Robust Control for Anti-lock Braking System Considering Dead Zone

M2015SC018 Ryouhei UKAI
Supervisor:Isao TAKAMI

Abstract

In this study, a method is proposed to guarantee the robust stability of Anti-lock Braking System (ABS) considering the dead zone. The objective of ABS is to control wheel slip to maximize the friction coefficient between the wheel and the road for any given road surface while the car is controllable. The ABS dynamics has strong nonlinearity, and it depends on uncertain parameters. These are constantly changing car speed and variations of the road surface friction such as the wet, dry and snow road. In addition, the dead zone appears in the relationship between the brake pad and the brake disc. These uncertain parameters and the dead zone have a bad influence on control. The robust stability for those uncertain parameters are guaranteed by using the matrix polytopic representation. The dead zone is dealt with a function to cancel the nonlinearity of the dead zone. Finally, the effectiveness of the proposed method is verified by simulations and experiments.

1 Introduction

ABS is an abbreviation for Anti-lock Braking System. ABS is an equipment to prevent the locking of the wheels in braking operation when sudden braking. By preventing wheel lock, it is able to prevent the following problems. From wheel slip, worsening of brake and unstableness of car body posture happen. Also, the maneuverability is lost and can't go around the curve even if the handle is turned. Namely, the objective of ABS is to ensure the braking force, car body posture, and maneuverability. However, there exist uncertain factors in controlling ABS. These are constantly changing car speed and variations of the road surface friction such as the wet, dry and snow road. These uncertain characteristics have a bad influence on control of ABS. The ABS dynamics depends on the slip rate and road surface friction. Also, this slip rate depends on the wheel speed and the car speed. So, the ABS dynamics depends on the car speed and uncertain characteristics such as the road surface friction. From this, the main difficulty of designing ABS control is how to deal with the strong nonlinearity and uncertain characteristics. For such difficulty, many research exist such as nonlinear PID-type control[1] and Fuzzy control[2] approaches. On the other hand, model based approaches are presented. In several papers, Sliding mode control[3] and Gain scheduling control[4] for ABS are applied. In this study, the nonlinear system is linearized[4] by using Taylor expansion around the equilibrium point (λ^*, τ_1^*). Here, λ^* is the reference slip rate, and τ_1^* is the equilibrium braking torque to keep λ^*. From relation of the cornering force and the friction coefficient, the reference slip rate is 0.2[5]. A design method is proposed to guarantee the robust stability for uncertain parameters such as the road surface friction and the car speed. In addition, the dead zone appears in the relationship between the brake pad and the brake disc. This dead zone have a bad influence on control[6][7]. So, a method is proposed to deal with the dead zone using a function to cancel nonlinearity of the dead zone. The matrix polytopic representation for the system with these parameters are proposed. The problem is formulated by solving a finite set of linear matrix inequalities (LMIs). Finally, the effectiveness of the proposed method is verified by simulations and experiments.

2 Control Target

The model of the simplified ABS experimental device used in this study is shown in Fig.1. The control target is the ABS experiment machine which is the quarter model of the real wheel made by INTECO company. The upper wheel simulates the car wheel, and the lower wheel simulates the road. The upper wheel has the brake and the lower wheel has the motor. The motor of the lower wheel turns to appointed speed and stops. Then, the brake of the upper wheel works. The objective of this study is to control wheel slip to maximize the friction coefficient between the wheel and the road for any given road surface. Here, the value of slip rate that maximize the friction coefficient between the wheel and the road for any given road surface is 0.2[5].

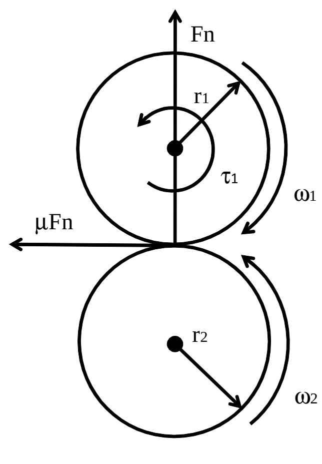

Figure 1 The simplified ABS experimental device

3 Modeling

The physical constants and variables of this study are shown in Table.1. The dynamical equations of the rotational motion of the upper wheel and the lower wheel are shown as Eq.(1) and Eq.(2).

$$J_1\dot{\omega}_1 = F_n r_1 \mu(\lambda) - \tau_1 \quad (1)$$

$$J_2\dot{\omega}_2 = -F_n r_2 \mu(\lambda) \quad (2)$$

The slip rate is defined by Eq.(3) as the function of the car velocity and the wheel velocity.

