Gain Scheduling Control for ABS with Estimation of Friction Coefficient by Unscented Kalman Filter

M2014SC008 Hiroshi KATAOKA
supervisor : Isao TAKAMI

Abstract

Anti-lock Brake System(ABS) is strongly nonlinear. In my study, the system is linearized and state equation is derived. However, the system has rational terms. Using descriptor representation and linear fractional transformation, equivalent system that has only linear terms is derived. The gain scheduling control based on Lyapunov function is applied for the ABS. Friction coefficient that is one of scheduling parameters is measured by unscented kalman filter since the friction coefficient can not be measured on real time. The effectiveness of the proposed method is illustrated by simulations.

1 Introduction

Anti-lock braking system (ABS) prevents cars from slip by locking wheels in brake operation at low friction road surface, or when suddenly braking. It is well known that when slip rate is nearly around 0.2, friction coefficient for lateral force and longitudinal force between tire and road are high enough[2]. By keeping the slip rate 0.2, braking distance can be shortened and skidding can be prevented. Since the slip rate depends on car velocity and wheel velocity, the ABS dynamics depends on them and the friction coefficient of road. The main difficulty in the design of ABS is caused by its strongly nonlinear and uncertain characteristics. For such difficulty, many contributions for ABS can be found in the literature. For example, PID-type approaches[1] and composite control[2] are reported. Extremum seeking control based on numerical optimization is presented in [3], [4]. The fuzzy control which has learning ability to compensate for adverse road condition is proposed[5]. On the other hand, model based approaches are presented. In several papers, sliding mode control for ABS is applied[6],[7],[8]. Gain scheduling control is proposed[9],[10]. It is well known that GS control has a potential to deal with large variation range and to improve the control performance.

Recently, unscented kalman filter (UKF) is proposed to estimate uncertain parameters[11]. It is nonlinear kalman filter. Extend kalman filter is one of the nonlinear kalman filter. However, Extend kalman filter is required to linearize nonlinear system. Since linearized system doesn't require in algorithm of UKF, UKF can be applied for nonlinear system as that.

In this paper, a design method is proposed to guarantee the robust stability for ABS. Since the car velocity and friction coefficient affects the dynamics, the variations of these parameters have to be considered. The polytopic representation for the system with these parameters is proposed. Since the system described in framework of state space equation has rational terms of car velocity and friction coefficient, guaranteeing the robust stability for the system with car velocity and friction coefficient by using the polytopic representation is difficult. By using descriptor representation and linear fractional transformation (LFT), the system which is affine with respect to car velocity and friction coefficient can be obtained. Then, the GS controller based on linear control theory can be designed. The robust stability and the robust performance are guaranteed theoretically by solving LMIs. The GS controller whose scheduling parameters are car velocity and friction coefficient is designed. However, friction coefficient can not be measured on real time. To measure the friction coefficient, UKF is proposed. The measured friction coefficient is assumed as scheduling parameters. Then, GS controller is applied for ABS and the effectiveness of the proposed method is illustrated by simulations.

2 Modeling and Control Target

The model of the simplified ABS experimental device used in this study is shown in Fig. 1. It is the one wheel model that is 1/4 scale of the real car. The upper wheel simulates the car wheel, and the lower wheel simulates the road. The point A is the axis of rotation of the balance lever, which supports the axis of the upper wheel. Table 1 shows the physical constants and variables used in this study.

