Disturbance Compensation of 3 DoF Helicopter using Disturbance Observer

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Abstract

The objective of this paper is wind disturbance compensation for a three degree of freedom helicopter (3DoF Helicopter). In this paper, a mathematical model and LQ controller is designed.Furthermore, a disturbance is estimated using disturbance observer based on full-order observer.Then the estimated disturbance is converted into an input voltage, and compensated by feedbacking this input. Finally, the effectiveness of distirbance compensation is examined by simulations and experiments.

1 Introduction

Recently, helicopters are used for purpose such as transportation of items, rescue, etc. However, there are some problems. For example, the flight is unstable due to the influence of wind, and the model has a strong nonlinear characteristic. Therefore, in recent studies, model predictive control, robust control and Kalman filter are used for objective of stable control[1]. For example, extended kalman filter (EKF)[2] [3] and unscented kalman filter (UKF)[4] are used for estimation of state and attitude control. In addition, gain scheduled (GS) theory are used for fault-tolerant control.

In this study, the objective is the control of 3DoF helicopter and the compensation of a wind disturbance. Therefore, the robust stubility for the plant system is guaranteed by LQ control design, and the observer to estimate a wind disturbance is designed. This observer is based on full-order observer. Then a estimated disturbance is converted into feedback input, and compensated about influence of the disturbance. Finally, in this paper, the effectiveness of disturbance compensation is examined by simulations and experiments.

2 Modeling

The experimental device of 3DoF helicopter used in this study is shown in Fig.1. This airframe is twin rotors, and it is called as a tandem rotor helicopter. It is able to control by the elevation angle, the pitch angle and the travel angle. The elevation angle is the altitude of the airframe, the pitch angle is inclination of the body, and the travel angle is the rotational angle about horizontal. The DC motors are attached to the front and rear on the airframe, and the thrust force is generated by the voltage difference between the front motor and the back motor. In addition, the front propeller is counterclockwise and the back motor is counterwise because of torque cancellation. Physical parameters of the helicopter are shown in Table.1. Three an-

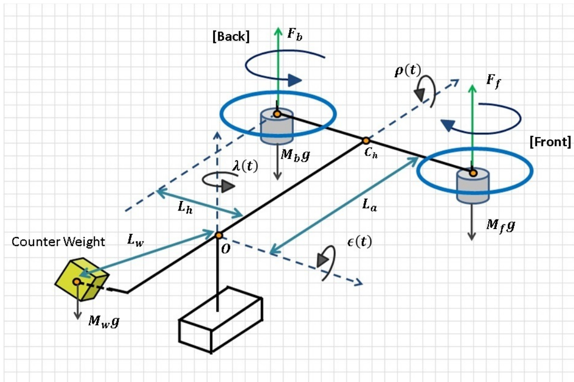

Figure 1 The 3DoF helicopter

Table 1 Physical parameter

Detail	Symbol	Unit
Acceleration due to gravity	g	[m/s^2]
Thrust factor	Kt	[N/V]
Mass of front motor	M_f	[kg]
Mass of back motor	M_b	[kg]
Mass of Counterweight	M_h	[kg]
Distance from center of airframe to fulcrum	L_a	[m]
Distance from center of airframe to a rotor	L_h	[m]
Distance from fulcrum to counterweight	L_w	[m]

gles (elevation angle, pitch angle, travel angle) are used, and these are defined as:

- ϵ: The elevation angle which is rotated around the elevation axis is defined as zero when the helicopter is hovering on horizontal.
- ρ: The pitch angle which is rotated around the pitch axis is defined as zero when the airframe is horizontal.
- λ: The travel angle which is rotated around the travel axis is defined as zero at the sensor's initial angle value about travel.

The unit of these angles are radian, and input voltage about two motors are defined as V_f[V], V_b[V] respectively.