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} \quad (3)$$

Table 1 The physical constants and variables

Radius of the upper wheel	r_1	[m]
Radius of the lower wheel	r_2	[m]
Moment of inertia of the upper wheel	J_1	[kgm^2]
Moment of inertia of the lower wheel	J_2	[kgm^2]
Vertical force	F_n	[N]
Angular velocity of the upper wheel	ω_1	[rad/s]
Angular velocity of the lower wheel	ω_2	[rad/s]
Braking torque	τ_1	[Nm]
Friction coefficient between wheels	μ	[–]
Slip rate	λ	[–]
Duty rate	u_d	[–]

The following equation is obtained from Eq.(3).

$$\dot{\lambda} = -\frac{r_1}{r_2\omega_2}\dot{\omega}_1 + \frac{r_1\omega_1}{r_2\omega_2^2}\dot{\omega}_2 \tag{4}$$

The dynamical equation of the slip rate is derived from Eq.(1), Eq.(2), Eq.(3) and Eq.(4).

$$\dot{\lambda} = -\frac{1}{\omega_2}\left(\frac{{r_1}^2F_n\mu(\lambda)}{r_2J_1} + \frac{r_2F_n\mu(\lambda)(1-\lambda)}{J_2}\right) + \frac{1}{\omega_2}\frac{r_1}{r_2J_1}\tau_1 \tag{5}$$

The road friction coefficient is defined by Eq.(6)[8] to consider variations of the road surface friction such as the wet road, the dry road and the snow road.

$$\mu(\lambda) = \alpha \arctan(52\lambda) \tag{6}$$

Here, α is a varying parameter by the road conditions. The relationship between α and the road conditions are shown in Table.2 and Fig.2.

Table 2 The relationship between α and the road conditions

Road condition	α
Dry road	0.437
Wet road	0.155
Snow road	0.070

Eq.(7) is obtained from Eq.(5) and Eq.(6).

$$\dot{\lambda} = -\frac{1}{\omega_2}f(\lambda) + \frac{1}{\omega_2}g\tau_1 \tag{7}$$

Here, $f(\lambda)$ and g are as follows.

$$f(\lambda) = \frac{{r_1}^2F_n(\alpha\arctan(52\lambda))}{r_2J_1} + \frac{r_2F_n(\alpha\arctan(52\lambda))(1-\lambda)}{J_2} \tag{8}$$

$$g = \frac{r_1}{r_2J_1} \tag{9}$$

The behavior around equilibrium point (λ^*, τ_1^*) is considered. Eq.(10) is obtained by substituting the equilibrium point in Eq.(7).

$$0 = -\frac{1}{\omega_2}f(\lambda^*) + \frac{1}{\omega_2}g\tau_1^* \tag{10}$$

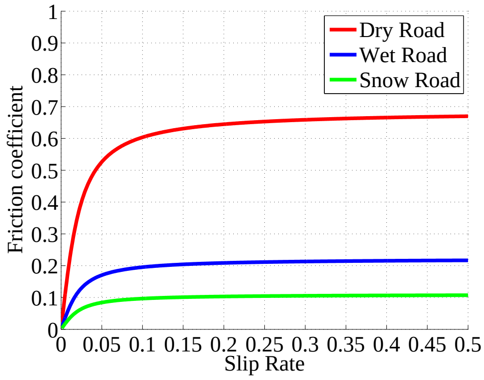

Figure 2 The relationship between α and the road conditions

Here, λ^* is the reference slip rate, and τ_1^* is the equilibrium braking torque to keep the reference slip rate. The equilibrium braking torque τ_1^* is derived from Eq.(10).

$$\tau_1^* = -\frac{f(\lambda^*)}{g} \tag{11}$$

Using Taylor expansion around the equilibrium point (λ^*, τ_1^*), this nonlinear model is linearized as follows [4].

$$\dot{\lambda} \simeq \dot{\lambda}(\lambda^*, \tau_1^*) = \frac{\partial\dot{\lambda}}{\partial\lambda}|_{\lambda=\lambda^*}(\lambda-\lambda^*) + \frac{\partial\dot{\lambda}}{\partial\tau_1}|_{\tau_1=\tau_1^*}(\tau_1-\tau_1^*) \tag{12}$$

