Gain-Scheduling Control for Crane via Parameter Dependent Lyapunov Function Considering All Varying Parameters in Dynamics

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1 Introduction

In this paper, Gain-Scheduling (GS) control via parameter dependent Lyapunov function for crane is synthesized. A purpose of cranes system is to lift and carry a load to assigned positions. Cranes are required to transport the load fast, accurately and to reduce oscillations. In previous research, a method to robust control of cranes to suppress effects by variation of the rope length have been reported[1]. For improvement of cranes efficiency, cranes should be operated in movement of the load horizontal and vertical directions simultaneously. It has been reported that decentralized control is effective for cranes[2]. We have reported a method to synthesis GS control via parameter dependent Lyapunov function in concept of treating complete dynamics[3]. The crane model not to ignore velocity and acceleration of the rope length in the dynamics has been derived. However, a mass of the load is not constant and included in the dynamics. Variation of the mass of the load is not considered. By using parameter dependent Lyapunov function, a controller becomes less conservative synthesis in comparison with adopting parameter independent Lyapunov function[4]. However, there is Lyapunov matrix derivative. For this problem, a estimation method of Lyapunov matrix derivative has been reported[5].

In this paper, a GS controller via parameter dependent Lyapunov function which guarantees variation of the rope length, its velocity, acceleration and the mass of the load is synthesized. All varying parameters in the dynamics which is related to operating the crane are taken into account. The rope length, its velocity and acceleration are treated as time-varying parameters. The mass of the load is different in every operation, it is treated as uncertain but time-invariant parameter. Time-varying parameters are assumed to measurable, they are treated as scheduling parameters.

Linear Parameter Varying (LPV) system for scheduling parameters is obtained. Robust stability with vertexes of varying parameters is guaranteed. By using parameter dependent Lyapunov function, there are product of scheduling parameters and derivative of Lyapunov matrix. For these problem, descriptor representation, Linear Fractional Transformation (LFT) and a estimation method of Lyapunov matrix derivative are adopted. The former problem of parameter dependent Lyapunov function can be solved by adopting descriptor representation. We show that adopting descriptor representation and LFT, the controller performance is improved from a viewpoint of evaluated function. By several simulations, it is shown that the controller is effective.

2 Controlled plant

A schematic diagram of the crane is shown in Fig. 1.

A position of the trolley \( \xi [\text{m}] \), a swing angle of the load \( \phi [\text{rad}] \) and the rope length \( l_p [\text{m}] \) are measurable. In addition, it is assumed that velocity and acceleration of the rope length can be obtained. The controlled output is the horizontal position of the load \( y [\text{m}] (y = \xi - l \sin \phi) \). The control input is a current of a jib motor \( I_j [\text{A}] \). The mass of the load is \( m_p \). The mass of the trolley is \( m_j \), the jib motor equivalent moment of inertia is \( J_p \), the ratio of the jib motor gear is \( K_{g,j} \), the radius of the jib motor gear is \( r_{j,p} \), the torque constant of the jib motor is \( K_t \) and the gravitational acceleration is \( g \).

This paper, the followings are assumed. i) The rope is rigid rod without the mass. ii) The load is a material \( \frac{m}{m_p} \). iii) The friction between the trolley and the boom.

iv) The friction between the trolley and the boom is ignored. Let generalized coordinate be \( q = [\xi \quad \phi \quad \ddot{\xi} \quad \ddot{\phi}]^T \), \( \phi \) and its velocity are assumed as small enough. \( \sin \phi \approx \phi \), \( \cos \phi \approx 1 \), \( \phi^2 \approx 0 \). Let \( m_j \) be \( m_j + \frac{J_j}{r_{j,p}^2} \). The mathematical model is obtained as Eq.(1)

\[
E \dddot{x} + F \dot{x} + Gq = HI_j
\]

Let state variable be \( x_j = [q^T \quad \dot{q}^T]^T \), output be \( y \) and input be \( u = I_j \). Then, the state space representation of the crane is obtained as Eq.(2) and Eq.(3).

\[
A_j = \begin{bmatrix} I & 0 \\ E & 0 \end{bmatrix}^{-1} \begin{bmatrix} I & 0 \\ -G & -F \end{bmatrix}
\]

\[
\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & m_j(l - g) & 0 & 0 \\ m_j(l - g) & 0 & m_j \end{bmatrix}
\]