2.1 Kinematic Equations

The experimental helicopter are added mainly to the front motor, the back motor, and the counterweight. Therefore, kinematic equations are obtained by using these three mass point. The kinematic equations are represented by Euler-Lagrange as follows.

$$\begin{cases} p_1\ddot{\epsilon}(t) + p_8 - p_{10} = (V_f(t) + V_b(t))K_t L_a \\ p_3\ddot{\rho}(t) + p_6\ddot{\epsilon}(t) + p_9 = (V_f(t) - V_b(t))K_t L_h \\ (p_2 + p_4)\ddot{\lambda}(t) = -(V_f(t) + V_b(t))K_t L_a \rho(t) \end{cases} \quad (1)$$

$$\begin{aligned} &p_1 = M_f L_a^2 + M_b L_a^2 + M_w L_w^2,\ p_2 = M_w L_w^2, \\ &p_3 = M_f L_h^2 + M_b L_h^2,\ p_4 = M_f L_a^2 + M_b L_a^2, \\ &p_6 = M_f L_a L_h - M_b L_a L_h, \\ &p_8 = (M_f + M_b) g L_a, \\ &p_9 = (M_f - M_b) g L_h,\ p_{10} = M_w g L_w \end{aligned}$$

Here, let input voltage for hovering be u_{f0}, u_{b0}, and assume that $u_{f0} = u_{b0}$. The input voltage u_{f0}, u_{b0} are given by Eq.(2).

$$u_{f0} = u_{b0} = \frac{1}{2K_t L_a}(M_f L_a + M_b L_a - M_w L_w)g \quad (2)$$

In addition, let input be $u(t) = [u_f(t),\ u_b(t)]^T$, and Eq.(3) are obtained.

$$\begin{cases} p_1\ddot{\epsilon}(t) = (u_f(t) + u_b(t))K_t L_a \\ p_3\ddot{\rho}(t) + p_6\ddot{\epsilon}(t) = (u_f(t) - u_b(t))K_t L_h \\ (p_2 + p_4)\ddot{\lambda}(t) = -(u_{f0} + u_{b0})K_t L_a \rho(t) \end{cases} \quad (3)$$

The state space representation of 3DoF helicopter is given by Eq.(4).

$$x(t) = [\epsilon\ \ \rho\ \ \lambda\ \ \dot{\epsilon}\ \ \dot{\rho}\ \ \dot{\lambda}]^T$$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (4)$$

$$A = E^{-1}A_0,\ B = E^{-1}B_0$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & e_3 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ b_1 & b_1 \\ b_2 & -b_2 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} e_1 &= (M_f + M_b)L_a^2 + M_w L_w^2 \\ e_2 &= (M_f + M_b)L_h^2 \\ e_3 &= (M_f + M_b)L_a^2 + M_w L_w^2 \end{aligned} \quad (5)$$

$$\begin{aligned} a_1 &= -(u_{f0} + u_{b0})K_t L_a \\ b_1 &= K_t L_a,\ b_2 = K_t L_h \end{aligned} \quad (6)$$

3 Control Design

In this section, controller synthesis and linear matrix inequalities (LMI) are described. Furthermore, disturbance observer based on full order observer is designed in this chapter's section.

3.1 Extended System

In the target values we desired, extended system is represented in order to follow without deviation. Let $e_\epsilon(t), e_\lambda(t)$ are deviations, g_ϵ, g_λ are integral values of these deviations (interval[0, t]). The extended system representation is given as follows.

$$x_e(t) = [g_\epsilon\ \ g_\lambda\ \ \epsilon\ \ \rho\ \ \lambda\ \ \dot{\epsilon}\ \ \dot{\rho}\ \ \dot{\lambda}]^T$$

$$\dot{x}_e(t) = A_e x(t) + B_e u(t) \quad (7)$$

$$A_e = \begin{bmatrix} O_{2\times2} & -C \\ O_{6\times2} & A \end{bmatrix},\ B_e = \begin{bmatrix} O_{2\times2} \\ B \end{bmatrix}$$

3.2 LQ Control Design

Let state space representation be considered as follows.

$$\begin{cases} \dot{x} = Ax(t) + Bu(t), \\ y = Cx(t) \end{cases} \quad (8)$$

The evaluation function J is defined by Eq.(9).

$$J = \int_0^\infty (x(t)^T Q x(t) + u(t)^T R u(t))dt \quad (9)$$

Here Q is a weight matrix for state variables, and R is a weight matrix for inputs. The weight matrices Q, R satisfies the following equation.

$$\begin{cases} Q = Q^T \succeq 0 \\ R = R^T \succ 0 \end{cases} \quad (10)$$

The state feedback controller $u(t) = Kx(t)$ that minimizes the evaluation function J is derived, and LMI conditions are given as follows.