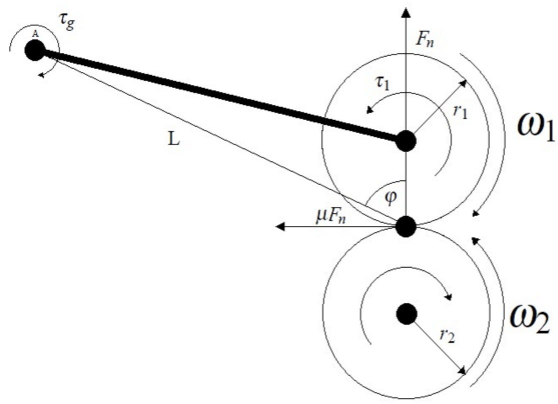

Fig. 1 Simplified Diagram of the ABS Experimental Device

Table 1 The Physical Constants and Variables

ω_1	Angular velocity of the upper wheel
ω_2	Angular velocity of the lower wheel
τ_1	Braking torque
r_1	Radius of the upper wheel
r_2	Radius of the lower wheel
J_1	Moment of inertia of the upper wheel
J_2	Moment of inertia of the lower wheel
F_n	Normal force
μ	Coefficient of friction between wheels
τ_g	Torque acting on the balance lever
L	Distance to the contact between wheels from the axis of rotation of the balance lever
φ	Angle between L and the normal of contact at wheels
λ	Slip rate

The dynamical equations of the rotational motion of the upper wheel and lower wheel are shown by Eq.(1) and Eq.(2).

$$J_1\dot{\omega}_1 = F_n r_1 \mu - \tau_1 \tag{1}$$
$$J_2\dot{\omega}_2 = -F_n r_2 \mu \tag{2}$$

The slip rate is defined by Eq.(3) as the function of car velocity and wheel velocity.

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} \tag{3}$$

The sum of torques corresponding to the point A is given as follows.

$$F_n L(\sin\varphi - \mu\cos\varphi) = \tau_g + \tau_1 \tag{4}$$

The following equation is obtained from Eq.(3).

$$\dot{\lambda} = -\frac{r_1}{r_2\omega_2}\dot{\omega}_1 + \frac{r_1\omega_1}{r_2\omega_2^2}\dot{\omega}_2 \tag{5}$$

Finally, the dynamical equation of slip rate is derived from Eq.(1), Eq.(2), Eq.(4) and Eq.(5) as follows.

$$\dot{\lambda} = \frac{1}{\omega_2}f(\lambda) + \frac{1}{\omega_2}g(\lambda)\tau_1,\ \omega_2 \neq 0 \tag{6}$$

The behavior around equilibrium point (λ^*, τ_1^*) is considered. Here, λ^* is the reference slip rate, and τ_1^* is the equilibrium braking torque to keep λ^*.

$$\tau_1^* = -\frac{f(\lambda^*)}{g(\lambda^*)} \tag{7}$$

Eq.(8) is obtained by substituting the equilibrium point in Eq.(6).

$$\frac{1}{\omega_2}f(\lambda^*) + \frac{1}{\omega_2}g(\lambda^*)\tau_1^* = 0 \tag{8}$$

Using Taylor expansion around the equilibrium point, nonlinear model Eq.(6) can be linearized[9].

$$\dot{\lambda} \simeq \dot{\lambda}(\lambda^*, \tau_1^*) + \frac{\partial\dot{\lambda}}{\partial\lambda}|_{\lambda=\lambda^*}(\lambda - \lambda^*) + \frac{\partial\dot{\lambda}}{\partial\tau_1}|_{\tau=\tau_1^*}(\tau_1 - \tau_1^*) \tag{9}$$

The linearized equation is derived by substituting Eq.(7) into Eq.(9).

$$\begin{aligned}\dot{\lambda} = &\left(\frac{c_1\mu^2 + c_2\mu}{\omega_2(c_6\mu^2 + c_7\mu + c_8)}\right)(\lambda - \lambda^*)\\ &+\left(\frac{c_3\mu^2 + c_4\mu + c_5}{\omega_2(c_6\mu^2 + c_7\mu + c_8)}\right)(\tau_1 - \tau_1^*)\end{aligned} \tag{10}$$

Here, c_1, c_2, $\cdots$, c_8 are constants. Note that the dynamics of $\dot{\lambda}$ depends on ω_2 and μ.

