Robust LQ Control for Anti-lock Braking System with Tracking Velocity Model

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Abstract

In this paper, the tracking velocity model for anti-lock braking system (ABS) is proposed. Furthermore, the robust stability for car velocity and the coefficient of friction between tire and road is guaranteed by using polytopic representation. Then the robust controller is obtained by solving a finite set of Linear Matrix Inequalities (LMIs). Finally, the effectiveness of the proposed method is illustrated by simulations and experiments.

1 Introduction

Anti-lock Braking System (ABS) prevents cars from slipping by locks of wheels in brake operations. It is well known that friction coefficient for lateral force and longitudinal force between tire and road are high enough when slip rate is nearly around 0.2[1]. By keeping the slip rate 0.2, skidding can be prevented. Since the slip rate is represented as the function of car velocity and wheel velocity, the ABS dynamics depends on these velocities and the coefficient of friction.

Many contributions for ABS can be found in the literature. Proportional integral derivative (PID) type approach[2], nonlinear PID type approach[3] and fuzzy control[4] are reported. On the other hand, model based approach are presented. Sliding mode control is applied[5]. However, this algorithm is quite complicated. On the other hand, linear control theory enables to design easily a feedback law by describing a plant as linear state equation. The performance of the designed controller is evaluated easily. In particular, LQ control can yield better performance with smaller input by minimizing a cost function. Hence applying LQ control to ABS is effective. Here, I consider slip rate follow 0.2 from 0. When car velocity is 50[km/h], wheel velocity follows 40[km/h] from 50[km/h], that means, deceleration is 10[km/h]. When car velocity is 10[km/h], wheel velocity follows 8[km/h] from 10[km/h], that means, deceleration is 2[km/h]. Therefore, the brake torque following reference slip rate 0.2 is varied by initial value of car velocity. However, in the model based on slip rate, reference is constant in any car velocity. Therefore, there is a difference of control performance by initial value of car velocity because control input is not varied by car velocity.

In this study, the tracking velocity model is proposed. The tracking velocity model controls slip rate by tracking wheel velocity to reference wheel velocity. In the tracking velocity model, reference wheel velocity is varied by car velocity. Therefore, in high velocity, control input is large. In low velocity, control input is small. Therefore, the tracking velocity model is expected to improve control performance because control input is varied by car velocity.

Furthermore, the dynamics of ABS depends on the uncertain parameters, which are car velocity and the coefficient of friction between tire and road. The robust stability is required to design ABS with respect to these uncertain parameters. The robust stability for the system with uncertain parameters is guaranteed theoretically by using matrix polytopic representation. Then the robust LQ controller is obtained by solving a finite set of Linear Matrix Inequalities (LMIs). Finally, the effectiveness of the proposed method is illustrated by simulations and experiments.

2 Control Target and Modeling

2.1 Equation of Motion

The model of the simplified ABS experimental device used in this study is shown in Fig.1. It is one wheel model that is 1/4 scale of the real vehicle. The upper wheel simulates the car wheel, and the lower wheel simulates the road. A control law is designed to keep the optimal slip rate 0.2 by operating the braking torque applied to the upper wheel. Table 1 shows physical constants and variables used in this study.

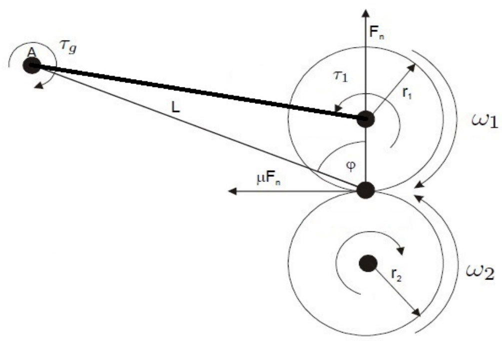


Fig. 1 Schematic Diagram of ABS Experimental Device

Table 1 Physical Parameters

Parameter	Symbol	Unit
Angular velocity of the upper wheel	ω_1	[rad/s]
Angular velocity of the lower wheel	ω_2	[rad/s]
Velocity of the upper wheel	V_r	[m/s]
Velocity of the lower wheel	V	[m/s]
Radius of the upper wheel	r_1	[m]
Radius of the lower wheel	r_2	[m]
Moment of inertia of the upper wheel	J_1	[kgm^2]
Moment of inertia of the lower wheel	J_2	[kgm^2]
Normal force	F_n	[Nm]
Braking torque	τ_b	[Nm]
Slip rate	λ	
Coefficient of friction between wheels	$\mu(\lambda)$	

The dynamical equations of the rotational motion of the upper and lower wheels are shown as Eq.(1) and (2).

$$J_1\dot{\omega}_1 = F_n r_1 \mu(\lambda) - \tau_b \qquad (1)$$

$$J_2\dot{\omega}_2 = -F_n r_2 \mu(\lambda) \qquad (2)$$

Slip rate is defined as Eq.(3) by car velocity and wheel velocity.

