Partial Gain Scheduling Control of Crane

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Abstract

This paper proposes a method to design partial gain scheduling (GS) controller for a crane. The partial GS controller has scheduled gains and fixed gains of state feedback. Selection of the scheduled gains is determined by pendulum property. From the pendulum property, angular velocity of the load depends on rope length. However, trolley position does not depend strongly on the rope length. Thus the scheduled gains are applied for the swing angle and the angular velocity of the load. The fixed gains are for the other states. To obtain a linear descriptor varying (LPV) system for designing the partial GS controller, redundant descriptor representation is adopted for state space representation of the crane, where the time-varying parameters are the rope length, its velocity and acceleration. The designed controller guarantees the robust stability for time varying parameters. The problem of the controller design can be formulated as solving a finite sets of linear matrix inequality (LMI). The effectiveness of the proposed method is illustrated by simulations.

1 Introduction

Cranes are mechanical system that lift and carry loads to assigned positions. Cranes are not only required to transport the load fast and accurately, but also required to reduce oscillations. However, it is difficult for all operators to fulfill these conditions. In this study, the load of the crane is moved to the horizontal and vertical directions simultaneously. It is reported that decentralized control is effective for cranes [1]. Swing of the load occurs by hoisting rope. It is desirable to design a robust controller considering variation of the rope length because it affects the swing angle of the load. In previous studies [2], robust controllers to suppress an influence of variation of the rope length have been reported. A gain scheduled controller is designed by selecting the rope length as the scheduling parameters [3]. In [4], using linear fractional transformation (LFT) and redundant descriptor representation, an linear varying parameter (LPV) system with not ignoring the rope length’s velocity and acceleration is constructed. In consequence, gain scheduling (GS) controller via parameter dependent Lyapunov function that depends on scheduling parameter is designed. However, the control design of the GS controller is very complicated, it is difficult to implement the controller for crane depending on computer’s performance. Moreover, the control performance of the robust controller deteriorates depending on the range of varying parameter.

In this study, a state feedback partial gain scheduling controller for crane systems is proposed. The partial GS controller has scheduled gains and fixed gains. Selection of the scheduled gains is determined by pendulum property. From the pendulum property, the rope length gives large influence on swing angle of the load because angular velocity of the load depends on rope length. However, the influence of rope length on the trolley position is small. The scheduled gains are given for the swing angle and the angular velocity of the load. The fixed gains are given for the other states. The scheduled gains and the fixed gains are obtained by different method respectively. First, the scheduled gains are derived. The scheduled gains are obtained by designing GS controller from a LPV system based on state variable of only swing angle and angular velocity. Then, the LPV system and a LPV system based on all state variables are equivalent on a property of crane’s dynamics. Next, the fixed gains are derived. The fixed gains are obtained by designing robust LQ controller from the system based on all state variables. The partial GS controller is designed by adding a matrix of the scheduled gains and a matrix of the fixed gains. Then, the matrix of the scheduled gains is transformed to the same size as that of the fixed gains. To obtain their respective LPV systems, redundant descriptor representation is adopted. The designing problem of their respective controllers can be formulated as a finite set of linear matrix inequalities (LMIs). The effectiveness of the proposed method is illustrated by simulations.

2 Modeling

The schematic diagram of the jib crane is shown in Fig. 1.

![Fig. 1 Crane model](image)

The jib crane consists of four parts, a tower, a trolley, a jib, and a load. The jib crane can lift and carry the load simultaneously in this study. Sensors of the crane can measure position of the trolley $\xi$ [m], swing angle of the load $\gamma$ [rad] and the rope length $l$ [m]. These are time-varying parameters. The input is a electric current of a jib motor $I_j$ [A]. The output is the horizontal position of the load $y$ [m] ($y = \xi - l \sin \gamma$). The physical constants of the crane are shown in Table 1.