3 Polytopic Representation

In this section, I show the process to obtain ABS system which is multi-affine for uncertain parameters μ and ω_2. The robust stability for these parameters is guaranteed by using polytopic representation. The system described in the framework of the state space representation has rational terms of these parameters. It is difficult to use polytopic representation for the system which has rational terms of uncertain parameters. Therefore, the system which is multi-affine for uncertain parameters μ and ω_2 is obtained by using the descriptor representation and LFT.

3.1 Transformation to Polynomial

State equation is obtained from Eq.(10). In order to track the output of the system to the optimal value without error, I add one integrator to the state variable. Let state variable $x(t) = [x_1(t) \;\; x_2(t)]^T = [\int(\lambda - \lambda^*)dt \;\; \lambda - \lambda^*]^T$ and input $u(t) = \tau_1 - \tau_1^*$. Then the state equation is obtained as follows.

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned} \tag{11}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & \frac{a}{e} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \frac{b}{e} \end{bmatrix}, c = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$e = \omega_2(c_6\mu^2 + c_7\mu + c_8)$$
$$a = c_1\mu^2 + c_2\mu$$
$$b = c_3\mu^2 + c_4\mu + c_5$$

Here Eq.(11) has rational terms of μ and ω_2. It is difficult to use polytopic representation. Eq.(11) is transformed to descriptor form to transform from rational to polynomial. Descriptor equation is obtained as follows.

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{12}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & e \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix}, B = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

Note that Eq.(12) is polynomial for uncertain parameters μ and ω_2.

3.2 Putting All Uncertain Parameter and Varying Parameter into One Matrix

Eq.(12) becomes polynomial for uncertain parameters μ and ω_2. However, there exists these parameters in matrices E, A and B. If Eq.(12) is transformed to an ordinary state space representation $\dot{x}(t) = E^{-1}Ax(t) + E^{-1}Bu(t)$, matrices $E^{-1}A$ and $E^{-1}B$ have rational terms of μ and ω_2. It is difficult to use polytopic representation. I put all uncertain parameters into one matrix by expanding dimension of matrices. Uncertain parameters μ and ω_2 are put into matrix A_d by defining $x_d(t) = [x(t) \;\; \dot{\lambda} \;\; u]^T$.

$$E_d\dot{x}_d(t) = A_d x_d(t) + B_d u(t) \tag{13}$$

$$E_d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_d = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & a & -e & b \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$$

Note that there exist the all uncertain parameters and μ and ω_2 in only A_d.

3.3 Transformation to Malti-affine

Only the matrix A_d has uncertain parameters. However, there are higher order terms of μ in it. In this case, it is still difficult for the matrix to use the polytopic representation. LFT is applied to transform the high order terms of μ to first order terms of μ.

Let $\tilde{x}_d$ be new descriptor variable $\tilde{x}_d(t) = [x_d^T(t) \quad z_\delta(t)]^T$, and the descriptor equation which is transformed by LFT is the following equation. Then, the system becomes multi-affine for μ and ω_2.

$$\tilde{E}_d\dot{\tilde{x}}_d(t) = \tilde{A}_d\tilde{x}_d(t) + \tilde{B}_d u(t)$$
$$tildeE_d = \begin{bmatrix} E_d & 0 \\ 0 & 0 \end{bmatrix}, \tilde{A}_d = \begin{bmatrix} A_n & B_\delta\Delta \\ C_\delta & -I \end{bmatrix}, \tilde{B}_d = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$$
$$A_n = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & c_2\mu & -(c_7\mu + c_8)\omega_2 & c_4\mu + c_5 \\ 0 & 0 & 0 & -1 \end{bmatrix}, B_\delta = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$
$$C_\delta = [\ 0 \quad c_1\mu \quad -c_6\mu\omega_2 \quad c_3\mu\ ] \qquad \Delta = \mu$$

A_n, C_δ and $B_\delta\Delta$ don't have polynomial but only multi-affine terms of μ and ω_2. The ABS model which is multi-affine for ω_2 and μ can finally be obtained.

