Adaptive Control for Jib Crane with Nonlinear Uncertainties

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1 Introduction

This paper presents a robust LQ control system with a Model Reference Adaptive Control (MRAC) law for a jib crane. Our approaches show that the robust control performance is improved in the presence of nonlinear uncertainties by adding the MRAC law into the usual robust control system. The proposed system is synthesized as follows. Firstly, the process to design a robust LQ controller in the framework of the redundant descriptor representation is considered. The robust LQ controller is designed for uncertainties, which can be linearly treated in controller synthesis. Secondly, the adaptive law with σ-modification is designed into the robust LQ control loop. The adaptive law is considered for nonlinear uncertainties. The feature of this study is to deal with nonlinear uncertainties, which can not be linearly treated in robust LQ controller synthesis, by adding the adaptive law. The exponential stability for the homogeneous system is analyzed though solving quadratic stability condition. Finally, the effectiveness of the proposed system is verified by comparing with the robust LQ controller without the MRAC law in simulations with using the jib crane. The notation $A > 0$ stands for positive definite matrix. The notation $\mathrm{He}\{A\}$ stands for $A^{\mathrm{T}} + A$.

2 Problem Formulation

2.1 Robust LQ Controller Synthesis

Consider a continuous time single-input single-output system described by:

$$\begin{cases} E(\delta)\dot{x}(t) = A(\delta)x(t) + B(\delta)u(t) \\ y(t) = C(\delta)x(t), \end{cases} \tag{1}$$

$$E(\delta) = E_0 + \sum_{i=1}^{k} \delta_i E_i,\ A(\delta) = A_0 + \sum_{i=1}^{k} \delta_i A_i,$$
$$B(\delta) = B_0 + \sum_{i=1}^{k} \delta_i B_i,\ C(\delta) = C_0 + \sum_{i=1}^{k} \delta_i C_i,$$

where $E_0, E_i, A_0, A_i \in \Re^{n\times n}$, $B_0, B_i \in \Re^{n\times m}$, $C_0, C_i \in \Re^{p\times m}$. Eq. (1) has affine perturbation in each coefficient matrix, where $\delta_i \in \Re$ is perturbation elements which satisfy $|\delta_i| \leq 1$. Generally, Eq. (1) is transformed to the state space representation by premultiplying E^{-1}. However, it is difficult to deal with perturbation elements in the state space representation when elements δ exist as not affine by premultiplying E^{-1}. As one of the methods to solve the problem, the redundancy of descriptor representation is adopted to more easily deal with elements δ. Let $x_d = [x^{\mathrm{T}}\ \dot{x}^{\mathrm{T}}\ u]^{\mathrm{T}}$ be descriptor variables. Then Eq. (1) is described as:

$$\begin{cases} \hat{E}_d\dot{x}_d(t) = \hat{A}_d(\delta)x_d(t) + \hat{B}_d u(t) \\ y(t) = \hat{C}x_d(t), \end{cases} \tag{2}$$

$$\hat{E}_d = \mathrm{diag}\{I, 0, 0\}, \hat{C} = [C(\delta) \quad 0 \quad 0],$$
$$\hat{A}_d = \begin{bmatrix} 0 & I & 0 \\ A(\delta) & -E(\delta) & B(\delta) \\ 0 & 0 & -I \end{bmatrix}, \hat{B}_d = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}.$$

By using the descriptor variables, it can be seen that perturbation elements δ are integrated into one coefficient matrix. One integrator is added inside the controlled loop to track the output of the plant to the reference without error. For Eq. (2), let $y, r, e_p := y - r$, and z be observable output, reference, error and integrated value of e_p, respectively. Letting state be $\tilde{x}_d = [x_d^{\mathrm{T}}\ z]^{\mathrm{T}}$, the augmented system with integrator is obtained as Eq. (3).

$$\tilde{E}_d\dot{\tilde{x}}_d(t) = \tilde{A}_d\tilde{x}_d(t) + \tilde{B}_d u(t) \tag{3}$$

$$\tilde{E}_d = \begin{bmatrix} \hat{E}_d & 0 \\ 0 & 0 \end{bmatrix}, \tilde{A}_d = \begin{bmatrix} \hat{A}_d & 0 \\ -\hat{C} & 0 \end{bmatrix}, \tilde{B}_d = \begin{bmatrix} 0 \\ \hat{B}_d \end{bmatrix}$$

To derive a stabilizing state feedback controller $u = -K\tilde{x}_d$, the following cost function is considered.

$$J = \int_0^\infty (\tilde{x}_d(t)^{\mathrm{T}} Q\tilde{x}_d(t) + u(t)^{\mathrm{T}} Ru(t))dt \tag{4}$$

Here $Q \in \Re \geq 0$ and $R \in \Re > 0$ are given weighting matrices. To minimize the cost function (4), the following LMI conditions are considered [1] [2].