Figure 1 Crane model
\[
B_j = \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{K_j}{m_j l_p} \end{bmatrix} = \left[ \begin{array}{c} 0 \\ \frac{K_j}{m_j l_p} \end{array} \right] \\
C_j = \begin{bmatrix} 1 & -l_p & 0 & 0 \end{bmatrix}
\]

3 Controller synthesis

Scheduling parameters are not only the rope length \( l_p \), but also the velocity \( l_p \) and the acceleration \( l_\dot{p} \). The stability of the closed system should be guaranteed theoretically for variation of the rope length, its the velocity and acceleration. Operation time can be shortened because it is possible to move the trolley and to hoist the rope at same time. An important purpose of controlling crane system is that the load follows references without error. The integral of the error \( x_e \) is added to the state variable \( x_j \) to make the controlled output follows the reference \( r \). The augmented system and state variable \( x(t) \) are assigned to Eq.(4), (5).

\[
\begin{align*}
  x(t) &= \begin{bmatrix} x_{e}(t)^T & q(t)^T & \dot{q}(t)^T \end{bmatrix}^T \\
  x_e &= \int_{t_0}^{t} \xi(e) d\tau, e = r - y \\
  \dot{x} &= A_c x + B_c u \\
  A_c &= \begin{bmatrix} 0 & -C_p l_p \\ 0 & A_p \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ B_p \end{bmatrix}
\end{align*}
\]

3.1 Matrices Transformation

Stability conditions with state space representation are obtained as Eq.(6), (7).

\[
\begin{align*}
  X(\theta) &= X(\theta)^T > 0 \\
  \text{Hc}\{A_c(\theta)X(\theta) + B_c(\theta)Y(\theta)\} - \dot{X}(\theta) < 0
\end{align*}
\]

Where, state feedback controller \( u = K(\theta)x \) and feedback gain \( K(\theta) = Y(\theta)X(\theta)^{-1} \). The coefficient matrices \( A_c \) and \( B_c \) include scheduling parameters. When GS control system is synthesized with a framework of the state space representation of the crane, there are products of scheduling parameters in the coefficient matrices and variable matrix \( Y(\theta) \), scheduling parameters in the coefficient matrices and Lyapunov matrix \( X(\theta) \). Linear Matrix Inequality (LMI) conditions for deriving GS controller can not be solved. Furthermore, the matrix \( A_c \) includes \( l \) and \( \frac{1}{l} \). If a method in framework of polytopic is adopted for Eq.(5), then synthesized controller becomes conservative. In this paper, descriptor representation is adopted. Coefficient matrix \( A_c \) which includes an affine system for \( l, \dot{l}, \ddot{l} \) and coefficient matrix \( B_c \) which does not include scheduling parameters are obtained. The parameter box Eq.(8) is defined by vertexes which consist of upper and lower bounds of the varying parameters. The scheduling parameters are \( \theta = [l_p, \dot{l}_p, \ddot{l}_p] \) and its derivatives are \( \dot{\theta} \). The time-invariant parameter is \( m_p \).

\[
\Theta = \{ \eta \mid \theta = \theta_0 | \theta_1, \theta_2, \theta_3, \theta_4 : \theta_i \in \{ \theta_0, \bar{\theta}_0 \} \} \quad (8)
\]

\[
\begin{align*}
  \theta_0 &= m_p, \theta_1 = l, \theta_2 = \dot{l}, \theta_3 = \ddot{l}, \theta_4 = \ddot{\theta}, (i = 0, ..., 4)
\end{align*}
\]

Let descriptor variable be \( x_d = [x^T \quad \dot{q}^T]^T \) and matrix \( J \) be \([1 \quad -\dot{\theta}]\). Eq.(9) is obtained from Eq.(2).

\[
\begin{align*}
  E_d \ddot{x}_d &= A_d(\theta)x_d + B_d u \\
  E_d &= \text{block diag}(I, I, 1, 0) \\
  A_d &= \begin{bmatrix} 0 & -J & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & -G & -F & -E \end{bmatrix}, B_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ H \end{bmatrix}
\end{align*}
\]