The linearized equation is derived from Eq.(11) and Eq.(12).

$$\dot{\lambda} = \frac{1}{\omega_2}(\alpha C_1(\lambda-\lambda^*) + C_2(\tau_1-{\tau_1}^*)) \tag{13}$$

Let β be $\beta = \frac{1}{\omega_2}$, Eq.(13) is rewritten as follows.

$$\dot{\lambda} = \alpha\beta C_1(\lambda-\lambda^*) + \beta C_2(\tau_1-{\tau_1}^*) \tag{14}$$

Here, C_1,C_2 are constants.

3.1 Dead Zone

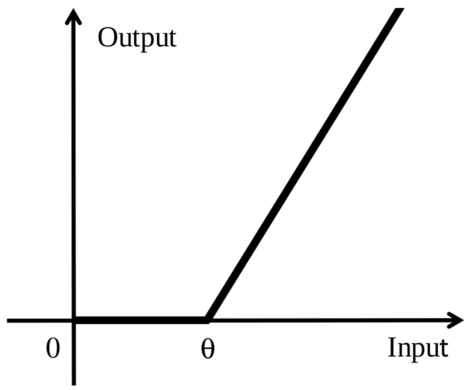

Figure 3 The dead zone function

The dead zone appears in the relationship between the brake pad and the brake disc. The dead zone function is shown in Fig.3. The dead zone have a bad influence on control. The dead zone is dealt with a function to cancel the nonlinearity of the dead zone. The relationship between braking torque τ_1 and duty rate u_d is shown as Eq.(15).

$$\tau_1(u_d) = \begin{cases} 0 & (0 \leqq u_d \leqq \theta) \\ a(u_d - \theta) & (u_d \geqq \theta) \end{cases} \quad (15)$$

The new input is defined as follows.

$$\tilde{u}_d = u_d + \theta \quad (16)$$

Here, θ is obtained from Eq.(1), (2) and Eq.(15).

$$\theta = u_d - \frac{1}{a}\left(\frac{r_1}{r_2} J_2 \dot{\omega}_2 + J\dot{\omega}_1\right) \quad (17)$$

The characteristic of the dead zone disappears.

$$\tau_1(\tilde{u}_d) = \begin{cases} 0 & (\tilde{u}_d = 0) \\ a\tilde{u}_d & (\tilde{u}_d \geqq 0) \end{cases} \quad (18)$$

4 Controller Synthesis

4.1 State Space Representation

In this section, state space representation is derived from Eq.(14). In order to track output of the system to optimal value without error, an integrator added to the state variable. The state variable $x(t)$ and input $u(t)$ are defined as follows.

$$\begin{aligned} x(t) &= [x_1(t)\ x_2(t)]^T \\ &= [\int (\lambda - \lambda^*)dt\ (\lambda - \lambda^*)]^T \quad (19) \\ u(t) &= a\tilde{u}_d - {\tau_1}^* \quad (20) \end{aligned}$$

Then the state space representation is obtained as follows.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (21)$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & \alpha\beta C_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \beta C_2 \end{bmatrix} \quad (22)$$

4.2 Polytopic Representation

The polytopic representation is used to guarantee the robust stability for the uncertain parameters such as the road surface friction and the car velocity. The ranges of α,β are assumed as follows.

$$\alpha \in [\alpha_{min}, \alpha_{max}] \quad (23)$$

$$\beta \in [\beta_{min}, \beta_{max}] \quad (24)$$

Let A_1, A_2, A_3 and A_4 be the vertex matrices for the variation range of matrix A.

$$\begin{aligned} A_1 = A(\alpha_{min}, \beta_{min}), A_2 = A(\alpha_{min}, \beta_{max}) \\ A_3 = A(\alpha_{max}, \beta_{min}), A_4 = A(\alpha_{max}, \beta_{max}) \quad (25) \end{aligned}$$

4.3 LQ Control Design

In this study, LQ controller is designed based on LMI. To derives a stabilizing state feedback controller $u(t) = Kx(t)$, the cost function J is defined as follows.

$$J = \int_0^\infty (x(t)^T Qx(t) + u(t)^T Ru(t))dt \quad (26)$$

Here, $Q \geqq 0$, and $R > 0$ are weight matrices for state variables, and input. The LMI conditions to minimize the cost function and to satisfy the stability conditions are shown as follows.

