Gain-Scheduled Control of Active Magnetic Bearing System via Parameter Dependent Lyapunov Function

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Abstract

This paper proposes a gain-scheduled (GS) state feedback control for active magnetic bearing (AMB) systems via parameter dependent Lyapunov function. The objective of this study is to stabilize rotor attitude of the AMB system that is influenced by gyro effect. Gyro effect corresponds to rotational speed of the rotor that is a time-varying parameter. Hence, the speed is treated as a scheduling parameter in the GS controller. The robust stability for the system with the time-varying parameter is guaranteed theoretically by using polytopic representation. The problem of the GS controller design can be formulated as liner matrix inequality (LMI) conditions from Riccati's differential equation. However, it is difficult to solve the designing problem at vertexes of polytope because the LMI conditions are not affine for the time-varying parameter. In oder to avoid this difficulty, Lyapunov matrix for the AMB system is restricted to make LMI conditions affine. Finally, the effectiveness of proposed method is illustrated by simulations.

1 Introduction

Active magnetic bearing systems stabilize rotor attitude without contact using electromagnetic force. The advantage of using the system is absence of friction. From this advantage, the rotational speed becomes high, while the rotational motion generates gyroscopic effect. Then, the system tends to be unstable by gyroscopic effect. Thus, Control of the AMB system is required to consider this phenomenon. The robust control system which suppresses the influence of gyroscopic effect have been stated in literature [1]. Recently, GS control system based on parameter dependent Lyapunov function has been published stating that redundant descriptor representation and LFT are applied to state-space representation [2].

This study proposes a gain scheduled state feedback control based on parameter dependent Lyapunov function for AMB. The objective of this study is to stabilize the rotor attitude of the system that is influenced by gyro effect. A linear parameter-varying (LPV) representation of the AMB system has terms representing gyro effect. The terms depend on the time-varying parameter which is the rotational speed. Therefore, the speed is treated as a scheduling parameter in the proposed GS controller. The robust stability for the LPV system with the time-varying parameter is guaranteed theoretically by using polytopic representation. The problem of the GS controller design can be formulated as LMI conditions from Riccati's differential equation. However, it is difficult to solve the design problem at vertexes of polytope because LMI conditions are not affine for the scheduling parameter. In order to avoid this difficulty, the structure of Lyapunov matrix is restricted to make LMI conditions affine for the scheduling parameter. As a result, the GS control based on parameter dependent Lyapunov function is derived without usage of descriptor representation, LFT and the sum-of-squares technique. The effectiveness of proposed GS controller is illustrated by comparing with robust LQ (RLQ) controller in simulations.

2 Motion Equation

In this section, state-space representation of an AMB system is derived. The system levitates and supports a rotor without contact by using magnetic force. The schematic diagram of the system is shown in Fig. 1. In this study, the AMB system has the four degree of

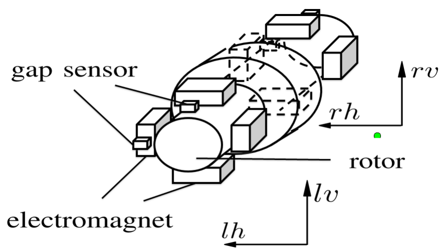

Figure 1: AMB system

freedom. The system consists of a rotor, four pairs of electromagnets and two gap sensors attached at each end. The pair of electromagnets face to each other and generate the levitation force corresponding to control input. The levitation force is generated in vertical and horizontal direction. The gap sensors measure the distance between the electromagnet and the rotor. Let equilibrium point of the rotor be a center point between the pair of electromagnets. At the equivalent point, the each distance between the rotor and electromagnets is the same. Each physical parameter of the AMB system is shown in Table 1. The subscript j of g_j and f_j is

Table 1: Physical parameters

parameter	symbols
Perturbation from the equilibrium point of the rotor	g_j [m]
Levitation force of electromagnets	f_j [N]
Mass of rotor	m [kg]
Rotational speed of rotor	ω [rad/s]
Distance from the center of gravity to the end of the rotor	l_m [m]
Moment of the X axis	J_x [Nm]
Moment of the Y axis	J_y [Nm]
Distance between the sensor and the rotor in the equilibrium state	G_0 [Nm]
Levitation force constant	k
Acceleration of gravity	g [m/s^2]

$j \in \{lv, rv, lh, rh\}$. Here, subscripts "l", "r", "v" and "h" represent the left-hand of the rotor, right-hand side of the rotor, the vertical direction and horizontal direction, respectively. For example, the vertical direction at the left-hand side of the rotor is represented as "lv". Both the perturbation g_j and the rotational speed ω are measurable parameter in operation.