Theorem 1 *If there exist $X_{11} > 0$, X_d and Y_d such that Eq. (5)- (7) hold, the close loop system is stabilized by the state feedback $u(t) = Y_d X_d^{-1}\tilde{x}_d(t) = YX_{11}^{-1}x(t) := -Kx(t)$.*
minimize γ subject to;

$$\begin{bmatrix} \mathrm{He}[\tilde{A}_d X_d - \tilde{B}_d Y_d] & X_d(Q^{\frac{1}{2}})^{\mathrm{T}} & Y_d(R^{\frac{1}{2}})^{\mathrm{T}} \\ Q^{\frac{1}{2}}X_d & -I & 0 \\ R^{\frac{1}{2}}Y_d & 0 & -I \end{bmatrix} < 0, \tag{5}$$

$$X_d = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}, Y_d = [Y \quad 0] \tag{6}$$

$$\begin{bmatrix} Z & I \\ I & X_{11} \end{bmatrix} > 0, \mathrm{trace}[Z] < \gamma. \tag{7}$$

Furthermore, through maximizing the trace of X_{11}, J is minimized.

Note the structure of Lyapunov matrix X_d and that of variable matrix Y_d. Structure of the Lyapunov matrix X_d is naturally restricted by the structure of matrix $\tilde{E}_d$ because Lyapunov function $V(\tilde{x}_d) = \tilde{x}_d^{\mathrm{T}}\tilde{E}_d^{\mathrm{T}}X_d^{-1}\tilde{x}_d = \tilde{x}_d^{\mathrm{T}}X_d^{-\mathrm{T}}\tilde{E}_d\tilde{x}_d$ is considered. Synthesized controller is divided as:

$$K = [\ \underbrace{K_{x1} \ \cdots \ K_{xn}}_{K_x} \ \ K_I\ ], \tag{8}$$

where $K_I \in \Re$ is integral gain and $K_x \in \Re^{m\times n}$ is state feedback gain.

2.2 Adaptive Controller with σ-modification Synthesis

Controller synthesis of the adaptive controller with σ-modification is explained [3]. Consider a single-input single-output system described by:

$$\begin{cases} \dot{x}_s(t) = A_s x_s(t) + B_s(u(t) + W^{\mathrm{T}}\phi(x_s(t))) \\ y_s(t) = C_s x_s(t), \end{cases} \tag{9}$$

where x_s is the state vector, u is the input, y_s is the output, W is an uncertain parameter vector, ϕ is a known set of smooth basis function, and matrices A_s, B_s, C_s are known. Let

$$u = u_{nom} - u_{ad}. \tag{10}$$

The robust LQ controller:

$$u_{nom} = -K_x x_s + K_I \int (r - y_s)dt, \tag{11}$$

is assumed to be designed in the closed loop system with $W = 0$. The reference model for desired system behavior is described as:

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m \int (r - y_m)dt \\ y_m(t) = C_m x_m(t), \end{cases} \tag{12}$$

where $A_m = A_s - B_s K_x$ is Hurwitz and $B_m = B_s K_I$. u_{ad} is an adaptive signal to approximately cancel the uncertainty $W^{\mathrm{T}}\phi(x_s)$ that is given by:

$$u_{ad} = \widehat{W}(t)^{\mathrm{T}}\phi(x_s(t)). \tag{13}$$

Here estimated $\widehat{W}(t)$ for the uncertain parameter vector W in Eq. (9) is updated by:

$$\dot{\widehat{W}}(t) = -\gamma_s \phi(x_s(t))e(t)^{\mathrm{T}} P B_s - \sigma \widehat{W}(t), \tag{14}$$

where $\gamma_s > 0$ is the adaptive gain, σ is σ-modification gain. Then $P > 0$ is obtained by solving the following Lyapunov inequality:

$$A_m^{\mathrm{T}} P + P A_m < 0. \tag{15}$$

Let the tracking error be $e(t) = x_m(t) - x_s(t)$. The tracking error dynamics are described by:

$$\dot{e}(t) = A_m e(t) + B_s \widetilde{W}(t)^{\mathrm{T}}\phi(x_s(t)), \tag{16}$$

where $\widetilde{W}(t) = \widehat{W}(t) - W$ is the weight estimation error. Eq. (14) is equivalent to:

$$\dot{\widetilde{W}}(t) = -\gamma_s \phi(x_s(t))e(t)^{\mathrm{T}} P B_s - \sigma \widetilde{W}(t) - \sigma W(t). \tag{17}$$