4 GS Control Design

I design a gain scheduling controller. The parameter box Θ is defined by vertexes which are upper bound and lower bound of μ and ω_2. Let scheduling parameters θ_1, θ_2 be θ_1=ω_2, θ_2=μ. Since I use parameter Lyapunov dependent function, parameter box includes derivative of its.

$$\Theta = \{[\theta_1\ ,\theta_2\ ,\theta_3\ ,\theta_4] : \theta_i \in \{\underline{\theta}_i,\ \bar{\theta}_i\}\}(i = 1, 2) \quad (14)$$
$$\theta_1 = \omega_2,\ \theta_2 = \mu, \theta_3 = \dot{\omega}_2,\ \theta_4 = \dot{\mu}$$

To derive a stabilizing state feedback $u(t) = \tilde{K}_d(\theta)x(t)$, I consider minimizing the following quadratic cost function.

$$J = \int_0^\infty (\tilde{x}_d(t)^T Q\tilde{x}_d(t) + u(t)^T Ru(t))dt \qquad (15)$$

Here $Q \geq 0$ is a weight matrix for state variables, and $R > 0$ is a weight matrix for inputs. From Lyapunov's stability theorem, if there exists P(θ) satisfying the following matrix inequalities, the descriptor system is stable and $J \prec trace(\tilde{E}_d P(\theta))$ is guaranteed[12].

$$\tilde{E}_d P(\theta) = (\tilde{E}_d P(\theta))^T \succeq 0 \qquad (16)$$
$$P(\theta)(\tilde{A}_d(\theta) + \tilde{B}_d\tilde{K}_d(\theta)) + (\tilde{A}_d(\theta) + \tilde{B}_d\tilde{K}_d(\theta))^T P(\theta) + Q + \tilde{K}_d(\theta)^T R\tilde{K}_d(\theta) \prec 0 \qquad (17)$$

Let $\tilde{X}_d(\theta) := P(\theta)^{-1}$. Then, let $\tilde{Y}_d(\theta) := \tilde{K}_d(\theta)\tilde{X}_d(\theta)$. Although I extended the dimension of the system by using descriptor variables, I can design the controller that uses original states without extended variables by giving these restrictions. In view of structure of $\tilde{E}_d$, I restrict $\tilde{X}_d(\theta)$ and variable matrix $\tilde{Y}_d(\theta)$ as follows.

$$\tilde{X}_d(\theta) = \begin{bmatrix} X_{11}(\theta) & 0 & 0 \\ X_{21}(\theta) & X_{22}(\theta) & X_{23}(\theta) \\ X_{31}(\theta) & X_{32}(\theta) & X_{33}(\theta) \end{bmatrix}, \tilde{Y}_d(\theta) = [\ Y(\theta) \quad 0 \quad 0\ ] \quad (18)$$

The stability is guaranteed by solving LMIs with the scheduling parameters θ at vertexes of the parameter box. Let matrices $\tilde{X}_d(\theta), X_{11}(\theta)\ \tilde{Y}_d(\theta), Y(\theta)$ be as follows, the matrices are represented by Θ_i(i=1,...,16).

$$\tilde{X}_d(\theta) = \tilde{X}_{d0} + \theta_1\tilde{X}_{d1} + \theta_2\tilde{X}_{d2} \qquad (19)$$
$$\tilde{X}_{11}(\theta) = \tilde{X}_{11,0} + \theta_1\tilde{X}_{11,1} + \theta_2\tilde{X}_{11,2} \qquad (20)$$
$$\tilde{Y}_d(\theta) = \tilde{Y}_{d0} + \theta_1\tilde{Y}_{d1} + \theta_2\tilde{Y}_{d2} \qquad (21)$$
$$Y(\theta) = Y_0 + \theta_1 Y_1 + \theta_2 Y_2 \qquad (22)$$
$$\Theta_1 = (\underline{\theta}_1, \underline{\theta}_2, \underline{\theta}_3, \underline{\theta}_4), \Theta_2 = (\bar{\theta}_1, \underline{\theta}_2, \underline{\theta}_3, \underline{\theta}_4)...$$
$$\Theta_{15} = (\bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3, \underline{\theta}_4), \Theta_{16} = (\bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3, \bar{\theta}_4) \quad (23)$$

The cost function J shown in Eq.(15) is adopted to derive the stabilizing state feedback GS controller. Then, multi-affine restriction is applied for Eq.(17). LMI condition to derive the stabilizing state feedback GS controller is as follows.