To obtain the motion equations of the AMB system, coordinate axes X, Y, and Z are introduced to the rotor. Fig. 2 shows a schematic diagram of the rotor with the introduced X, Y, and Z axes. Here, the origin of three

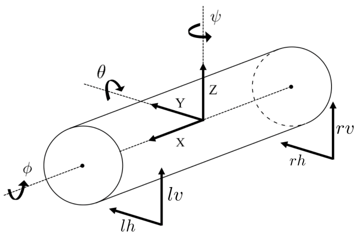

Figure 2: Rotor

axes means the center of gravity of the rotor at the equilibrium state. Let x, y, z, ϕ, θ and ψ be the position of the center of gravity on X, Y and Z axes, the rotation angle about X, Y and Z, respectively. Translational motion and rotational motion of the Y and Z axes of the rotor are described as Eq.(1) - (4).

$$m\ddot{y} = f_{lh} + f_{rh} \tag{1}$$
$$m\ddot{z} = f_{lv} + f_{rv} - mg \tag{2}$$
$$J_y\ddot{\psi} = -J_x\omega\dot{\theta} - l_m f_{lh} + l_m f_{rh} \tag{3}$$
$$J_y\ddot{\theta} = J_x\omega\dot{\psi} + l_m f_{lv} - l_m f_{rv} \tag{4}$$

Using time-varying parameter ω, the gyroscopic effect is represented by the first term on the right-hand side of Eq.(3) and (4). In Eq.(1) - (4), the position of the center of gravity y, z, the rotation angle θ, ψ and levitation force f_j of the electromagnet are described as Eq.(5) - (7) by using the perturbation g_j and control input i_j, respectively.

$$y = \frac{1}{2}(g_{rh} + g_{lh}),\ z = \frac{1}{2}(g_{lv} + g_{rv}) \tag{5}$$
$$\theta \approx \frac{g_{lv} - g_{rv}}{2l_m},\ \psi \approx \frac{g_{rh} - g_{lh}}{2l_m} \tag{6}$$
$$f_j = k\frac{4I_0I_j}{G_0^2} + K_{xj}g_j + K_{ij}i_j \tag{7}$$
$$K_{xj} = k\frac{4(I_0^2 + I_j^2)}{G_0^3}, K_{ij} = k\frac{4I_0}{G_0^2}$$

Here, let I_0 and I_j be bias current and steady-state current, respectively. The steady-state current I_j is corresponding to the gravitational force mg. $I_0 \pm (I_j + i_j)$ is a total of input current for the electromagnets. Let state variable $x(t)$ and input variable $u(t)$ be Eq.(8) and (9).

$$x(t) = [g_{lv}\ g_{rv}\ g_{lh}\ g_{rh}\ \dot{g_{lv}}\ \dot{g_{rv}}\ \dot{g_{lh}}\ \dot{g_{rh}}]^T \tag{8}$$
$$u(t) = [i_{lv}\ i_{rv}\ i_{lh}\ i_{rh}]^T \tag{9}$$

From Eq.(1) - (9), the state-space representation of AMB system is derived as Eq.(10).

$$\begin{cases} \dot{x}(t) = A(\omega(t))x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{10}$$

$$A(\omega) = A_0 + \omega A_1$$

$$A_0 = \begin{bmatrix} O_{4\times4} & I_{4\times4} \\ \hat{A}_0 & O_{4\times4} \end{bmatrix}, \hat{A}_0 = \begin{bmatrix} a_1 & a_2 & 0 & 0 \\ a_3 & a_4 & 0 & 0 \\ 0 & 0 & a_5 & a_6 \\ 0 & 0 & a_7 & a_8 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} O_{4\times4} & O_{4\times4} \\ O_{4\times4} & \hat{A}_1 \end{bmatrix}, \hat{A}_1 = \begin{bmatrix} 0 & 0 & -a_9 & a_9 \\ 0 & 0 & a_9 & -a_9 \\ a_9 & -a_9 & 0 & 0 \\ -a_9 & a_9 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} O_{4\times4} \\ B_1 \end{bmatrix}, B_1 = \begin{bmatrix} b_1 & b_2 & 0 & 0 \\ b_3 & b_4 & 0 & 0 \\ 0 & 0 & b_5 & b_6 \\ 0 & 0 & b_7 & b_8 \end{bmatrix}, C = I_{8\times8}$$

