Partial Gain Scheduling Control of Crane

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Abstract

This paper proposes a method to design partial gain scheduling (GS) controller for a crane. The partial GS controller has scheduled gains and fixed gains of state feedback. Selection of the scheduled gains is determined by pendulum property. From the pendulum property, angular velocity of the load depends on rope length. However, trolley position does not depend strongly on the rope length. Thus the scheduled gains are applied for the swing angle and the angular velocity of the load. The fixed gains are for the other states. To obtain a linear parameter varying (LPV) system for designing the partial GS controller, redundant descriptor representation is adopted for state space representation of the crane, where the time-varying parameters are the rope length, its velocity and acceleration. The designed controller guarantees the robust stability for time varying parameters. The problem of the controller design can be formulated as solving a finite sets of linear matrix inequality (LMI). The effectiveness of the proposed method is illustrated by simulations.

1 Introduction

Cranes are mechanical system that lift and carry loads to assigned positions. Cranes are not only required to transport the load fast and accurately, but also required to reduce oscillations. However, it is difficult for all operators to fulfill of these conditions. In this study, the load of the crane is moved to the horizontal and vertical directions simultaneously. It is reported that decentralized control is effective for cranes [1]. Swing of the load occurs by hoisting rope. It is desirable to design a robust controller considering variation of the rope length because it affects the swing angle of the load. In previous studies [2], robust controllers to suppress an influence of variation of the rope length have been reported. A gain scheduled controller is designed by selecting the rope length as the scheduling parameters [3]. In [4], using linear fractional transformation (LFT) and redundant descriptor representation, an linear varying parameter (LPV) system with not ignoring the rope length's velocity and acceleration is constructed. In consequence, gain scheduling (GS) controller via parameter dependent Lyapunov function that depends on scheduling parameter is designed. However, the control design of the GS controller is very complicated, it is difficult to implement the controller for crane depending on computer's performance. Moreover, the control performance of the robust controller deteriorates depending on the range of varying parameter.

In this study, a state feedback partial gain scheduling controller for crane systems is proposed. The partial GS controller has scheduled gains and fixed gains. Selection of the scheduled gains is determined by pendulum property. From the pendulum property, the rope length gives large influence on swing angle of the load because angular velocity of the load depends on rope length. However, the influence of rope length on the trolley position is small. The scheduled gains are given for the swing angle and the angular velocity of the load. The fixed gains are given for the other states. The scheduled gains and the fixed gains are obtained by different method respectively. First, the scheduled gains are derived. The scheduled gains are obtained by designing GS controller from a LPV system based on state variable of only swing angle and angular velocity. Then, the LPV system and a LPV system based on all state variables are equivalent on a property of crane's dynamics. Next, the fixed gains are derived. The fixed gains are obtained by designing robust LQ controller from the system based on all state variables. The partial GS controller is designed by adding a matrix of the scheduled gains and a matrix of the fixed gains. Then, the matrix of the scheduled gains is transformed to the same size as that of the fixed gains. To obtain their respective LPV systems, redundant descriptor representation is adopted. The designing problem of their respective controllers can be formulated as a finite set of linear matrix inequalities (LMIs). The effectiveness of the proposed method is illustrated by simulations.

2 Modeling

The schematic diagram of the jib crane is shown in Fig. 1.

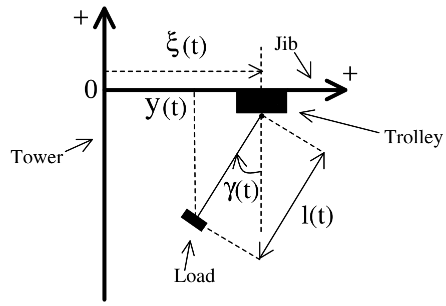

Fig. 1 Crane model

The jib crane consists of four parts, a tower, a trolley, a jib, and a load. The jib crane can lift and carry the load simultaneously in this study. Sensors of the crane can measure position of the trolley ξ [m], swing angle of the load γ [rad] and the rope length l [m]. These are time-varying parameters. The input is a electric current of a jib motor I_j [A]. The output is the horizontal position of the load y [m] ($y = \xi - l \sin\gamma$). The physical constants of the crane are shown in Table 1.