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} = \frac{V - V_r}{V} \quad (3)$$

Road friction coefficient $\mu(\lambda)$ is given as Eq.(4)[6].

$$\mu(\lambda) = \alpha \tan^{-1}(80\lambda) \quad (4)$$

Here, α is a varying parameter by road conditions. The relationship between α and road conditions are shown in Table2 and Fig.2.

Table 2 The relationship between α and road conditions

α	road conditon
0.45	Dry road
0.2	Wet road
0.065	Ice road

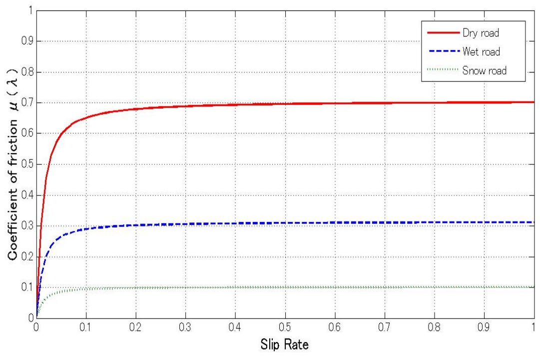


Fig. 2 The relationship between α and road conditions

2.2 Tracking Velocity Model

The proposed method controls slip rate by tacking wheel velocity to reference wheel velocity. Therefore, consider tracking error as Eq.(5).

$$z = V_r - V_r^* \quad (5)$$

Here, reference wheel velocity is defined as follows from Eq.(3).

$$V_r^* = (1 - \lambda^*)V, \lambda^* = 0.2 \quad (6)$$

Eq.(7) is derived from Eq.(1), (2), (3) and (5).

$$\dot{z} = \left(\frac{r_1^2}{J_1} + \frac{r_2^2}{J_2}(1 - \lambda^*)\right)F_n\mu(\lambda) - \frac{r_1}{J_1}\tau_b \quad (7)$$

To apply linear control theory, nonliner term in Eq.(7) is linearized around equilibrium point (V_r^*, τ_b^*). Here, τ_b^* is the equilibrium braking torque to keep V_r^*. Using Taylor expansion around the equilibrium point, this nonlinear model is linearized as follows.

$$\dot{z} = -\frac{\alpha}{V}c_1(V_r - V_r^*) - \frac{r_1}{J_1}(\tau_b - \tau_b^*) \quad (8)$$

Here, c_1 is a constant.

2.3 Extended System

In order to track the output of the system to the optimal value without error, an integrator is added to the state variable. Let the state variable be $x(t) = [V_r - V_r^* \quad \int(V_r - V_r^*)dt]^T$ and control input be $u(t) = \tau_b - \tau_b^*$. Then the state space representation is obtained as Eq.(9).

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$
$$A = \begin{bmatrix} -\frac{\alpha}{V}c_1 & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} -\frac{r_1}{J_1} \\ 0 \end{bmatrix}$$

3 Controller Synthsis

3.1 Polytopic Representation

The polytopic representation is used to guarantee the robustness for car velocity and the coefficient of friction. For Eq.(9), let β be $\beta=\frac{1}{V}$. Lower bound and upper bound of uncertain parameters are given as follows.

$$\alpha \in [\alpha_{min}, \alpha_{max}] \quad (10)$$
$$\beta \in [\beta_{min}, \beta_{max}] \quad (11)$$

Let A_1, A_2, A_3 and A_4 be the vertex matrices for the variation range of matrix A.

$$A_1 = A(\alpha_{max}, \beta_{max}), A_2 = A(\alpha_{min}, \beta_{min})$$
$$A_3 = A(\alpha_{max}, \beta_{min}), A_4 = A(\alpha_{min}, \beta_{max})$$

3.2 LQ Control Design

To derive a stabilizing state feedback $u(t) = Kx(t)$, consider to minimize the following cost function.

$$J = \int_0^\infty (x(t)^T Qx(t) + u(t)^T Ru(t))dt \quad (12)$$

Here $Q \geq 0$ is a weight matrix for state variables, and $R > 0$ is a weight matrix for inputs.