$$a_1 = \frac{K_{xlv}}{m} + \frac{K_{xlv}l_m^2}{J_y},\ a_2 = \frac{K_{xrv}}{m} - \frac{K_{xrv}l_m^2}{J_y}$$
$$a_3 = \frac{K_{xlv}}{m} - \frac{K_{xlv}l_m^2}{J_y},\ a_4 = \frac{K_{xrv}}{m} + \frac{K_{xrv}l_m^2}{J_y}$$
$$a_5 = \frac{K_{xlh}}{m} + \frac{K_{xlh}l_m^2}{J_y},\ a_6 = \frac{K_{xrh}}{m} - \frac{K_{xrh}l_m^2}{J_y}$$
$$a_7 = \frac{K_{xlh}}{m} - \frac{K_{xrh}l_m^2}{J_y},\ a_8 = \frac{K_{xrh}}{m} + \frac{K_{xrh}l_m^2}{J_y},\ a_9 = \frac{J_x}{2J_y}$$
$$b_1 = \frac{K_{ilv}}{m} + \frac{K_{ilv}l_m^2}{J_y},\ b_2 = \frac{K_{irv}}{m} - \frac{K_{irv}l_m^2}{J_y}$$
$$b_3 = \frac{K_{ilv}}{m} - \frac{K_{ilv}l_m^2}{J_y},\ b_4 = \frac{K_{irv}}{m} + \frac{K_{irv}l_m^2}{J_y}$$
$$b_5 = \frac{K_{ilh}}{m} + \frac{K_{ilh}l_m^2}{J_y},\ b_6 = \frac{K_{irh}}{m} - \frac{K_{irh}l_m^2}{J_y}$$
$$b_7 = \frac{K_{ilh}}{m} - \frac{K_{ilh}l_m^2}{J_y},\ b_8 = \frac{K_{irh}}{m} + \frac{K_{irh}l_m^2}{J_y}$$

Since the state matrix A in Eq.(10) depends on time-varying parameter ω, AMB system is described by Liner parameter-varying (LPV) representation.

3 Control System Design

In this section, GS controller based on parameter dependent Lyapunov function for the LPV system Eq.(10) is designed. The scheduling parameter of the designed controller is rotational speed ω. The robust stability for the system depending on time-varying parameter can be guaranteed theoretically by solving linear matrix inequality (LMI) conditions with polytopic representation.

3.1 Polytopic Representation

Let lower and upper bounds of time-varying parameter ω be ω_1 and ω_2. The parameter is represented as following equation.

$$\omega \in [\ \underline{\omega}\ ,\ \overline{\omega}\ ] = [\omega_1, \omega_2] \tag{11}$$

From Eq.(11), polytopic representation of the matrix $A(\omega)$ in Eq.(10) can be described as Eq.(12).

$$A(\omega) = \lambda A(\omega_1) + (1 - \lambda)A(\omega_2) \quad (0 \leq \lambda \leq 1) \tag{12}$$

By using the polytopic representation Eq.(12), designing problem of the GS controller via parameter dependent Lyapunov function can be formulated as a finite set of LMI conditions in the next subsection.

3.2 Quadratic Stability

Let a GS controller gain and a state feedback $u(t)$ for the LPV system Eq.(10) be $K(\omega)$ and Eq.(13).

$$u(t) = K(\omega)x(t) \tag{13}$$

To guarantee the quadratic stability for the system, the parameter dependent Lyapunov function $V(t) = x^T(t)P(\omega(t))x(t) \succ O$ is considered, where $P(\omega) = P^T(\omega) \succ O$ is Lyapunov matrix. Then, the stability condition for the system can be described as Eq.(14).

$$\dot{V}(t) = x^T(t)(\mathrm{He}\{P(\omega)A_{cl}(\omega)\} + \dot{P}(\omega))x(t) \prec O \quad (14)$$
$$A_{cl}(\omega) := A(\omega) + BK(\omega)$$

If there exist $P(\omega)$ and $K(\omega)$ such that Eq.(14)holds, then the closed loop system Eq.(10) is stabilized by the state feedback $u(t) = K(\omega)x(t)$.