minimize: γ

subject to :

$$X > 0$$

$$\begin{bmatrix} Z & I \\ I & X \end{bmatrix} > 0$$

$$trace(Z) < \gamma$$

$$\begin{bmatrix} -He\{AX + BF\} & (Q_h X)^T & (F)^T \\ Q_h X & I & O \\ F & O & -R^{-1} \end{bmatrix} < 0$$

3.3 Disturbance Observer

Disturbance observer estimates disturbance invading in the system as one of the states. Therefore, disturbance observer is based on full order observer, and can obtain observer gain. In this study, wind is assumed disturbance, and disturbance observer is designed. In addition, the disturbance is dealt as wind blowing from top to bottom. Here let disturbance w_d[Nm] is state variables. The extended system for disturbance observer is represented as follows.

$$x_d(t) = [x(t) \;\; w_d(t)]^T$$

$$\begin{cases} \dot{x}_d(t) = A_d x_d(t) + B_d u(t) \\ y(t) = C_d x_d(t) \end{cases} \tag{11}$$

$$A_d = \begin{bmatrix} A & D \\ O_{1\times 6} & O_{1\times 1} \end{bmatrix}, \; B_d = \begin{bmatrix} B \\ O_{1\times 2} \end{bmatrix}$$

$$C_d = \begin{bmatrix} I_{3\times 3} & O_{3\times 4} \end{bmatrix}, \; D = E^{-1} \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}^T$$

The observer system is given by Eq.(13), and $\eta(t)$ is the deviation about outputs, G is the observer gain.

$$x_d(t) = [x(t) \;\; w_d(t)]^T \tag{12}$$

$$\begin{cases} \dot{\hat{x}}_d(t) = A_d \hat{x}(t) + B_d u(t) - G\eta(t), \\ y(t) = C_d \hat{x}(t), \; \eta(t) = \tilde{y}(y) - y(t) \end{cases} \tag{13}$$

The estimated disturbance is converted into inputs, and feedbacked. The converted inputs u_d is as follows.

$$u_d(t) = \begin{bmatrix} \frac{1}{(M_f+M_b)L_a^2 + M_w L_w^2} \\ \frac{1}{(M_f+M_b)L_a^2 + M_w L_w^2} \end{bmatrix} w_d(t) \tag{14}$$

The schematic diagram of the disturbance observer is shown as Fig.3.3.

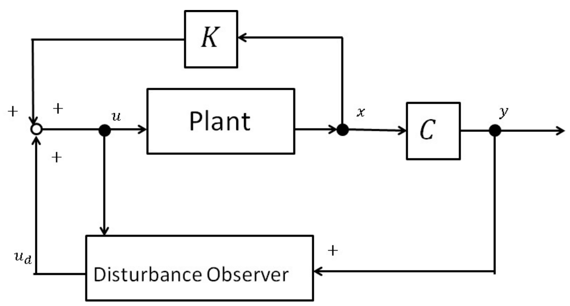

Figure 2 Schematic diagram of disturbance observer

4 Simulation

In this simulation, wind disturbance blowing from top to bottom is added when the helicopter is hovering, and the value of the disturbance is about 0.647Nm. Fig.3 shows the estimated disturbance. The simulation result of the elevation with disturbance compensation and without compensation are shown in Fig.4. In these result, the wind disturbance is estimated, and the effectiveness of the disturbance compensation is verified.

5 Experiment

The wind disturbance blowing from top to bottom is added when the helicopter is hovering, and value of the disturbance is about 0.647Nm. In the experiment, a burden of clay is used instead of the wind disturbance, and the mass of the burden is about 100g. Fig.5 shows the experiment result about the estimated disturbance, and the result of the elevation with disturbance compensation and without compensation are shown in Fig.6. The voltage about the front motor is shown in Fig.7. The