The LMI conditions to minimize the cost function and to satisfy the stability conditions are shown as follows.

Lemma 1 : *If there exist X and Y satisfying the follow LMIs, the system is stabilized by $u(t) = Kx(t) = YX^{-1}x(t)$.*
minimize :γ
subject to $X \succ 0$

$$\begin{bmatrix} He[A_iX + BY] & X(Q^{\frac{1}{2}})^T & Y^T(R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}}X & -I & 0 \\ R^{\frac{1}{2}}Y & 0 & -I \end{bmatrix} \prec 0 \quad (13)$$
$$(i = 1, 2, 3, 4)$$
$$\begin{bmatrix} W & I \\ I & X \end{bmatrix} \succ 0 \quad (14)$$
$$trace(W) < \gamma \quad (15)$$

Here,

$$X = P^{-1}, X^{-1} \prec W, Y = KX, J < \gamma$$

4 Simulation

4.1 Validity of Proposed Method

In this section, the validity of the proposed method is illustrated by simulations. In this study, the range of road conditions is assumed as from ice road (α=0.065) to dry road (α=0.45), and the range of the car velocity is assumed as from 10[km/h] to 50[km/h]. Simulation cases are shown in Table.3.

Table 3 Simulation Conditions

	Road Condition	Initial Value of Car Velocity[km/h]
Case1	Dry	50
Case2	Dry	20
Case3	Ice	50
Case4	Ice	20

4.1.1 Simulation at Case1

The simulation result of slip rate at the case1 is shown in Fig.3. The simulation result of car velocity and wheel velocity are shown in Fig.4. As can be seen in these figures, wheel velocity is tracked reference wheel velocity and slip rate is kept around reference slip rate 0.2.

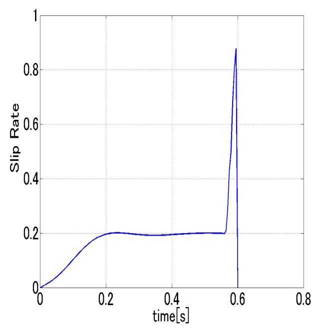


Fig. 3 Case1:Slip Rate

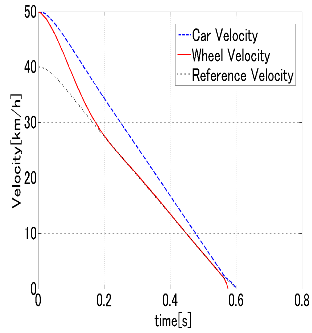


Fig. 4 Case1:Velocity of Car and Wheel

4.1.2 Simulation at Case2

The simulation result of slip rate at the case2 is shown in Fig.5. The simulation result of car velocity and wheel velocity are shown in Fig.6. Slip rate is not kept around 0.2. However, the car stops within 0.4 seconds. Since the car stops at short time, the function of ABS is fulfilled.