3.3 GS Control System Design

In order to derive the stabilizing state feedback $u(t) = K(\omega)x(t)$ for the LPV system Eq.(10), the following quadratic stability condition Eq.(15) is considered.

$$\dot{V}(t) \prec -x^T(Q + K(\omega)^T RK(\omega))x(t) \quad (15)$$

Here, $Q \succeq 0$ and $R \succ 0$ are weight matrices for the state variable and input variable, respectively. Note that the inequality Eq.(15) is equivalent to Riccati's differential inequality. Therefore, this inequality condition can be regarded as problem of LQ optimal regulator that minimizes the following cost function J.

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \quad (16)$$

For this problem, following results is well-known.

Theorem 1 : *Letting $X(\omega) := P^{-1}(\omega)$ and $Y(\omega) := K(\omega)X(\omega)$, the inequality condition Eq.(15) is rewritten as following matrix inequality* [3].

$$M - \dot{X}(\omega) + X(\omega)QX(\omega) + Y^T(\omega)RY(\omega) \prec O \quad (17)$$
$$M := He\{A(\omega)X(\omega) + BY(\omega)\}$$

Theorem 2 : *If there exist matrix $P(\omega) = X^{-1}(\omega)$ such that the inequality Eq.(15) holds, a upper bound on the cost function is represented as following equation* [4].

$$J < trace(P) = trace(X^{-1}) \quad (18)$$

In this study, the GS controller via parameter dependent Lyapunov function is designed for the LPV system Eq.(10) with polytopic representation Eq.(12). Matrices $X(\omega)$ and $Y(\omega)$ are defined as follows.

$$\begin{aligned} X(\omega) &= X_0 + \omega X_1 \quad (19)\\ &= \begin{bmatrix} X_{A0} & X_{B0}^T \\ X_{B0} & X_{C0} \end{bmatrix} + \omega \begin{bmatrix} X_{A1} & X_{B1}^T \\ X_{B1} & X_{C1} \end{bmatrix} \\ Y(\omega) &= Y_0 + \omega Y_1 \quad (20)\\ &= [\ Y_{A0} \quad Y_{B0}\ ] + \omega [\ Y_{A1} \quad Y_{B1}\ ] \end{aligned}$$

Let a differential of rotational speed ω be $\alpha(t) \in [\underline{\alpha}, \overline{\alpha}] = [\alpha_1, \alpha_2]$. By using matrix $X(\omega)$ in Eq.(19), the matrix $\dot{X}(\omega)$ in inequality condition Eq.(17) can be represented as Eq.(21).

$$\dot{X}(\omega) = \alpha X_1 = S(\alpha) \quad (21)$$

Likewise, by using matrix $X(\omega)$, $A(\omega)X(\omega)$ is described as Eq.(22).

$$\begin{aligned} A(\omega)X(\omega) &= (A_0 + \omega A_1)(X_0 + \omega X_1) \quad (22)\\ &= A_0X_0 + \omega(A_0X_1 + A_1X_0) + \omega^2 A_1X_1 \end{aligned}$$

Note that $He\{A(\omega)X(\omega)\}$ in the condition Eq.(17) has square term of scheduling parameter ω. In this case, it is difficult to derive the GS control by solving the conditions at vertexes of polytope because it is not affine for scheduling parameter ω. To solve the condition Eq.(17), the following convexity condition Eq.(23) must be held.

$$\begin{aligned} &\frac{\partial^2}{\partial\omega^2}\mathrm{He}\{A(\omega)X(\omega)\} \succeq O \quad (23)\\ &\Leftrightarrow \mathrm{He}\{A_1X_1\} \succeq O \end{aligned}$$

By using Schur complement, the designing problem of proposed GS controller for the LPV system Eq.(10) can be formulated as a finite set of LMI conditions from Theorem 1, 2 and the convexity condition Eq.(23).