Lemma 1 *If there exists* $X_{11}(\Theta_i)\succ 0$, $\tilde{X}_d(\Theta_i)$, $\tilde{Y}_d(\Theta_i)$ *satisfying the following LMI, the system is stable.*
minimize :γ
subject to $X_{11}(\Theta_i) \succ 0$

$$\begin{bmatrix} M(\Theta_i) - \tilde{E}_d\dot{\tilde{X}}_d(\Theta_i) & \tilde{X}_d(\Theta_i)^T(Q^{\frac{1}{2}})^T & \tilde{Y}_d(\Theta_i)^T(R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}}\tilde{X}_d(\Theta_i) & -I & 0 \\ R^{\frac{1}{2}}\tilde{Y}_d(\Theta_i) & 0 & -I \end{bmatrix} \prec 0 \quad (24)$$
$$\begin{bmatrix} W & I \\ I & X_{11}(\Theta_i) \end{bmatrix} \succ 0 \quad (25)$$
$$trace(W) < \gamma \quad (26)$$
$$M(\Theta_i) := He[\tilde{A}_d(\Theta_i)\tilde{X}_d(\Theta_i) + \tilde{B}_d\tilde{Y}_d(\Theta_i)] \quad (i = 1, ..., 16)$$

The gain scheduling controller $\tilde{K}_d(\theta)$ with framework of the descriptor representation is given as follow.

$$\tilde{K}_d(\theta) = [Y(\theta)X_{11}(\theta)^{-1} \quad 0 \quad 0] \qquad (27)$$

Finally, the controller $K(\theta) = Y(\theta)X_{11}(\theta)^{-1}$ with framework of state space representation is obtained.

5 Estimation of Friction Coefficient

In this paper, the GS controller whose scheduling parameters are friction coefficient and car velocity is designed. However, friction coefficient can not be measured on real time. UKF is able to apply for nonlinear system. Therefore, UKF is applied to estimate friction coefficient. To apply the UKF, system to estimate the friction coefficient is considered. From Eq.(2), (3), the following equations is derived.

$$\dot{\omega}_2 = -\frac{r_2 M_g S}{J_2} - \frac{r_2 S M_1}{J_2} \qquad (28)$$
$$S = \frac{\mu}{L(sin\phi - \mu cos\phi)} \qquad (29)$$

From Eq.(28), nonlinear state space representation to estimate the friction coefficient is built and S is function of μ. To estimate the friction coefficient, S is added to state variable. Let state variable $x_z(t)$ be $x_z(t)$=[$\omega_2(t)$ $S(t)$ $]^T$ and input $u_z(t)$ =τ_1.

$$\dot{x}_z(t) = Ax_z(t) + B_z S(t)u_z(t) \qquad (30)$$
$$y(t) = C_z(t)x_z(t)$$
$$A_z = \begin{bmatrix} 0 & \frac{M_g r_2}{J_2} \\ 0 & 0 \end{bmatrix}, B_z = [\ \frac{r_2}{J2} \quad 0\ ]^T$$
$$C_z = [\ 1 \quad 0\ ]^T$$

The nonlinear state space representation (30) is discretized. Since μ is varying parameter, S is also varying parameter. S is defined as follows.

$$S(k+1) = S(k) + v(k) \qquad (31)$$

v is process noise. The discretized nonlinear state space representation (32) is as follows. Let state variable

$x_{zd}(t)$ be $x_{zd}(t)$=$[\omega_2(k) \quad S(k)\ ]^T$ and input $u_{zd}(t)$ =τ_1.