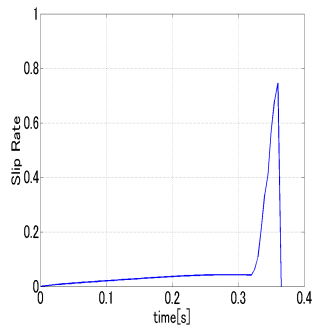


Fig. 5 Case2:Slip Rate

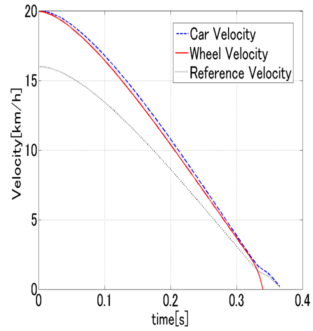


Fig. 6 Case2:Velocity of Car and Wheel

4.1.3 Simulation at Case3

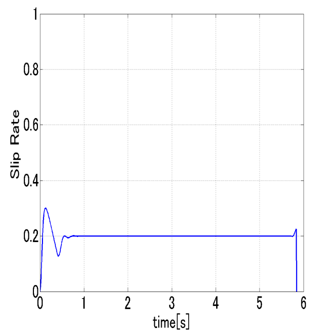


Fig. 7 Case3:Slip Rate

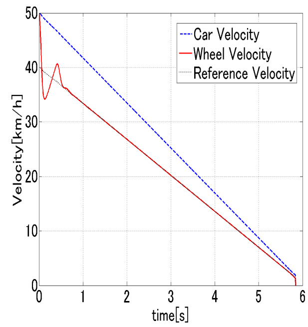


Fig. 8 Case3:Velocity of Car and Wheel

The simulation result of slip rate at the case3 is shown in Fig.7. The simulation result of car velocity and wheel velocity are shown in Fig.8. There exist overshoot of slip rate at first but it is controlled to 0.2 from Fig.7.

4.1.4 Simulation at Case4

The simulation result of slip rate at the case4 is shown in Fig.9. The simulation result of car velocity and wheel velocity are shown in Fig.10. There exist overshoot of slip rate at first but it is controlled to 0.2 as can be seen in Fig.9.

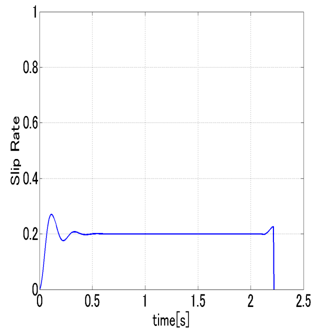


Fig. 9 Case4:Slip Rate

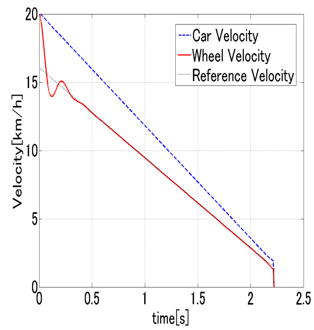


Fig. 10 Case4:Velocity of Car and Wheel

From the simulation results, slip rate is stabilized. The robust stability is guaranteed in the range.

4.2 Effectiveness of Proposed Method

In this section, the effectiveness of the proposed method is illustrated by simulations. The tracking velocity model is compared with the model based on slip rate by using robust LQ controller. Simulation cases are case1,3 and 4. Because the car stops at very short time, Case2 doesn't compare.

4.2.1 Simulation at Case1

The simulation result of slip rate at the case1 is shown in Fig.11. The simulation result of the braking torque at the case1 is shown in Fig.12. As can be seen in Fig.11, both proposed method and conventional method show similar performances.

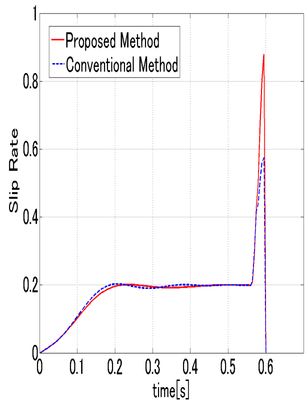


Fig. 11 Case1:Slip Rate

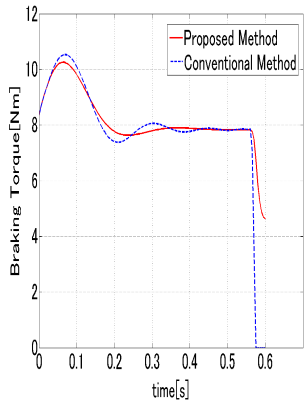
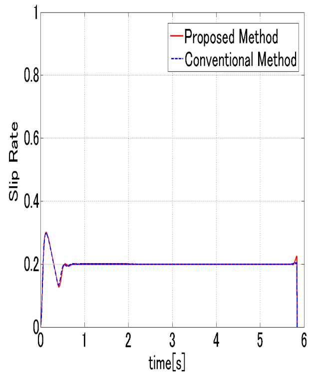


Fig. 12 Case1:Brake Torque

4.2.2 Simulation at Case3

The simulation result of slip rate at the case3 is shown in Fig.13. The simulation result of the braking torque at the case3 is shown in Fig.12. As can be seen in these figures, both proposed method and conventional method show similar performances.