<table>
<thead>
<tr>
<th>Physical constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of load $m_p$</td>
<td>[kg]</td>
</tr>
<tr>
<td>Mass of trolley $m_t$</td>
<td>[kg]</td>
</tr>
<tr>
<td>Acceleration of gravity $g$</td>
<td>[m/s$^2$]</td>
</tr>
<tr>
<td>Jib motor radius $r_j$</td>
<td>[m]</td>
</tr>
<tr>
<td>Jib motor gear box efficiency $\eta_j$</td>
<td>[-]</td>
</tr>
<tr>
<td>Jib motor gear ratio $K_{j,p}$</td>
<td>[-]</td>
</tr>
<tr>
<td>Jib motor torque constant $K_{j,t}$</td>
<td>[Nm/A]</td>
</tr>
<tr>
<td>Jib motor equivalent moment of inertia $J_{eq}$</td>
<td>[kgm$^2$]</td>
</tr>
</tbody>
</table>

In this study, we make following assumptions, (i) the rope is rigid rod without the mass, (ii) the load is a...
material point, (iii) the load moves horizontally along the jib, (iv) the friction of the trolley can be ignored, (v) the swing angle and its velocity are small enough.

In Euler-Lagrange equations, $\sin \gamma$, $\cos \gamma$, and $\dot{\gamma}$ can be approximated to $\gamma$, $1$, and $0$ by the assumption (v).

Let $m_{1}$ be $m_{p} + m_{t} + J_{p} \frac{K_{s}}{2}$. When an equation of motion with respect to $\xi$ and $\gamma$ is obtained, an equation of motion with only $\gamma$ (i.e., the equation without $\xi$) can be derived as follows:

$$
(m_{1} - m_{p})\ddot{\gamma} = -2(m_{1} - m_{p})\dot{\gamma} - (m_{g} - m_{p})\gamma + K_{i,j} \dot{I}_{j}.
$$

The state variable of the crane system for the scheduled gains is only the swing angle and the angular velocity. However, the state variable of the crane system for the fixed gains is not only those but also the trolley position and its velocity. Thus, two state space representations are obtained for the scheduled gains and the fixed gains. First, the state space representation for the scheduled gain is shown. Let state variable be $\dot{x} = [\gamma \dot{\gamma}]^{T}$ and input be $u_{1} = I_{j}$. The state space representation of the crane is obtained as follows:

$$
\dot{x} = E(l)^{-1}A(\dot{\gamma}, \dot{\gamma})x + E(l)^{-1}Bu_{1} \tag{2}
$$

$$
\dot{E} = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & -E_{1} & 0 \\ 0 & 0 & 1 \\
\end{array} \right], \quad \dot{A} = \left[ \begin{array}{ccc} 0 & 1 & 0 \\ m_{p}l - m_{g} & 2(m_{p} - m_{g}) & 1 \\
\end{array} \right], \quad \dot{B} = \left[ \begin{array}{ccc} 0 & 1 \\ K_{i,j} & 0 \\
\end{array} \right].
$$

Next, the state space representation for the fixed gain is shown. Let generalized coordinate be $q = [\xi \gamma]^{T}$, state variable be $x = [q \dot{q}]^{T}$, output be $y$ and input be $u$.

The state space representation of the crane is obtained as follows:

$$
\dot{x} = E(l)^{-1}A(\dot{\gamma})x + E(l)^{-1}Bu \tag{3}
$$

$$
E = \left[ \begin{array}{ccc} I_{2} & 02x2 \\ O_{2x2} & E_{1} \\
\end{array} \right], \quad E_{1} = \left[ \begin{array}{ccc} m_{1} & -m_{p} \\ 1 & l \\
\end{array} \right], \quad y = C(l)x \tag{4}
$$

$$
A = \left[ \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 0 & 0 \\ m_{p}l & 2m_{p}l & 1 \\
\end{array} \right], \quad B = \left[ \begin{array}{ccc} 0 & 0 \\ K_{i,j} & 0 \\
\end{array} \right], \quad C = [C_{1} \quad O_{1xn}] \\
$$