Let $\zeta(t) = [e(t)^{\mathrm{T}}\ \widetilde{W}(t)^{\mathrm{T}}]^{\mathrm{T}}$. Then the error dynamics composed of the tracking error and the weight estimation error is described as follows.

$$\dot{\zeta}(t) = \underbrace{\begin{bmatrix} A_m & B_s\phi(x_s(t))^{\mathrm{T}} \\ -\gamma_s\phi(x_s(t))B_s^{\mathrm{T}}P & -\sigma I \end{bmatrix}}_{\bar{A}(x_s)} \zeta(t) + \underbrace{\begin{bmatrix} 0 \\ -I \end{bmatrix}}_{\bar{B}} \sigma W \tag{18}$$

For Eq. (18), quadratic stability analysis is discussed. Let $\bar{A}(x_s)$ in Eq. (18) be decomposed as follows:

$$\bar{A}(x_s) = \underbrace{\begin{bmatrix} A_m & 0 \\ 0 & -\sigma I \end{bmatrix}}_{A_{r0}} + \underbrace{\begin{bmatrix} 0 & B_s\phi(x_s(t))^{\mathrm{T}} \\ -\gamma_s\phi(x_s(t))B_s^{\mathrm{T}}P & 0 \end{bmatrix}}_{A_r(x_s)}. \tag{19}$$

In this study, we note that the basis funciton $\phi(x_s) = [\phi_1(x_s),\ \cdots,\ \phi_j(x_s)]^{\mathrm{T}}$ is known, and hence we can calculate the domain, i.e., $\phi_j(x_s) \in [\min(\phi_j(x_s)), \max\phi_j(x_s)] = [\underline{\phi}_j, \overline{\phi}_j]$. Then matrix $A_r(x_s)$ is described as follows.

$$A_r(x_s) = \sum_{j=1}^{n} \phi_j(x_s) A_{rj} \tag{20}$$

From Eq. (20), Eq. (19) is rewritten as follows.

$$\bar{A}(x_s) = A_{r0} + \sum_{j=1}^{n} \phi_j(x_s) A_{rj} = \bar{A}(\rho(t)) \tag{21}$$

Eq. (21) is affine with respect to $\rho(t) = \phi(x_s(t))$. Then the following relationship is obtained.

$$\dot{\zeta} = \bar{A}(x_s)\zeta = \bar{A}(\rho(t))\zeta \tag{22}$$

The exponential stability for the homogeneous system in Eq. (22) is checked as a feasibility problem of the following LMI [3].

Theorem 2 *The system in Eq. (22) is quadratically stable for perturbation ρ if there exists $X = X^{\mathrm{T}} > 0$ such that Eq. (23) holds.*

$$\bar{A}(\rho)^{\mathrm{T}} X + X\bar{A}(\rho) < 0. \tag{23}$$

Note that the quadratic stability is analyzed by solving LMI conditions for all perturbations ρ.

3 Application to Jib Crane

The simplified model of the jib crane used in this study is shown in Figure 1. The input I_j[A] is a current of the

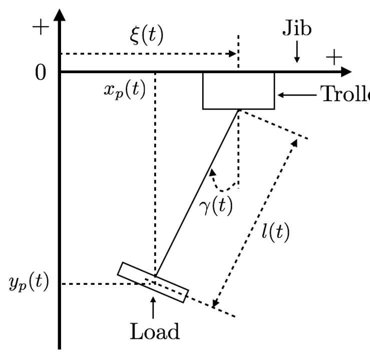

Figure 1 Simplified diagram of jib crane

jib motor and the output is the horizontal position of the load. Let the horizontal position of the load and the vertical position of the load be $x_p(t) = \xi(t) - l(t)\sin(\gamma(t))$ and $y_p(t) = -l(t)\cos(\gamma(t))$, where $\xi(t)$ is the position of the trolley, $l(t)$ is the rope length and $\gamma(t)$ is the swing angle of the load, respectively. Note that the hoisting system of the rope length is controlled by an another independent controller which is not discussed in this paper. The controller for the horizontal position of the load must be robust for the rope length $l(t)$ which is measured. The nonlinear friction F_n shown in Figure 2 is considered in this study. Let the viscous friction,