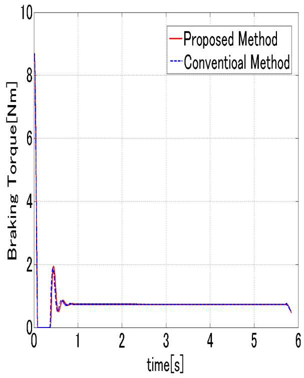
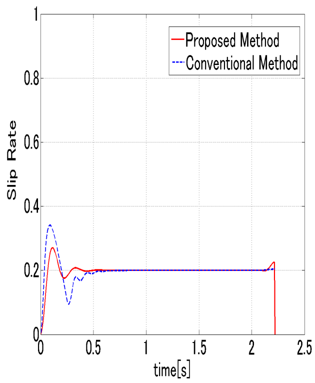


Fig. 13 Case1:Slip Rate   Fig. 14 Case1:Brake Torque

4.2.3 Simulation at Case4

The simulation result of slip rate at the case4 is shown in Fig.15. The simulation result of the braking torque at the case4 is shown in Fig.16. As can be seen in these figures, slip rate of proposed method becomes smaller overshoot and converges more quickly on reference slip rate 0.2 than conventional method.



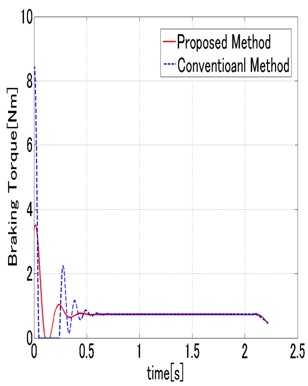


Fig. 15 Case1:Slip Rate   Fig. 16 Case1:Brake Torque

When car velocity is high, braking torque both proposed method and conventional method are similar as can be seen in Fig.14. However, when car velocity is low, braking torque of proposed method is smaller than conventional method as can be seen in Fig.16. From this simulation results, proposed method is varied by car velocity. Therefore, proposed method is better than conventional method.

5 Experiment

In this section, the validity of the proposed method is illustrated by experiments. The simulation and experiment results of slip rate is shown in Fig.17. The simulation and experiment results of car velocity and wheel velocity are shown in Fig.18.

When car velocity is more than 15[km/h](t<1.6[s]), slip rate is controlled around the optimal value 0.2. However, slip rate oscillates when car velocity is less than 15[km/h]. It is concluded that ABS works well at high velocity and the proposed method is useful.

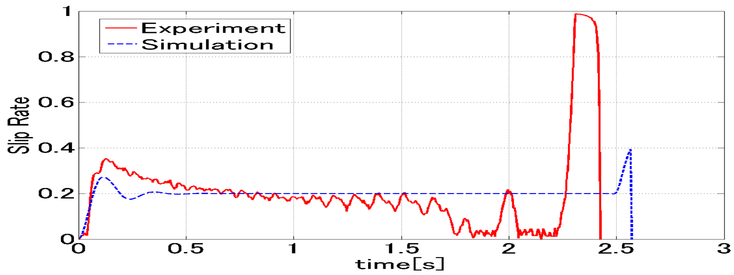


Fig. 17 Slip Rate

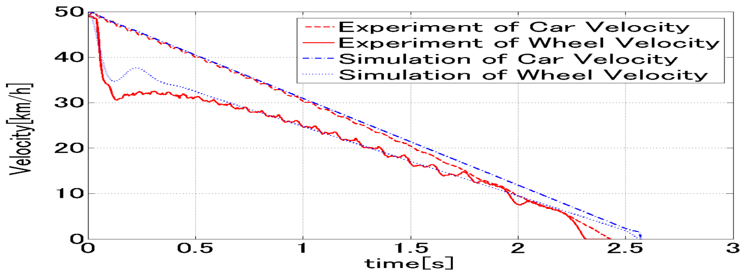


Fig. 18 Velocity of Car and Wheel

6 Conclusion

In this study, the tracking velocity model for anti-lock braking system (ABS) is proposed. Furthermore, a method to guarantee the robust stability for car velocity and the friction coefficient is proposed. The effectiveness of the proposed method is illustrated by comparing the model based on slip rate in simulations. The tracking velocity model improves the control performance. The validity of the proposed method is illustrated by simulations and experiments. Slip rate is kept around the optimal value 0.2 when car velocity is more than 15[km/h] from experiment results. This means ABS works well at high velocity by the proposed method.

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