$$
C_{1} = [1 \quad -l].
$$

An important purpose of controlling crane system is transporting the load to the target without error. To eliminate study error, servo control system is constructed. Let the error and integral of the error be $e$ and $e_{r}$. To make the output $y$ follow a reference $r$, a state variable $x_{h}$ consists of the state variable $x$ and $e_{r}$. The servo control system is as follows:

$$
\dot{x}_{h} = E_{h}(l)^{-1}A_{h}(\dot{\gamma}, \dot{\gamma})x_{h} + E_{h}(l)^{-1}Bu_{h} \tag{5}
$$

$$
E_{h} = \left[ \begin{array}{ccc} E & O_{4x1} \\ O_{1x4} & 1 \\
\end{array} \right], \quad A_{h} = \left[ \begin{array}{ccc} A(l, \dot{\gamma}) & O_{4x1} \\ -C(l) & 0 \\
\end{array} \right], \quad B_{h} = \left[ \begin{array}{ccc} 0 & 1 \\ K_{i,j} & 0 \\
\end{array} \right] \tag{6}
$$

$$
x_{h}(t) = \int_{0}^{t} e(\tau)d\tau, \quad e = r - y \\
\dot{u}(t) = u(t) - u(\infty).
$$

3 Control Designing

In this paper, the controller based on an optimal regulator theory is designed.

3.1 Parameter Box

The time-varying parameters are $l$, $\dot{l}$, and $\ddot{l}$. The parameter box Eq.(7) is defined by upper and lower bounds of time-varying parameters $l$, $\dot{l}$, and $\ddot{l}$.

$$
\Theta = \{[\theta_{1}, \theta_{2}, \theta_{3}] : \theta_{i} \in [\underline{\theta}_{i}, \overline{\theta}_{i}]\} \tag{7}
$$

$$
\theta_{i} = [l, \dot{l}, \ddot{l}] \quad (i = 1, 2, 3)
$$

3.2 Control Designing for Scheduled Gain

3.2.1 Descriptor Representation

The time-varying parameters are included in the matrix $A$ in the state space representation Eq.(2). The coefficient matrix $E(l)^{-1}A$ and $E(l)^{-1}B$ include rational terms of the time-varying parameters. By using redundant descriptor representation [5], the state space representation is rewritten as descriptor representation with only linear terms of the time-varying parameters. Let descriptor variable be $\dot{x}_{d} = [\gamma \dot{\gamma}]^{T}$, Eq.(8) is obtained from Eq.(2).

$$
\dot{E}_{d}\dot{x}_{d} = A_{d}(\theta)\dot{x}_{d} + \dot{B}_{d}u_{1} \tag{8}
$$

$$
\dot{E}_{d} = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\
\end{array} \right], \quad \dot{B}_{d} = \left[ \begin{array}{ccc} 0 & 0 \\ K_{i,j} & 0 \\
\end{array} \right] \\
$$

$$
A_{d} = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ m_{p}\theta_{3} - m_{j}g & 2(m_{p} - m_{j})\theta_{2} & (m_{p} - m_{j})\theta_{1} \\
\end{array} \right].
$$

3.2.2 Stability Conditions

The stability conditions of the descriptor system Eq.(8) is shown. The system is stabilized by state feedback controller $u_{1} = K_{d}(\theta)\dot{x}_{d}$ and feedback gain $K_{d}(\theta) = \dot{Y}_{d}(\theta)\dot{X}_{d}^{-1}$. Considering structure of the matrix $\dot{E}_{d}$, candidates of Lyapunov matrix $\dot{X}_{d}$ and variable matrix $\dot{Y}_{d}(\theta)$ are restricted as Eq.(9).

$$
\dot{X}_{d} = \left[ \begin{array}{ccc} X & O_{2xn} \\ \dot{X}_{d21} & \dot{X}_{d22} \\
\end{array} \right] \tag{9}
$$

$$
\dot{Y}_{d}(\theta) = \dot{Y}_{d0} + \sum_{i=1}^{3} \theta_{i}\dot{Y}_{di} = \left[ \begin{array}{ccc} \dot{Y}(\theta) & 0 \\
\end{array} \right] \tag{10}
$$

