friction Estimation by using the SRUKF

The UKF is the method which is able to maintain the accurate estimation even if the observed system has a strong nonlinearity. The statistic model of the nonlinear system is approximated by using the probability distribution function at a few sample points called the sigma points which are decided based on the standard deviation.

Similarly to the SRUKF, the EKF can estimate state variable which has nonlinearity. The nonlinear equation is linearized using the Jacobin matrix at every sampling time. Compared to the SRUKF, the EKF has the advantage which is low calculation amount, although the strong nonlinearity and discontinuity cannot be considered. In this paper, Eq. (9) has the absolute term $|x_2(k)|$ that means the equation has discontinuity. Therefore, it is difficult to derive the Jacobian matrix. On the other hand, the Cholesky decomposition is needed to calculate sigma points in the UKF. There is a case that the covariance matrix becomes negative definite by the cancellation of significant digits. The QR decomposition is applied to derive the square root matrix in the SRUKF. The square root matrix can be derived when the matrix is not positive definite. In this study, the SRUKF is applied to improve stability. The nonlinear friction $\hat{F}_n(k)$ is estimated by Eq. (10) using the SRUKF. The SRUKF algorithm is shown as follows.

- Calculation of the sigma points $X_i(k-1)$

$$\chi_0(k-1) = \hat{x}(k-1) \quad (11)$$

$$\chi_i(k-1) = \hat{x}(k-1) + \sqrt{n+\lambda}\,(S_{xx}(k-1))_i \quad (12)$$

$$\chi_{n+i}(k-1) = \hat{x}(k-1) - \sqrt{n+\lambda}\,(S_{xx}(k-1))_i \quad (13)$$

Here, S_{xx} is the square root matrix of the covariance matrix by using the QR decomposition, $(S_{xx}(k-1))_i$ is the i-th column vector of the square root of the covariance matrix and λ is the scaling parameter.

- Derivation of the prior estimation value

$$\chi_i^-(k) = f(X_i(k-1), u(k-1)) \quad (14)$$

$$\hat{x}^-(k) = \sum_{i=0}^{2n} W_i X_i^-(k) \quad (15)$$

$$M_{xx}(k) = \left[\sqrt{w_i}(\chi_i^-(k) - \hat{x}^-(k)\right] \quad (16)$$

$$S_{xx}(k) = f_{qr}([M_{xx}(k), \sqrt{v_d(k)}]^T) \quad (17)$$

Here, f is the nonlinear function, f_{qr} is the QR decomposition function, W_i is the wight matrix and S_{xx} is the upper triangular matrix derived by the QR decomposition. S_{xx} is equivalent to the square root matrix of the covariance matrix.

- Recalculation of the sigma points

$$\chi_0(k) = \hat{x}(k) \quad (18)$$

$$\chi_i(k) = \hat{x}(k) + \sqrt{n+\lambda}\,(S_{xx}(k))_i \quad (19)$$

$$\chi_{n+i}(k) = \hat{x}(k) - \sqrt{n+\lambda}\,(S_{xx}(k))_i \quad (20)$$

- Calculation of the state and output covariance matrices

$$M_{xx}(k) = [\sqrt{w_i}(\chi^i(k) - \hat{x}(k))] \quad (21)$$

$$M_{xy}(k) = [\sqrt{w_i}(\gamma_i^-(k) - \hat{y}(k+1))] \quad (22)$$

$$S_{yy}(k) = f_{qr}([M_{yy}(k+1), \sqrt{w_d(k)}]^T) \quad (23)$$

$$P_{xy}(k) = M_{xx}(k)M_{yy}^T(k) \quad (24)$$

Here, $\gamma_i^-(k) = h(X_i^-(k))$, $\hat{y}^- = \sum_{i=0}^{2n} W_i\gamma_i^-(k)$, S_{yy} is the square root matrix of output covariance matrix and P_{xy} is the covariance matrix of state and output.

- Derivation of the Kalman gain $g(k)$ and the predicted state $\hat{x}(k)$

$$\hat{x}(k) = \hat{x}^-(k) + g_k(k)(y(k) - \hat{y}^-(k)) \quad (25)$$

$$S_{xx}(k) = f_{qr}([M_{xx}(k) - M_{yy}(k)g_k(k)^T, \sqrt{w_d(k)}g_k(k)^T]^T) \quad (26)$$

$$g_k(k) = P_{xy}^-(k)(S_{yy}(k)^T S_{yy}(k))^{-1} \quad (27)$$

Here, $y(k)$ is the measurement output of the ball screw system by a sensor. The nonlinear friction $\hat{F}_n(k)$ is estimated by Eq. (10) using the obtained state $\hat{x}(k)$.