$$x_{zd}(k+1) = A_{zd}x(k) + B_{zd}S(k)u_{zd}(k) + B_v v(k) \quad (32)$$
$$y(k) = C_z x_{zd}(k) + w(k)$$
$$A_{zd} = e^{A_z T_s}, B_{zd} = (\int_0^{T_s} e^{A_z \lambda} d\lambda) B_z$$
$$B_v = [\ 0 \quad 1\ ]^T$$

$v(k)$ is process noise, $w(k)$ is observation noise. T_s is sampling time. UKF can be applied for nonlinear system (32). let $\hat{x}(k)$ be a state estimate. By the following equation, optimal value of $\hat{x}(k)$ is decided.

$$\hat{x}(k) = \hat{x}^-(k) + g(k)\{y(k) - \hat{y}^-(k)\} \quad (33)$$

$\hat{x}^-(k)$ is a priori state estimate. $g(k)$ is a kalman gain. $\hat{y}^-(k)$ is a priori output estimate. Optimal value of $g(k)$ is decided by the following equation.

$$g(k) = \frac{P_{xy}^-(k)}{P_{yy}^-(k) + \sigma_w^2} \quad (34)$$

$P_{xy}^-(k)$ is a priori state, output error covariance matrix. $P_{yy}^-(k)$ is a priori output error covariance matrix. σ_w^2 is of covariance of observation noise $w(k)$. Since decision of optimal value of $g(k)$ determines the optimal value of $\hat{x}(k)$, friction coefficient can be estimated by Eq.(33), (34).

6 Simulation

I conducted the simulations of $\mu = 0.7$ and $\mu = 0.1$ when slip rate is 0.2. The range that robust stability of car velocity guaranteed in my study is from 10 to 50[km/h]. Here assuming that $\mu = 0.7$ is dry road and $\mu = 0.1$ is snowy road.

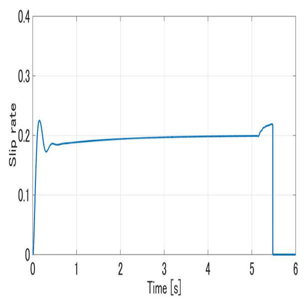

Fig. 2 Slip Rate at snowy

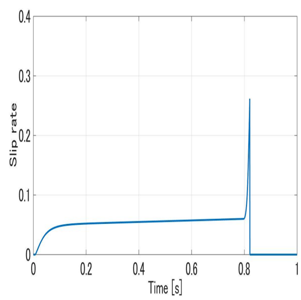

Fig. 3 Slip Rate at dry

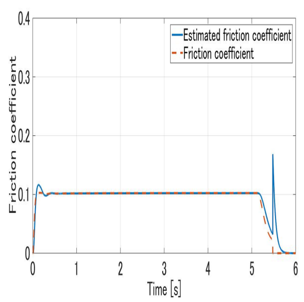

Fig. 4 Friction coefficient at snowy

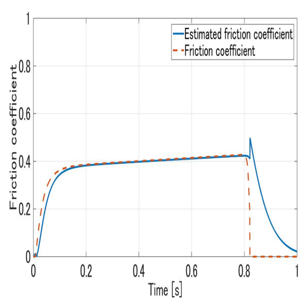

Fig. 5 Friction coefficient at dry

In Fig.4, 5, dot line shows friction coefficient value and solid line shows estimated value. If UKF is applied for ABS, slip rate can be controlled at optimal value in snowy road. However, slip rate doesn't reach to optimal value 0.2 in dry road. The car stops at 0.81 seconds in dry road. Since stopping time is short, it can be said that function of ABS is fulfilled. Friction coefficient can be estimated in both dry and snowy roads.

7 Conclusion

In this paper, a method to guarantee the robust stability for Anti-lock Braking System is shown. Since the car velocity and friction coefficient affect the dynamics, a method is designed to consider variation of these parameters. From the simulation result, the slip rate is controlled to optimal value in snowy road. Friction coefficient can be estimated accurately by unscented kalman filter. Therefore, the effectiveness of the proposed method is verified.

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