[Theorem] [6] The system Eq.(8) is stable by and state feedback controller, if $\dot{X}_{d}$ and $\dot{Y}_{d}(\theta)$ exist satisfying Eq.(10) and (11).

$$
\dot{E}_{d}\dot{X}_{d} = (\dot{E}_{d}\dot{X}_{d})^{T} \geq 0 \tag{10}
$$

$$
\text{He}(\dot{A}_{d}(\theta)\dot{X}_{d} + \dot{B}_{d}\dot{Y}_{d}(\theta)) < 0 \tag{11}
$$

For Eq.(8), an evaluated function $\dot{J}_{e}$ is given by Eq.(12).

$$
\dot{J}_{e} = \int_{0}^{\infty} (\dot{x}_{d}^{T}\dot{Q}_{d} + u_{1}^{T}\dot{R}_{u1})dt \tag{12}
$$

In order to minimize the evaluated function $\dot{J}_{e}$ in
Eq. (12), the minimum $\gamma_1$ that satisfies the following conditions is derived.
\[
\dot{X} > 0
\]
Eq. (26) is rewritten as $x_d = [x_d^T \ y_d^T]^T$. Descriptor equation of the crane is obtained as Eq. (27).
\[
E_d \dot{x}_d = A_d \dot{x}_d + B_d u_2
\]
\[
E_d = \begin{bmatrix} I_5 & O_{2 \times 5} \\ O_{2 \times 5} & O_{2 \times 2} \end{bmatrix}, \quad A_d = \begin{bmatrix} O_{2 \times 2} & I_2 & O_{2 \times 1} & O_{2 \times 2} \\ O_{2 \times 2} & I_2 & O_{2 \times 1} & O_{2 \times 2} \\ C_1 & O_{1 \times 2} & 0 & O_{1 \times 2} \\ A_{h21} & A_{h22} & O_{2 \times 1} & E_1 \end{bmatrix}
\]
\[
B_d = \begin{bmatrix} O_{5 \times 1} \\ 0 \end{bmatrix}
\]
3.3.2 Stability Conditions
The stability conditions of the descriptor system Eq. (27) is shown. The system is stabilized by the state feedback controller $u_2 = K_d x_d$. Considering structure of the matrix $E_d$, candidates of Lyapunov matrix $X_d$ and variable matrix $Y_d$ are restricted as Eq. (28).
\[
X_d = \begin{bmatrix} X \ Y \ O_{5 \times 2} \\ X_{d21} \ X_{d22} \end{bmatrix}, \quad Y_d = [Y \ O_{1 \times 2}]
\]
By Lyapunov’s stability theorem, if there exists $X_d$ satisfying the following matrix inequality, the system is stabilized [6].
\[
E_d X_d = (E_d X_d)^T \geq 0
\]
\[
\{ \text{He}[A_d x_d + B_d u_2] \} + \{C_d x_d + D_d u_2\} < 0
\]
\[
B_{dw} = \begin{bmatrix} B_w \\ O_{2 \times 5} \end{bmatrix}, \quad B_w = I_5, \quad C_d = [W_x \ O_{5 \times 2}], \quad D_d = \begin{bmatrix} O_{7 \times 1} \\ R_f^2 \end{bmatrix}
\]
\[
W_x = \begin{bmatrix} \tilde{Q}^2 \\ 0 \end{bmatrix}
\]
An evaluated function $||J_2||^2$ is given by Eq. (32).
\[
||J_2||^2 = \int_0^\infty z^T z dt
\]
In order to minimize the evaluated function $J_2$ of Eq. (32), the minimum $\gamma_2$ that satisfy the following conditions [4].
\[
E_d x_d = (E_d X_d)^T \geq 0
\]
\[
\text{He}[A_d x_d + B_d u_2] + \{C_d x_d + D_d u_2\}^T \{C_d x_d + D_d u_2\} < 0
\]
\[
B_{dw}^T X_d^{-1} B_w < W
\]
\[
\text{trace}(W) < \gamma_2^2
\]
The problem of designing the controller of the fixed gain is formulated as follows from Eq. (34) and Schur complement:
\[
\text{Lemma 2} : \text{If there exist matrices } X \text{ and } Y \text{ satisfying following LMIs, the descriptor system is stable.}
\]
\[
X > 0 \quad \int \begin{bmatrix} O_{2 \times 2} & I_2 & O_{2 \times 1} & O_{2 \times 1} \\ A_{h21} & A_{h22} & O_{2 \times 1} & 0 \\ C_1 & O_{1 \times 2} & 0 & 0 \\ 0 & 2 m_i + \hat{K}_i \hat{K}_i \end{bmatrix} < 0 \quad \int \begin{bmatrix} W \ B_w \ X \ \text{trace}(W) < \gamma_2^2 \end{bmatrix}
\]
From matrix $X_d$ and $Y_d$, the controller of the fixed gain can be obtained as follows:

$$K_d = Y_d X_d^{-1} = [K_h \ 0 \ 0] \quad (41)$$

$$K_h = Y X^{-1} = [K_1 \ K_2 \ K_3 \ K_4 \ K_5]. \quad (42)$$

By Eq. (25) and (42), the partial state feedback gain $K_{h2}(\theta)$ is as follows:

$$K_{h2}(\theta) = K_h + \hat{K}_h(\theta)$$

$$= [K_1 \ K_2 + \hat{K}_1(\theta) \ K_3 \ K_4 + \hat{K}_2(\theta) \ K_5]. \quad (43)$$

### 4 Design Result

In this section, numerical simulations are shown. By simulation, we compare three controllers, Robust LQ controller, partial GS controller, and GS controller. Upper and lower bounds of the varying parameters are assigned as $\theta_1 \in [0.1, 0.7]$, $\theta_2 \in [0, 0.2609]$, $\theta_3 \in [-2.022, 2.022]$. The reference is assigned 0.1 [m] for the horizontal position $y$ of the load with hoisting movement.

#### 4.1 Comparing Control Performance with Cost Function

By using cost function, each controller performance is compared. These cost function values are compared by simulink because an accurate upper bound of cost function of the partial GS controller cannot be calculated in its designing method.

<p>| Table 2 Comparing each control performance with cost function |
|----------------|----------------|----------------|-------------|</p>
<table>
<thead>
<tr>
<th>cost function value</th>
<th>Robust LQ</th>
<th>Partial GS</th>
<th>GS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1286</td>
<td>1.1569</td>
<td>1.0396</td>
<td></td>
</tr>
</tbody>
</table>

From Table 2, the cost function value of the partial GS controller is smaller than that of Robust LQ controller. Thus, the control performance of the partial GS controller is better than that of Robust LQ controller.

#### 4.2 Simulation

Simulation results of controllers are shown. This study focus on horizontal position of the load, because of effectiveness of decentralized control. To verify the controller guarantees robust stability for the rope length, the rope length is changed with time. Hoisting movement of rope length is shown in Fig. 2.

Result of the partial state feedback gain for state variable is shown in Fig. 3.

From Fig. 3, only the gains for swing angle and the angular velocity of the partial GS controller are certainly varied by varying of rope length. Next, result of horizontal position of the load is shown in Fig. 4.

### 5 Conclusion

In this paper, the partial gain scheduling (GS) controller which is easy to do implement to the crane’s system from GS controller is synthesized for crane. The influence of variation of rope length is large for the swing angle of the load by pendulum property. However, the influence of variation of rope length is small for trolley position of the crane. The partial GS controller has the scheduled gains and fixed gains. The scheduled gains are synthesized from the system with the partial state variable of swing angle and angular velocity. The fixed gains are synthesized from the system with all the state variable. The partial GS controller is designed by adding the scheduled gain to the fixed gain. The effectiveness of the partial GS controller is shown by simulation.

### References