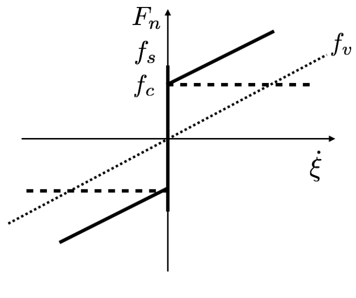

Figure 2 Friction model

the coulomb friction and the maximum static friction be f_v, f_c and f_s, respectively. The friction F_n is as follows:

$$F_n(\xi,\dot{\xi}) = \begin{cases} f_v\dot{\xi}(t) + \mathrm{sgn}(\dot{\xi}(t))f_c(\xi) & \dot{\xi}(t) \neq 0 \\ \mathrm{sgn}(I_j(t))f_s & \dot{\xi}(t) = 0, \end{cases} \quad (24)$$

where sgn$(\cdot)$ is the signum function. Note the structure of the coulomb friction coefficient $f_c(\xi)$. In our plant, we have already known that the contact surface and condition are varied by some experiments. The viscous friction coefficient f_v and the maximum static friction coefficient f_s are a constant, however, these value are unknown in controller synthesis. Let generalized coordinate be $q = [\xi \ \gamma]^{\mathrm{T}}$. The mathematical model of the jib crane is obtained as follows [4].

$$E(l)\ddot{q} + F(\dot{l})\dot{q} + G(\ddot{l})q = HI_j - F_n \quad (25)$$

$$E = \begin{bmatrix} m_j & -m_p l \\ -m_p & m_p l \end{bmatrix}, F = \begin{bmatrix} 0 & -2m_p\dot{l} \\ 0 & 2m_p\dot{l} \end{bmatrix}, G = \begin{bmatrix} 0 & -m_p\ddot{l} \\ 0 & m_p g \end{bmatrix},$$

$$H = [k_t \quad 0]^{\mathrm{T}}$$

For Eq. (25), let state vector be $x = [q \ \dot{q}]^{\mathrm{T}}$ and input be $u = I_j$. Then jib system is described as Eq. (1) with the following E_d, A_d, B_d and C.

$$E_d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & m_j & -m_p l \\ 0 & 0 & -m_p & m_p l \end{bmatrix}, A_d = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & m_p\ddot{l} & 0 & 2m_p\dot{l} \\ 0 & -m_p g & 0 & -2m_p\dot{l} \end{bmatrix},$$

$$B_d = [0 \quad 0 \quad k_t \quad 0]^{\mathrm{T}}, C = [1 \quad -l \quad 0 \quad 0] \quad (26)$$

One integrator is added inside the controlled loop. Let $y, r, e_p := y - r$, and z be observable output, reference, error and integrated value of e_p, respectively. Letting state as $x_p = [x^{\mathrm{T}} \ z]^{\mathrm{T}}$, matrices $\hat{E}_d, \hat{A}_d$ and $\hat{B}_d$ of the augmented system with integrator are obtained as follows.

$$\hat{E}_d = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & m_j & -m_p l & 0 \\ 0 & 0 & -m_p & m_p l & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \hat{A}_d = \begin{bmatrix} A_d & 0_{4\times 1} \\ -C & 0 \end{bmatrix},$$

$$\hat{B}_d = [0 \quad B_d] \quad (27)$$

Note that there exists the product of uncertain parameter into matrix $\hat{E}_d^{-1}\hat{A}_d$. Hence, let $\tilde{x}_d = [x^{\mathrm{T}} \ z \ \ddot{q}^{\mathrm{T}}]^{\mathrm{T}}$ as descriptor variables. Then the jib system is described as Eq. (3) with the following $\tilde{E}_d, \tilde{B}_d$ and $\tilde{A}_d$.

$$\tilde{E}_d = \mathrm{diag}\{1,1,1,1,1,0,0\}, \tilde{B}_d = [0 \ 0 \ \hat{B}_d]^{\mathrm{T}},$$

$$\tilde{A}_d = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & l & 0 & 0 & 0 & 0 & 0 \\ 0 & m_p\ddot{l} & 0 & 2m_p\dot{l} & 0 & -m_j & m_p l \\ 0 & -m_p g & 0 & -2m_p\dot{l} & 0 & m_p & -m_p l \end{bmatrix} \quad (28)$$