Table 1 Physical constants of the crane

Mass of load	m_p	[kg]
Mass of trolley	m_t	[kg]
Acceleration of gravity	g	[m/s^2]
Jib motor gear radius	$r_{j,p}$	[m]
Jib motor gear box efficiency	$\eta_{g,j}$	[-]
Jib motor gear ratio	$K_{g,j}$	[-]
Jib motor torque constant	$K_{t,j}$	[Nm/A]
Jib motor equivalent moment of inertia	J_ψ	[kgm^2]

In this study, we make following assumptions, (i) the rope is rigid rod without the mass, (ii) the load is a material point, (iii) the load moves horizontally along the jib, (iv) the friction of the trolley can be ignored, (v) the swing angle and its velocity are small enough.

In Euler-Lagrange equations, $\sin\gamma$, $\cos\gamma$, and $\dot{\gamma}^2$ can be approximated to γ, 1, and 0 by the assumption (v). Let m_j be $m_p + m_t + J_\psi \frac{K_{g,j}^2}{r_{j,p}^2}$. When an equation of motion with respect to ξ and γ is obtained, an equation of motion with only γ (i.e., the equation without ξ) can be derived as follows:

$$(m_j - m_p)l\ddot{\gamma} = -2(m_j - m_p)\dot{l}\dot{\gamma} - (m_j g - m_p\ddot{l})\gamma + K_{t,j}I_j. \quad (1)$$

The state variable of the crane system for the scheduled gains is only the swing angle and the angular velocity. However, the state variable of the crane system for the fixed gains is not only those but also the trolley position and its velocity. Thus, two state space representations are obtained for the scheduled gains and the fixed gains. First, the state space representation for the scheduled gain is shown. Let state variable be $\hat{x} = [\gamma\ \dot{\gamma}]^T$ and input be $u_1 = I_j$. The state space representation of the crane is obtained as Eq.(2).

$$\dot{\hat{x}} = \hat{E}(l)^{-1}\hat{A}(\dot{l},\ \ddot{l})\hat{x} + \hat{E}(l)^{-1}\hat{B}u_1 \quad (2)$$
$$\hat{E} = \begin{bmatrix} 1 & 0 \\ 0 & -(m_p - m_j)l \end{bmatrix},$$
$$\hat{A} = \begin{bmatrix} 0 & 1 \\ m_p\ddot{l} - m_j g & 2(m_p - m_j)\dot{l} \end{bmatrix},\ \hat{B} = \begin{bmatrix} 0 \\ K_{t,j} \end{bmatrix}.$$

Next, the state space representation for the fixed gain is shown. Let generalized coordinate be $q = [\xi \quad \gamma]^T$, state variable be $x = \left[q^T \quad \dot{q}^T\right]^T$, output be y and input be u. The state space representation of the crane is obtained as follows:

$$\begin{cases} \dot{x} = E(l)^{-1}A(\dot{l},\ \ddot{l})x + E(l)^{-1}Bu & (3) \\ y = C(l)x & (4) \end{cases}$$
$$E = \begin{bmatrix} I_2 & O_{2\times2} \\ O_{2\times2} & E_1 \end{bmatrix},\ E_1 = \begin{bmatrix} m_j & -m_p l \\ -1 & l \end{bmatrix}$$
$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & m_p\ddot{l} & 0 & 2m_p\dot{l} \\ 0 & -g & 0 & -2\dot{l} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ K_{t,j} \\ 0 \end{bmatrix},\ C = [C_1 \quad O_{1\times2}]$$
$$C_1 = [1 \quad -l].$$