Lemma 1 : *If there exist matrices $X(\omega) = X(\omega)^T$ and $Y(\omega)$ satisfying Eq.(24) - (27), the LPV system Eq.(11) is stabilized by the state feedback $u(t) = K(\omega)x(t) = Y(\omega)X(\omega)^{-1}x(t)$.*

$$He\{A_1X_1\} \succeq O \quad (24)$$
$$\begin{bmatrix} N - S(\alpha_t) & X(\omega_s)Q^{\frac{1}{2}} & Y(\omega_s)R^{\frac{1}{2}} \\ Q^{\frac{1}{2}}X(\omega_s)^T & -I & 0 \\ R^{\frac{1}{2}}Y(\omega_s)^T & 0 & -I \end{bmatrix} \prec 0 \quad (25)$$
$$\begin{bmatrix} W & I \\ I & X(\omega_s) \end{bmatrix} \succ 0, \quad (s = 1,\ 2), (t = 1,\ 2) \quad (26)$$
$$trace(W) < \gamma \quad (27)$$
$$N := He\{A(\omega_s)X(\omega_s) + BY(\omega_s)\}$$

Here, γ is upper bound of J. Though minimizing the γ, the cost function J is minimized.

To avoid the difficultly of convexity condition Eq.(23), the matrices X_{B1} and X_{C1} in X_B and X_C are restricted by considering the structure of state matrix A_1 in Eq.(10). The restrictions considering the matrix structure are described as Eq.(28).

$$X_{B1} = \begin{bmatrix} X_{b1} & X_{b2} \\ X_{b3} & X_{b4} \end{bmatrix},\ X_{C1} = \begin{bmatrix} X_{c1} & X_{c2}^T \\ X_{c2} & X_{c3} \end{bmatrix} \quad (28)$$
$$X_{bj} = \begin{bmatrix} x_{bj} & \widetilde{x}_{bj} \\ x_{bj} & \widetilde{x}_{bj} \end{bmatrix}, \quad X_{ck} = \begin{bmatrix} x_{ck} & x_{ck} \\ x_{ck} & x_{ck} \end{bmatrix}$$
$$(j = 1, 2, 3, 4), (k = 1, 2, 3)$$

Using the restrictions Eq.(28), A_1X_1 in LMI condition Eq.(25) is rewritten as following equation.

$$\begin{aligned} A_1X_1 &= \begin{bmatrix} 0_{4\times4} & 0_{4\times4} \\ 0_{4\times4} & \hat{A}_1 \end{bmatrix} \begin{bmatrix} X_{A1} & X_{B1}^T \\ X_{B1} & X_{C1} \end{bmatrix} \quad (29)\\ &= \begin{bmatrix} O_{4\times4} & O_{4\times4} \\ \hat{A}_1X_{B1} & \hat{A}_1X_{C1} \end{bmatrix} = O_{8\times8} \end{aligned}$$

As a results, $He\{A(\omega)X(\omega)\}$ in LMI condition Eq.(25) is represented as affine for scheduling parameter ω by the restrictions Eq.(28). Here, $\hat{A}_1X_{B1}$ in A_1X_1 Eq.(29) is described as following equation.

$$\begin{aligned} \hat{A}_1X_{B1} &= \begin{bmatrix} 0 & 0 & -a_9 & a_9 \\ 0 & 0 & a_9 & -a_9 \\ a_9 & -a_9 & 0 & 0 \\ -a_9 & a_9 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{b1} & \widetilde{x}_{b1} & x_{b2} & \widetilde{x}_{b2} \\ x_{b1} & \widetilde{x}_{b1} & x_{b2} & \widetilde{x}_{b2} \\ x_{b3} & \widetilde{x}_{b3} & x_{b4} & \widetilde{x}_{b4} \\ x_{b3} & \widetilde{x}_{b3} & x_{b4} & \widetilde{x}_{b4} \end{bmatrix} \\ &= O_{4\times4} \quad \succeq O \end{aligned}$$

Likewise, $He\{\hat{A}_1 X_{C1}\} \succeq O$ in Eq.(29) is held. Since the convexity condition Eq.(24) is held by the restrictions Eq.(28), the convexity condition is removed from LMI conditions. The GS controller for the LPV system Eq.(10) based on parameter dependent Lyapunov function can be derived as following equations by solving LMI conditions Eq.(25) - (27).

$$u(t) = K(\omega)x(t) = Y(\omega)X(\omega)^{-1}x(t) \tag{30}$$
$$= \begin{bmatrix} Y_A(\omega) & Y_B(\omega) \end{bmatrix} \begin{bmatrix} X_A(\omega) & X_B^T(\omega) \\ X_B(\omega) & X_C(\omega) \end{bmatrix}^{-1} x(t)$$