By introducing the descriptor variable, it can be seen that only matrix $\tilde{A}_d$ linearly depends on uncertain parameters $l, \dot{l}$ and $\ddot{l}$. Here, the parameter box (29) is defined by lower bounds and upper bounds of parameter $l, \dot{l}$ and $\ddot{l}$.

$$\Theta = \{\theta = [\theta_1, \theta_2, \theta_3]^{\mathrm{T}} : \theta_i \in \{\underline{\theta}_i, \overline{\theta}_i\}\}, \theta_1 = l, \theta_2 = \dot{l}, \theta_3 = \ddot{l} \quad (29)$$

The lower bounds and upper bound of $l, \dot{l}$ and $\ddot{l}$ are assumed as $\theta_1 = l \in [0.1, 0.7], \theta_2 = \dot{l} \in [-0.2609, 0.2609], \theta_3 = \ddot{l} \in [-2.0218, 2.0218]$ from some experiments. Then the matrix $\tilde{A}_d$ is described as follows:

$$\tilde{A}_d = \tilde{A}_{d0} + \sum_{i=1}^{3} \theta_i \tilde{A}_{di}. \quad (30)$$

To derive the stabilizing state feedback controller, the cost function (4) is considered. For the redundant descriptor system (28), the state feedback controller is obtained by solving LMI conditions shown in Theorem 1 at each vertex of matrix $\tilde{A}_d$. Let weighting matrices Q and R be as (31). The robust LQ controller is obtained as (32).

$$Q = \mathrm{diag}[2000 \ 500 \ 100 \ 100 \ 100 \ 0 \ 0], R = 1 \quad (31)$$

$$K = [\ \underbrace{0.1127 \ 0.2957 \ 0.1228 \ 0.1327}_{K_x} \ | \ \underbrace{-0.0236}_{K_I}\ ] \quad (32)$$

Then the obtained controller gain K is divided into the state feedback gain $K_x \in \Re^{1\times 4}$ and the integral gain $K_I \in \Re$.

From here, adaptive controller is designed. Let controlled plant as follows:

$$A_s = E_d^{-1}A_d = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m_p g}{M} & 0 & 0 \\ 0 & -\frac{m_j g}{Ml} & 0 & 0 \end{bmatrix}, \ B_s = E_d^{-1}B_d = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M}k_t \\ \frac{1}{Ml}k_t \end{bmatrix}, \quad (33)$$

where $M = 1/(m_j - m_p)$. The reference model with $A_m = A_s + B_sK_x, B_m = B_sK_I$ stabilized by the obtained state feedback controller K is employed. Note that uncertainties of $\dot{l}$ and $\ddot{l}$ are not considered in the reference model since there become many vertices in Theorem 2. Adaptive law with σ-modification (14) is employed. The matrix P is obtained by through solving Eq. (15) for uncertainty of l which exists in matrix A_m. $\phi(x_s(t)) \in \Re^{4\times 1}$ is the known set of smooth basis function. In this study, the nonlinear uncertainty $W^{\mathrm{T}}\phi(x_s(t)))$ is supposed as the nonlinear friction $F_n(\xi, \dot{\xi})$

shown in Eq. (24). The nonlinear friction $F_n(\xi,\dot{\xi})$ involves not the swing angle of the load $\gamma(t)$ and its velocity $\dot{\gamma}(t)$ but the position of the trolley $\xi(t)$ and its velocity $\dot{\xi}(t)$ by some experiments. The coulomb friction is considered as the function of the trolley position. The viscous friction coefficient is the unknown constant in controller synthesis. Thus, we considered that it is appropriate to choose the basis function for $f_v\dot{\xi}(t)$ and $\mathrm{sgn}(\dot{\xi}(t))f_c(\xi)$. Hence, region of basis function is decided on considering the position of the trolley ξ and its velocity $\dot{\xi}$. Let the basis function be

$$\phi(\hat{x}_s) = [\phi_1(\xi), 0, \phi_3(\dot{\xi}), 0]^{\mathrm{T}} \in [\min(\phi(\hat{x}_s)), \max\phi(\hat{x}_s)]. \tag{34}$$