An important purpose of controlling crane system is transporting the load to the target without error. To eliminate study error, servo control system is constructed. Let the error and integral of the error be e and x_e. To make the output y follow a reference r, a state variable x_h consists of the state variable x and x_e. The servo control system is as follows:

$$\begin{cases} \dot{x}_h = E_h(l)^{-1}A_h(\dot{l},\ \ddot{l})x_h + E_h(l)^{-1}B_h u & (5) \\ y = C_h x_h & (6) \end{cases}$$

$$E_h = \begin{bmatrix} E & O_{4\times1} \\ O_{1\times4} & 1 \end{bmatrix},\ A_h = \begin{bmatrix} A(\dot{l}, \ddot{l}) & O_{4\times1} \\ -C(l) & 0 \end{bmatrix},\ B_h = \begin{bmatrix} B \\ 0 \end{bmatrix}$$
$$x_h(t) = \begin{bmatrix} x(t) - x(\infty) \\ x_e(t) - x_e(\infty) \end{bmatrix} = \begin{bmatrix} q(t) - q(\infty) \\ \dot{q}(t) \\ x_e(t) - x_e(\infty) \end{bmatrix}$$
$$x_e = \int_0^t e(\tau)\mathrm{d}\tau, e = r - y$$
$$u(t) = u(t) - u(\infty).$$

$q(\infty)$, $x_e(\infty)$ and $u(\infty)$ are steady state values.

3 Control Designing

In this paper, the controller based on an optimal regulator theory is designed.

3.1 Parameter Box

The time-varying parameters are l, $\dot{l}$, and $\ddot{l}$. The parameter box Eq.(7) is defined by upper and lower bounds of time-varying parameters l, $\dot{l}$, and $\ddot{l}$.

$$\Theta = \{[\theta_1,\ \theta_2,\ \theta_3] :\ \theta_i \in \{\underline{\theta}_i,\ \overline{\theta}_i\}\} \quad (7)$$
$$\theta_1 = l,\ \theta_2 = \dot{l},\ \theta_3 = \ddot{l} \quad (i = 1, 2, 3)$$

3.2 Control Designing for Scheduled Gain

3.2.1 Descriptor Representation

The time-varying parameters are included in the matrix $\hat{A}$ in the state space representation Eq.(2). The coefficient matrix $\hat{E}(l)^{-1}\hat{A}$ and $\hat{E}^(l) - 1\hat{B}$ include rational terms of the time-varying parameters. By using redundant descriptor representation [5], the state space representation is rewritten as descriptor representation with only linear terms of the time-varying parameters. Let descriptor variable be $\hat{x}_d = [\gamma\ \dot{\gamma}\ \ddot{\gamma}]^T$, Eq.(8) is obtained from Eq.(2).

$$\hat{E}_d\dot{\hat{x}}_d = \hat{A}_d(\theta)\hat{x}_d + \hat{B}_d u_1 \quad (8)$$

$$\hat{E}_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\hat{A}_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ m_p\theta_3 - m_j g & 2(m_p - m_j)\theta_2 & (m_p - m_j)\theta_1 \end{bmatrix},\ \hat{B}_d = \begin{bmatrix} 0 \\ 0 \\ K_{t,j} \end{bmatrix}.$$

3.2.2 Stability Conditions

The stability conditions of the descriptor system Eq.(8) is shown. The system is stabilized by state feedback controller $u_1 = \hat{K}_d(\theta)\hat{x}_d$ and feedback gain $\hat{K}_d(\theta) = \hat{Y}_d(\theta)\hat{X}_d^{-1}$. Considering structure of the matrix $\hat{E}_d$, candidates of Lyapunov matrix $\hat{X}_d$ and variable matrix $\hat{Y}_d(\theta)$ are restricted as Eq.(9).