4 Simulation

In this study, the effectiveness of the GS controller is compared with robust LQ (RLQ) controller in the simulations. The RLQ controller guarantees the robustness for the variation of the rotational speed by fixed controller gain. The physical constants of AMB system are given as Table 2. In this study, rotational speed is changed from 0 to 2618.0 [rad/s] with an acceleration of 261.8[rad/s^2]. At 0 [rad/s], the initial state of the rotor x_0 is given as $x_0 = 0_{8\times1}$. In addition, pulse disturbance is given to the right end of the rotor on the direction of Z axis. The disturbance of 1.0 [N] and 0.5 seconds is given every 2.0 seconds. Figure.3 - Figure.5 show the simulation results. The red line shows the response of proposed GS controller. The blue line shows the response of RLQ controller. From the simulation results, both the controllers can stabilize the rotor position. However, both the state variables g_{lv} and g_{lh} by the GS controller are smaller than those by the RLQ controller. Cost function value J of the GS controller is smaller than that of the RLQ controller.

Table 2: Physical contents of AMB system

symbol	value
g	$9.8[m/s^2]$
m	$0.277064[kg]$
l_m	$0.1344295[m]$
J_x	$1.5 \times 10^{-5}[kgm^2]$
J_y	$1.343 \times 10^{-3}[kgm^2]$
G_0	$8.0 \times 10^{-4}[m]$
k	2.8000×10^{-7}
r	$2.375 \times 10^{-4}[m]$
I_{lv}, I_{rv}	$0.1553[A]$
I_{lh}, I_{rh}	$0[A]$

5 Conclusion

In this paper, the GS controller via parameter dependent Lyapunov function for the AMB system is designed. The system tends to be unstable by gyroscopic effect corresponding to the rotational speed. Thus, the rotational speed is selected as the scheduling parameter of the GS controller. The robust stability for rotational speed by using polytopic representation. The problem of the GS controller design can be formulated as solving a finite set of LMI conditions. However, it is difficult to solve the designing problem at vertexes of polytope because the LMI conditions are not affine for the scheduling parameter. In oder to avoid this difficulty, Lyapunov matrix for the AMB system is restricted to make LMI conditions affine. The simulation results of the GS controller are compared with these of RLQ controller. In the simulations, the cost function value J of the GS controller is smaller than that of the RLQ controller.

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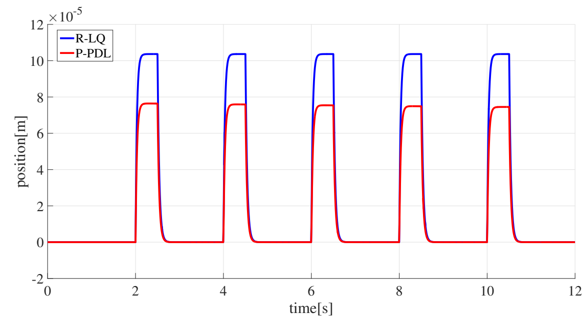

Figure 3: Position of g_{lv}

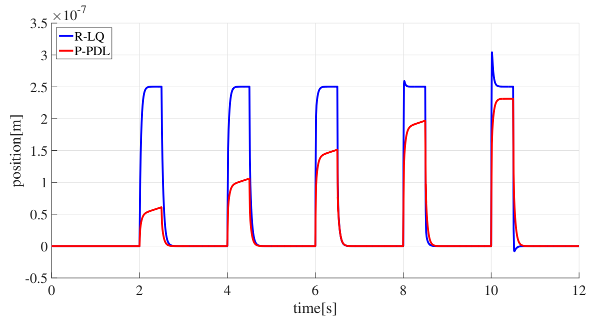

Figure 4: Position of g_{lh}

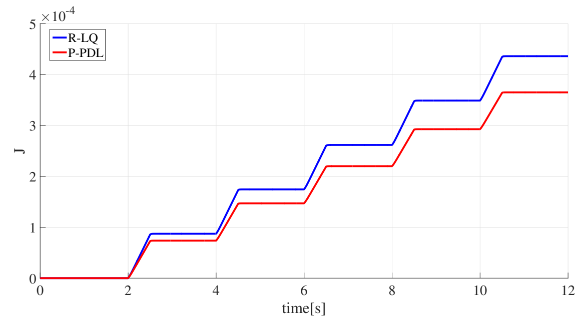

Figure 5: Cost function