Lemma 1 :
If there exist X and Y satisfy following LMI conditions, the system is stabilized by the state feedback $u(t) = Kx(t) = YX^{-1}x(t)$. And, by minimizing γ, the cost function J is also minimized.

$$minimize\ \gamma,\ subject\ to$$

$$\begin{bmatrix} He[A_iX + B_iY] & X^T(Q^{\frac{1}{2}})^T & Y^T(R^{\frac{1}{2}})^T \\ (Q^{\frac{1}{2}})X & -I & 0 \\ (R^{\frac{1}{2}})Y & 0 & -I \end{bmatrix} < 0 (27)$$

$$(i = 1, 2, 3, 4)$$

$$\begin{bmatrix} W & I \\ I & X \end{bmatrix} > 0 (28)$$

$$trace(W) < \gamma (29)$$

5 Simulation

In this section, the validity of the proposed method is illustrated by simulations. The proposed method is compared with the model not considering the dead zone by using robust LQ controller. Simulation results of slip rate in any road conditions are shown in Fig.4, 5 and 6.

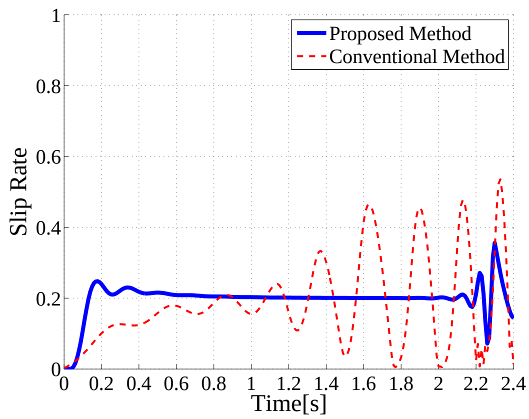

Figure 4 Slip Rate Simulation at the wet road

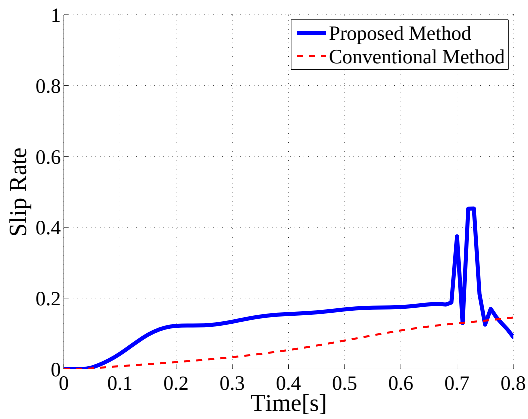

Figure 5 Slip Rate Simulation at the dry road

As can been seen in Fig.4 and Fig.6, the slip rate of proposed method converges more quickly around the optimal value 0.2 and does not oscillate. The simulation result of slip rate at the dry road is not converging around

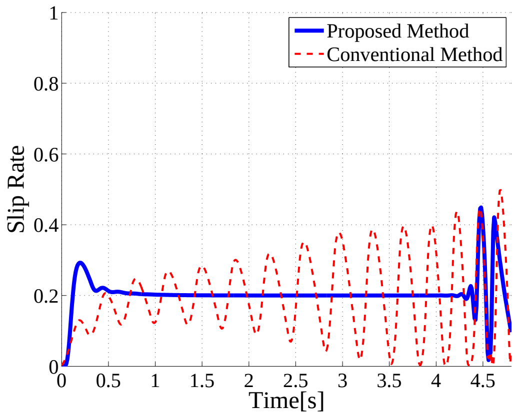

Figure 6 Slip Rate Simulation at the snow road

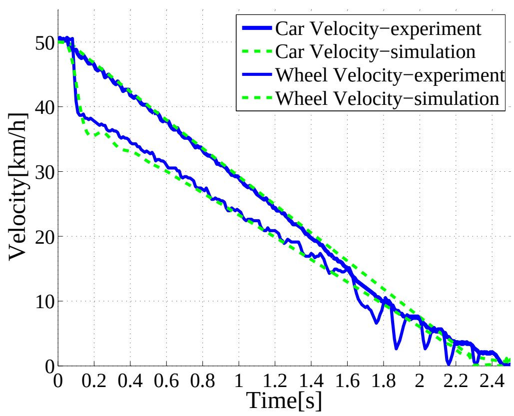

Figure 8 Velocity of Car and Wheel

the optimal value 0.2. However, the car speed stopped at 0.7 seconds. This value is very small. So, it can be said that the function of ABS is satisfied. Therefore, the proposed method is better than the conventional method.