$$\hat{X}_d = \begin{bmatrix} \hat{X} & O_{2\times1} \\ \hat{X}_{d21} & \hat{X}_{d22} \end{bmatrix} \quad (9)$$
$$\hat{Y}_d(\theta) = \hat{Y}_{d0} + \sum_{i=1}^{3}\theta_i\hat{Y}_{di} = \left[\hat{Y}(\theta) \quad 0\right]$$

[Theorem] [6] The system Eq.(8) is stable by and state feedback controller, if $\hat{X}_d$ and $\hat{Y}_d(\theta)$ exist satisfying Eq.(10) and (11).

$$\hat{E}_d\hat{X}_d = (\hat{E}_d\hat{X}_d)^T \geq 0 \quad (10)$$
$$\mathrm{He}\{\hat{A}_d(\theta)\hat{X}_d + \hat{B}_d\hat{Y}_d(\theta)\} < 0 \quad (11)$$

For Eq.(8), an evaluated function $\hat{J}_z$ is given by Eq.(12).

$$\hat{J}_z = \int_0^\infty (\hat{x}_d^T\hat{Q}\hat{x}_d + u_1{}^T\hat{R}u_1)dt \quad (12)$$

In order to minimize the evaluated function $\hat{J}_z$ in

Eq.(12), the minimum γ_1 that satisfies the following conditions is derived.

$$\hat{X} > 0 \tag{13}$$

$$\mathrm{He}\{\hat{A}_d(\theta)\hat{X}_d + \hat{B}_d\hat{Y}_d(\theta)\} + \hat{X}_d^T\hat{Q}\hat{X}_d + \hat{Y}_d(\theta)^T\hat{R}\hat{Y}_d(\theta) < 0 \tag{14}$$

$$\hat{X}^{-1} < W \tag{15}$$

$$trace(\hat{W}) < \gamma_1 \tag{16}$$

Matrices $\hat{A}_d(\theta)$, $\hat{Y}_d(\theta)$, and $\hat{Y}(\theta)$ can be represented by parameter box $\Theta_j(j = 1, ..., 8)$.

$$\Theta_1 = (\underline{\theta}_1, \underline{\theta}_2, \underline{\theta}_3), \Theta_2 = (\bar{\theta}_1, \underline{\theta}_2, \underline{\theta}_3), ...$$
$$..., \Theta_7 = (\bar{\theta}_1, \bar{\theta}_2, \underline{\theta}_3), \Theta_8 = (\bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3) \tag{17}$$

The problem of designing the controller of the scheduled gain is formulated as follows from Schur complement:

Lemma 1 *:If there exist matrices $\hat{X}$ and $\hat{Y}(\theta)$ satisfying following LMIs, the descriptor system (8) is stable.*

$$\hat{X} > 0 \tag{18}$$

$$\left[\begin{array}{c|c|c} He\{\hat{A}_d(\Theta_j)\hat{X}_d + \hat{B}_d\hat{Y}_d(\Theta_j)\} & \hat{Y}_d^T(\Theta_j)\hat{R}^{\frac{1}{2}} & \hat{X}_d^T\hat{Q}^{\frac{1}{2}} \\ \hline \hat{R}^{\frac{1}{2}}\hat{Y}_d(\Theta_j) & -1 & O_{1\times 3} \\ \hline \hat{Q}^{\frac{1}{2}}\hat{X}_d & O_{3\times 1} & -I_3 \end{array}\right] < 0 \tag{19}$$

$$\begin{bmatrix} \hat{W} & I_2 \\ I_2 & \hat{X} \end{bmatrix} > 0 \tag{20}$$

$$trace(\hat{W}) < \gamma_1. \tag{21}$$

$$(j = 1, ..., 8)$$

From matrix $\hat{X}_d$ and $\hat{Y}_d(\theta)$, the controller of the scheduled gain can be obtained as follows.