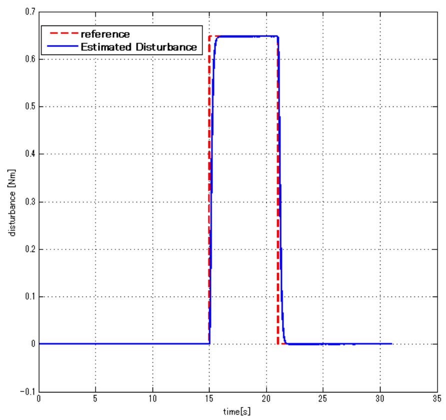

Figure 3 estimated disturbance

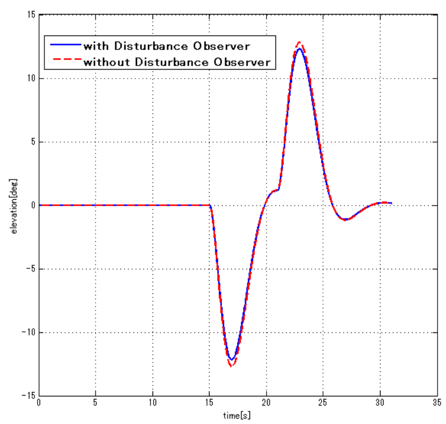

Figure 4 elevation angle

result of the elevation using observer about simulation and experiment are shown in Fig.8.

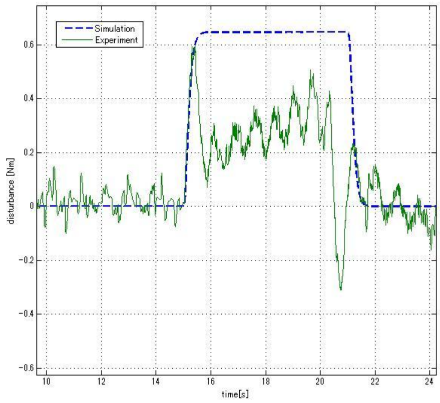

Figure 5 estimated disturbance

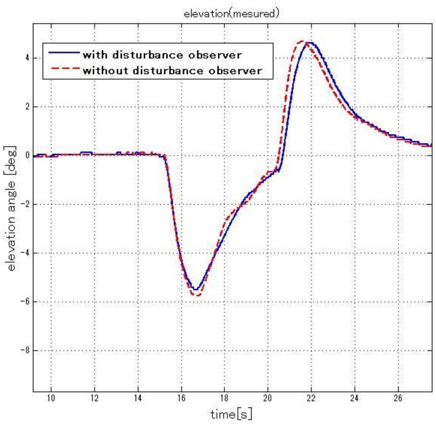

Figure 6 elevation angle

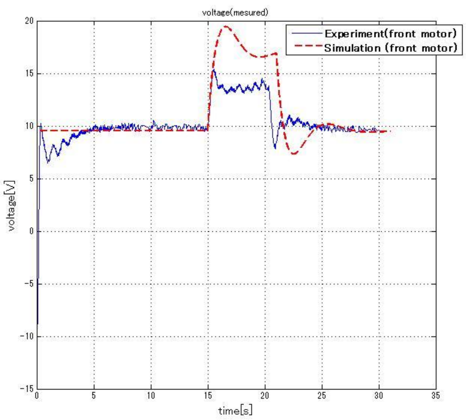

Figure 7 voltage (the front motor)

From the experimental results, the estimation of the disturbance and the effectiveness of the disturbance compensation are verified to some extent.

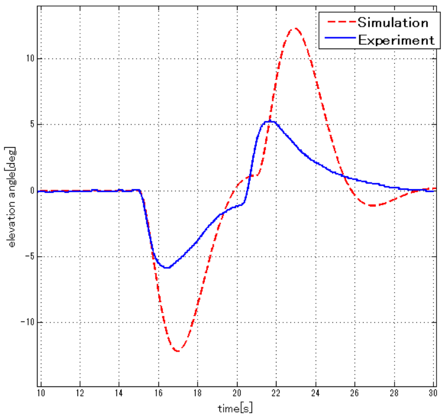

Figure 8 elevation angle (simulation and experiment)

However, the experimental results are better than the results of the simulation. The simulation models or characteristic of the experimental device (the motors) are considered to the cause.

6 Conclusion

In this study, the mathematical model of the 3DoF helicopter using Euler-Lagrange and LQ controller are designed. Furthermore, the wind disturbance observer to compensate disturbance is proposed. The estimation of the disturbance using disturbance observer is illustrated by simulations and experiments. The effectiveness of the disturbance compensation is examined by simulations and experiments. However, it is necessary to improve a part of the simulation models as my after works.

References

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