where $\phi_1(\xi) = \xi$ and $\phi_3(\dot{\xi}) = \dot{\xi}$. The lower bound and the upper bound of ξ and $\dot{\xi}$ are assumed as $\xi \in [-1.1, 1.1]$ and $\dot{\xi} \in [-0.3, 0.3]$ by some experiments. Eq. (18) is considered as LPV system with respect to $\phi(\hat{x}_s)$. For the lower and upper bound of $\phi_1(\xi)$ and $\phi_3(\dot{\xi})$, the matrix $\bar{A}(\hat{x}_s)$ is described as follows:

$$\bar{A}(\hat{x}_s) = A_{r0} + \sum_{i=1}^{2}\sum_{j=1}^{2} \phi_{1i}\phi_{3j}A_{rij} = \bar{A}(\rho(t)). \tag{35}$$

Let the adaptive gain γ_s and the σ-modification gain σ be as 5.5×10^{-5} and 5×10^{-5}, respectively. For (35), quadratic stability is analyzed by solving LMI condition shown in Theorem 2 at each vertex of $\phi(\hat{x}_s)$ and uncertainty of l which exists in matrix A_m.

4 Simulation

In this section, the effectiveness of the designed controller is verified by simulations. Furthermore, the usefulness of the proposed method is discussed by comparing with the robust LQ controller without the MRAC law. The coulomb friction coefficient f_c, the viscous friction coefficient f_v and the maximum static friction f_s are set as follows.

$$\begin{aligned} f_c &= \begin{cases} 2.2 & 0 \le \xi < 0.2 \\ 4.2 & 0.2 \le \xi \le 0.6 \\ 3.2 & 0.6 \le \xi, \end{cases} \\ f_v &= 6.2, \\ f_s &= 2.3 \end{aligned} \tag{36}$$

Note that these coefficients are unknown in controller synthesis. In this study, the rope length is controlled independently by the another controller. The time response of the horizontal position of the load for step response is shown in Figure 3. The swing angle for the load is shown in Figure 4. The solid line and the dotted line mean the proposed method and the robust LQ controller without the MRAC law, respectively. As can be seen in Figure 3, the load controlled by the proposed method converges smoothly. However, the robust LQ controller without the MRAC law yields the performance degradation by influence of the friction. As shown in Figure 4, the proposed method can suppress the oscillation of the load, while the load controlled by robust LQ controller without the MRAC law yields the performance degradation by the oscillation. This results mean influence of the friction is reduced by adding the MRAC law. In other words, the proposed system is effective for the friction which is not considered in the robust LQ controller synthesis.

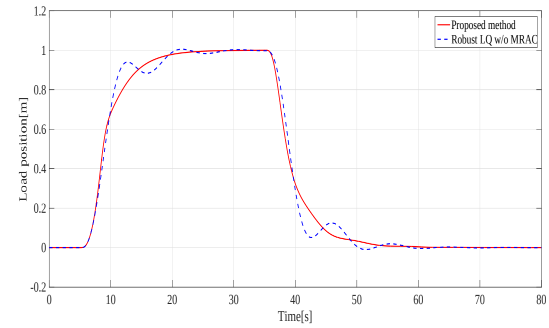

Figure 3 Horizontal position of load

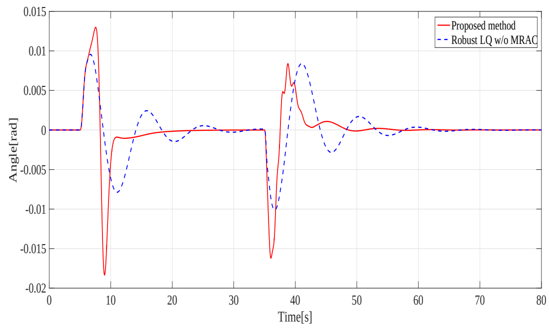

Figure 4 Swing angle γ of load

5 Conclusions

In this paper, the robust LQ control system adding the MRAC law with σ-modification is proposed for the jib crane. The stability for the composed system of the tracking error and the weight estimation error is analyzed based on LMI. The main results of this study are to deal with uncertainties, which can be linearly treated by the robust LQ controller, and to consider other nonlinear uncertainties with friction by adding the MRAC law. The effectiveness of the proposed method is verified by comparing with the robust LQ controller without the MRAC law in simulations. From simulations, the proposed method is better than the robust LQ controller without the MRAC law for the nonlinear friction. It can be said that the proposed method improves the control performance for nonlinear uncertainties with friction.

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