$$\hat{K}_d(\theta) = \begin{bmatrix} \hat{Y}(\theta)\hat{X}^{-1} & 0 \end{bmatrix} = \begin{bmatrix} \hat{K}_1(\theta) & \hat{K}_2(\theta) & 0 \end{bmatrix}. \tag{22}$$

3.3 Control Designing for Fixed Gain

Partial state feed back controller is defined as $u = K_{h2}(\theta)x_h$. Then, partial state feed back gain $K_{h2}(\theta)$ is as follows:

$$K_{h2}(\theta) = [K_{h21} \quad K_{h22}(\theta) \quad K_{h23} \quad K_{h24}(\theta) \quad K_{h25}]. \tag{23}$$

In this study, partial state feed back gain $K_{h2}(\theta)$ is derived by adding the fixed gain and the scheduled gain.Let the fixed gain be K_h and the scheduled gain be $\tilde{K}_h(\theta)$. $K_{h2}(\theta)$ is represented as $K_{h2}(\theta) = K_h + \tilde{K}_h(\theta)$. $\tilde{K}_h(\theta)$ is represented as an extension form of dimension of the scheduled gain $\hat{K}_h(\theta)$. K_h and $\tilde{K}_h(\theta)$ are defined as follows:

$$K_h = [K_1 \quad K_2 \quad K_3 \quad K_4 \quad K_5] \tag{24}$$

$$\tilde{K}_h(\theta) = \begin{bmatrix} 0 & \hat{K}_1(\theta) & 0 & \hat{K}_2(\theta) & 0 \end{bmatrix}. \tag{25}$$

When input u_1 is defined as obtained state feedback controller $u_1 = \tilde{K}_h(\theta)x_h$ by the scheduled gain $\tilde{K}_h(\theta)$ and input u_2 is defined as obtained state feedback controller $u_2 = K_hx_h$ by the fixed gain K_h, the partial state feedback controller $u = K_{h2}(\theta)x_h$ is represented as $u = (K_h + \tilde{K}_h(\theta))x_h = u_1 + u_2$. The variable of input u_1 is determined by Eq.(25). So, the fixed gain K_h of state feedback controller $u_2 = K_hx_h$ is derived. Eq.(5) is rewritten as Eq.(26).

$$\dot{x}_h = E_h(l)^{-1}A_{h2}(l,\ \dot{l},\ \ddot{l})x_h + E_h(l)^{-1}B_hu_2 \tag{26}$$

$$A_{h2} = \begin{bmatrix} O_{2\times 2} & I_2 & O_{2\times 1} \\ A_{h21} & A_{h22} & O_{2\times 1} \\ C_1 & O_{1\times 2} & 0 \end{bmatrix},\ A_{h21} = \begin{bmatrix} 0 & m_p\ddot{l} + K_{t,j}\hat{K}_1(\theta) \\ 0 & -g \end{bmatrix}$$

$$A_{h22} = \begin{bmatrix} 0 & 2m_p\dot{l} + K_{t,j}\hat{K}_2(\theta) \\ 0 & -2\dot{l} \end{bmatrix}.$$

3.3.1 Descriptor Representation

Eq.(26) is rewritten as descriptor representation. Let descriptor variable be $x_d = [x_h{}^T \ \ddot{q}^T]^T$. Descriptor equation of the crane is obtained as Eq.(27).

$$E_d\dot{x}_d = A_d(\theta)x_d + B_du_2 \tag{27}$$

$$E_d = \begin{bmatrix} I_5 & O_{5\times 2} \\ O_{2\times 5} & O_{2\times 2} \end{bmatrix},\ A_d = \begin{bmatrix} O_{2\times 2} & I_2 & O_{2\times 1} & O_{2\times 2} \\ O_{2\times 2} & O_{2\times 2} & O_{2\times 1} & I_2 \\ C_1 & O_{1\times 2} & 0 & O_{1\times 2} \\ A_{h21} & A_{h22} & O_{2\times 1} & E_1 \end{bmatrix}$$

$$B_d = \begin{bmatrix} O_{5\times 1} \\ K_{t,j} \\ 0 \end{bmatrix}.$$

3.3.2 Stability Conditions

The stability conditions of the descriptor system Eq.(27) is shown. The system is stabilized by the state feedback controller $u_2 = K_dx_d$. Considering structure of the matrix E_d, candidates of Lyapunov matrix X_d and variable matrix Y_d are restricted as Eq.(28).