4 Simulations and Experiments

In this section, the effectiveness of the proposed method is illustrated by simulations and experiments. The friction influence appears the most at the reversal action of the table. Therefore, the circle trajectory is described by using the x-y table driven by the ball screw system to show the influence of the friction remarkably. The reference of X axis is given as the sinusoidal wave signal and that of Y axis is given as the cosinusoidal wave signal. Here, the amplitude of the reference is 10^{-4} m and the frequency of the reference is $\pi/4$ Hz. The MPC is used to control the table that the friction is compensated by using the SRUKF. The sampling time T_s is set to 0.001 s.

The simulation and the experiment results are shown in Figure 2 and 3. The dotted curve shows the reference, the dashed curve shows the result when the system is only controlled by the MPC and the solid curve shows the result of the proposed method that the friction is compensated by using the SRUKF.

By comparing Figure 2 with Figure 3, the simulation result is remarkably similar to the experiment result. The modeling of the ball screw system and the friction are properly. Additionally, the identified parameters are appropriate. At the reversal action of the table by using only the MPC, the table dose not track the target trajectory and the quadrant glitches are observed. By using the proposed method for the friction estimation and compensation, the positioning accuracy is improved and the quadrant glitches are suppressed. Figure 4 shows the friction of X axis. The dotted line shows the reproduced friction calculated by the LuGre model in the simulation, the dashed line shows the estimated friction using the SRUKF in the simulation and the solid line shows estimated friction using the SRUKF in the the experiment. The friction is estimated and compensated effectively in simulations and experiments. As the estimated value is appropriate, the quadrant glitches do not appear using the proposed method in Figure 2 and Figure 3. From the above, the proposed method is able to compensate the varying friction.

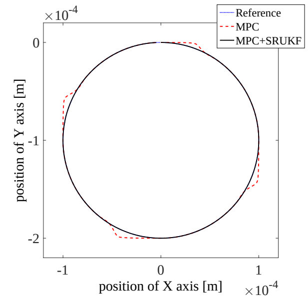

Figure 2 Simulation results

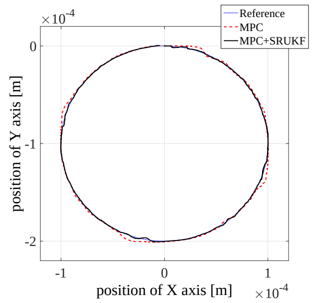

Figure 3 Experiment results

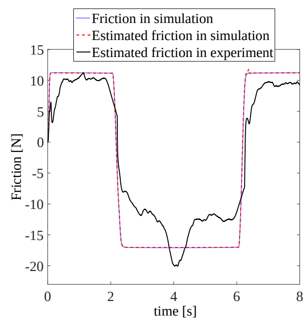

Figure 4 The estimated friction of X axis

5 Conclusion

In this paper, the counter control method for the ball screw system is proposed. The friction which deteriorates the positioning performance is estimated and suppressed by using the friction compensator based on the LuGre model and the SRUKF in real time. The asymmetric friction is estimated to define its parameters as an unknown parameter. The effectiveness of proposed method is illustrated by suppressing the quadrant glitches in the two axes ball screw system by simulations and experiments.

References

[1] C. Canudas, H. Olsson, K. J. Astrom and P. Lischinsky: "A new model for control of systems with friction." IEEE Transactions on Automatic Control, Vol. 40, No. 3, pp.419-425, 1995

[2] K. J. Astrom and C. Canudas de Wit: "Revisiting the LuGre friction model. " IEEE Control Systems Magazine, Institute of Electrical and Electronics Engineers, Vol. 28, No. 6, pp.101-114, 2008

[3] Farid Al-Bender, Vincent Lampaert and Jan Swevers: "The generalized Maxwell-slip model: A novel model for friction simulation and compensation." IEEE Transaction on Automatic Control, Vol. 50, No. 11, pp1883-1887, 2005

[4] Vincent Lampaert, Jan Swevers and Farid Al-Bender: "Modification of the Leuven integrated friction model structure." IEEE Transactions on Automatic Control, Vol. 47, No. 4, pp683-687, 2002

[5] D. Hoshino and J. Ishikawa: "Friction compensation using time variant disturbance observer based on the LuGre model." Transactions of the Japan society of Mechanical Engineers, Series C, Vol. 79, No. 805, pp.3206-3220, 2013

[6] Simon Julier, Jeffrey Uhlmann and Hugh F. Durrant-Whyte: "A new method for the nonlinear transformation of means and covariances in filters and estimators." IEEE Transactions on Automatic Control, Vol. 45, No. 3, pp477-482, 2000

[7] Rudolph Van der Merwe and Eric A. Wan: "The square-root unscented Kalman filter for state parameter-estimation." IEEE international Conference on Acoustics, Speech, and Signal Processing, Vol. 6, pp3461-3464, 2001.