6 Experiment

In this section, the effectiveness of the proposed method is illustrated by experiments. The experimental result and simulation result of slip rate are shown in Fig.7. Their result of car and wheel velocity are shown in Fig.8. The slip rate is controlled around the optimal value 0.2 when the car velocity is 50[km/h] to 18[km/h]. However, the slip rate and the wheel velocity oscillate when the car velocity is 18[km/h] to 0[km/h]. The result means that ABS works well when the car velocity is fast, and the locking of the car wheels can be prevented. By the result, it can be said that the effectiveness of the proposed method is verified.

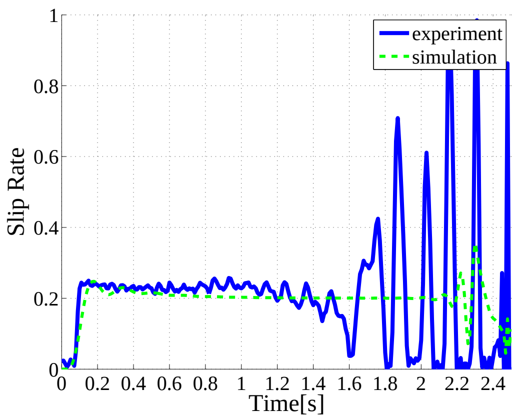

Figure 7 Slip Rate

7 Conclusion

In this study, a method to guarantee the robust stability for uncertain parameters such as the road surface friction and the car speed for anti-lock braking system (ABS) is proposed. Also, a method to deal with the dead zone for ABS is proposed. The validity of the proposed method is illustrated by comparing the model not considering the dead zone in simulations. In any road conditions, the proposed method is better than the conventional method. The effectiveness of the proposed method is illustrated by experiments. The slip rate is controlled around the optimal value 0.2 when the car velocity is 50[km/h] to 18[km/h]. The result means that ABS works well when the car velocity is fast, and the locking of the car wheels can be prevented. Furthermore, the effectiveness of the proposed method is verified.

References

[1] Fangjun Jiang and Zhiqiang Gao." An Application of Nonlinear PID Control to a Class of Truck ABS Problems." IEEE Conference on Decision and Control. Vol. 1, 1998.

[2] Jeffery R.Layne and Kevin M Passino." Fuzzy Learning Control for Antiskid Braking Systems." IEEE Transactions on Control Systems Technology 1.2 ,p122-129, 1993.

[3] Seunghwan Baek, Jeonghoon Song and Duksun Yun." Application of a Sliding Mode Control to Anti-lock Brake System." International Conference on Control, Automation and Systems, 2008.

[4] Idar Petersen, Tor A. Johansen, Jenis Kalkkuhl and Jens Ldemann." Wheel Slip Control in ABS Brakes Using Gain Scheduled Constrained LQR." Control Conference (ECC), European. IEEE, 2001.

[5] Makoto Yokoyama, Yoshiaki Iwata, Shinji Katayose, Masamichi Imamura and Makoto Nibe." Antilock Braking System Using Sliding Mode Control." The Japan Society of Mechanical Engineers, No.96-0678, 1997 (in Japanese).

[6] Leonardo Acho, Faycal Ikhouane, and Gisela Pujol." Robust Control Design for Mechanisms with Backlash."Journal of Control Engineering and Technology (JCET) 3.4 p175-180, 2013.

[7] Nizar J. Ahmad, Mahmud J. Alnaser ,Ebraheem Sultan and Khuloud A. Alhendi." Robust Adaptive Control for a Class of Systems with Deadzone Nonlinearity." Intelligent Control and Automation 6.01, 2015.

[8] Hiroyuki Inoue, Kenji Mori, Shogo Miyasaka, Takanori Fukao and Norihilo Adachi." Hybrid Brake Controller of a Vehicle for Velocity and Slip Servo." The Japan Society of Mechanical Engineers, No.03-8, 2003 (in Japanese).