$$X_d = \begin{bmatrix} X & O_{5\times 2} \\ X_{d21} & X_{d22} \end{bmatrix},\ Y_d = [Y \quad O_{1\times 2}]. \tag{28}$$

By Lyapunov's stability theorem, if there exists X_d satisfying the following matrix inequality, the system is stabilized [6].

$$E_dX_d = (E_dX_d)^T \geq 0 \tag{29}$$

$$\mathrm{He}\{A_d(\theta)X_d + B_dY_d\} < 0 \tag{30}$$

For a system of Eq.(27), a general plant as Eq.(31) considering LQ control specification is assigned.

$$\begin{cases} E_d\dot{x}_d = A_d(\theta)x_d + B_{dw}w + B_du_2 \\ \quad z = C_dx_d + D_du_2 \end{cases} \tag{31}$$

$$B_{dw} = \begin{bmatrix} B_w \\ O_{2\times 5} \end{bmatrix},\ B_w = I_5,\ C_d = [W_x \quad O_{8\times 2}],\ D_d = \begin{bmatrix} O_{7\times 1} \\ R^{\frac{1}{2}} \end{bmatrix}$$

$$W_x = \begin{bmatrix} Q^{\frac{1}{2}} \\ O_{2\times 5} \\ R\tilde{K}_h(\theta) \end{bmatrix}$$

An evaluated function $||J_z||^2$ is given by Eq.(32).

$$||J_z||^2 = \int_0^\infty z^Tzdt \tag{32}$$

In order to minimize the evaluated function J_z of Eq.(32), the minimum γ_2 that satisfy the following conditions [4].

$$E_dX_d = (E_dX_d)^T \geq 0 \tag{33}$$

$$\mathrm{He}\{A_d(\theta)X_d + B_dY_d\} + \{C_dX_d + D_dY_d\}^T\{C_dX_d + D_dY_d\} < 0 \tag{34}$$

$$B_w^TX^{-1}B_w < W \tag{35}$$

$$trace(W) < \gamma_2^2 \tag{36}$$

The problem of designing the controller of the fixed gain is formulated as follows from Eq.(34) and Schur complement:

Lemma 2 *:If there exist matrices X and Y satisfying following LMIs, the descriptor system is stable.*

$$X > 0 \tag{37}$$

$$\begin{bmatrix} He\{A_d(\Theta_j)X_d + B_dY_d\} & \{C_dX_d + D_dY_d\}^T \\ C_dX_d + D_dY_d & -I_8 \end{bmatrix} < 0 \tag{38}$$

$$\begin{bmatrix} W & B_w^T \\ B_w & X \end{bmatrix} > 0 \tag{39}$$

$$trace(W) < \gamma_2^2. \tag{40}$$

$$(j = 1, ..., 8)$$

From matrix X_d and Y_d, the controller of the fixed gain can be obtained as follows:

$$K_d = Y_d X_d^{-1} = [K_h \quad 0 \quad 0] \tag{41}$$

$$K_h = Y X^{-1} = [K_1 \quad K_2 \quad K_3 \quad K_4 \quad K_5]\,. \tag{42}$$

By Eq.(25) and (42), the partial state feedback gain $K_{h2}(\theta)$ is as follows:

$$\begin{aligned} K_{h2}(\theta) &= K_h + \tilde{K}_h(\theta) \\ &= \left[K_1 \quad K_2 + \hat{K}_1(\theta) \quad K_3 \quad K_4 + \hat{K}_2(\theta) \quad K_5\right] \end{aligned} \tag{43}$$

4 Design Result

In this section, numerical simulations are shown. By simulation, we compare three controllers, Robust LQ controller, partial GS controller, and GS controller. Upper and lower bounds of the varying parameters are assigned as $\theta_1 \in [0.1,\ 0.7]$, $\theta_2 \in [0,\ 0.2609]$, $\theta_3 \in [-2.022,\ 2.022]$. The reference is assigned 0.1 [m] for the horizontal position y of the load with hoisting movement.

4.1 Comparing Control Performance with Cost Function

By using cost function, each controller performance is compared. These cost function values are compared by simulink because an accurate upper bound of cost function of the partial GS controller cannot be calculated in its designing method.

Table 2 Comparing each control performance with cost function

	Robust LQ	Partial GS	GS
cost function value	1.4264	1.1569	1.0386

From Table 2, the cost function value of the partial GS controller is smaller than that of Robust LQ controller. Thus, the control performance of the partial GS controller is better than that of Robust LQ controller.

4.2 Simulation

Simulation results of controllers are shown. This study focus on horizontal position of the load, because of effectiveness of decentralized control. To verify the controller guarantees robust stability for the rope length, the rope length is changed with time. Hoisting movement of rope length is shown in Fig. 2.

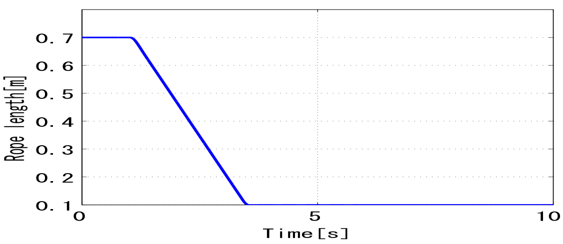

Fig. 2 rope length: hoisting

Result of the partial state feedback gain for state variable is shown in Fig. 3.

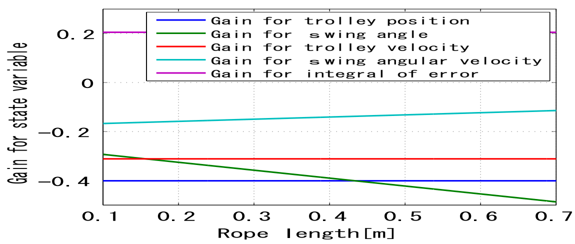

Fig. 3 Gain for state variable x_h

From Fig. 3, only the gains for swing angle and the angular velocity of the partial GS controller are certainly varied by varying of rope length. Next, result of horizontal position of the load is shown in Fig. 4.

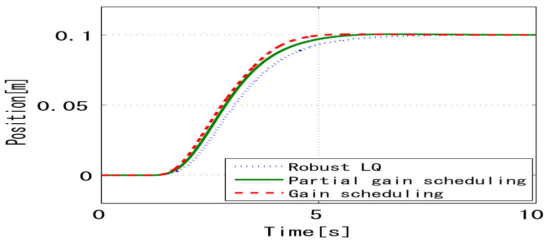

Fig. 4 Horizontal position of the load

As can be seen in Fig. 4, it is shown that the convergence of the horizontal position of the partial GS controller is fast than that of Robust LQ controller.

5 Conclusion

In this paper, the partial gain scheduling (GS) controller which is easy to do implement to the crane's system from GS controller is synthesized for crane. The influence of variation of rope length is large for the swing angle of the load by pendulum property. However, the influence of variation of rope length is small for trolley position of the crane. The partial GS controller has the scheduled gains and fixed gains. The scheduled gains are synthesized from the system with the partial state variable of swing angle and angular velocity. The fixed gains are synthesized from the system with all the state variable. The partial GS controller is designed by adding the scheduled gain to the fixed gain. The effectiveness of the partial GS controller is shown by simulation.

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