Where, \( I \) is the unit matrix which has the appropriate dimension. From Eq.(9), matrix \( H \) includes only constant. The coefficient matrix \( A_d \) includes only affine terms. If Eq.(9) includes scheduling parameters between first row and fifth row, then flexibility of the synthesized controller is fall. LFT is adopted into Eq.(9) for remove scheduling parameters between first row and fifth row. Let new descriptor variable be \( x_{if} = [x_{id}^T \quad z_i^T]^T \).

\[
\begin{align*}
  E_{if} \ddot{x}_{if} &= A_{if}(\theta)x_{if} + B_{if} u \\
  A_{if}(\theta) &= A_{if0} + \sum_{i=1}^{3} \theta_i A_{ifi} \\
  &= \begin{bmatrix} A_n & \frac{B_{1\Delta}}{C_\Delta} \\ \frac{C_\Delta}{B_{1\Delta}} & -I + D_{1\Delta} \end{bmatrix} \\
  &= \begin{bmatrix} A_{n11} & A_{n12} & B_{1\Delta} \\ A_{n21} & A_{n22} & B_{2\Delta} \\ C_{n1} & C_{n2} & -I + D_{1\Delta} \end{bmatrix} \\
  &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
  E_{if} &= \begin{bmatrix} 0 & 0 \end{bmatrix}, B_{if} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
  A_n &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_{1\Delta} = \begin{bmatrix} (\theta_0 + m_j) \theta_1 \\ \bar{\theta}_0 \theta_1 \\ 0 \\ g \theta_1 \\ -1 \end{bmatrix} \\
  B_{2\Delta} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, C_{\Delta} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{align*}
\]

Where, \( z_3 \) is virtual input. From Eq.(10), the coefficient matrix \( A_{if} \) includes only affine terms and does not include scheduling parameters between first row and fifth row.

3.2 Stability conditions

The stability conditions of descriptor system which is represented by Eq.(10) is related to LQ control specification for the closed loop system assigned by state feedback controller \( u = K_{if}(\theta)x_{if} \). Considering structure of the matrix \( E_{if} \), candidates of Lyapunov matrix \( X_{if}(\theta) \)
and variable matrix $Y_f(\theta)$ are restricted as Eq.(12).

$$X_f(\theta) = \begin{bmatrix} X(\theta) & 0 & 0 \\ X_{2,1}(\theta) & X_{2,2}(\theta) & X_{2,3}(\theta) \\ X_{3,1}(\theta) & X_{3,2}(\theta) & X_{3,3}(\theta) \end{bmatrix}$$ (12)

$$E_f \dot{X}_f = \begin{bmatrix} \dot{X}(\theta) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad Y_f(\theta) = [Y(\theta) \ 0]$$

[Theorem][6] The closed loop system with a plant Eq.(9) and state feedback is stable, if $X(\theta), X_f(\theta)$ and $Y_f(\theta)$ exist satisfying Eq.(13) and (14).

$$X(\theta) > 0$$ (13)

$$\text{He}(A_f(\theta)X_f(\theta) + B_fY_f(\theta)) - E_fX_f(\theta) < 0$$ (14)

Eq.(15) is obtained when $[B_3 \Delta(I - D_3 \Delta^{-1})]$ and its transportation are multiplied to Eq.(14) from both sides.

$$\text{He}(A_f(\theta)X_f(\theta) + B_fY_f(\theta)) - \dot{X}(\theta) < 0$$ (15)

$X(\theta)$ can be seen as Lyapunov matrix for the dimension of the state space representation. In Eq.(14), there are products of scheduling parameters and Lyapunov matrix $X_f(\theta)$. $X_f(\theta)$ is assigned as form of Eq.(16) and restricted as Eq.(17). The Eq.(14) become multi-affine for $\theta$. The stability is guaranteed by solving finite LMI conditions at vertexes of the scheduling parameter $\theta$ and derivative $\dot{\theta}$.

$$X_f(\theta) = X_f0 + \sum_{i=1}^{3} \theta_iX_{fi}$$ (16)

$$X_{fi} = \begin{bmatrix} X_{1.1-i} & \ldots & X_{1.5-i} & 0 & 0 \\ X_{5.1-i} & \ldots & X_{5.5-i} & 0 & 0 \\ X_{6.1-i} & \ldots & X_{6.6-i} & \ldots & \vdots \\ X_{10.1-i} & \ldots & X_{10.10-i} & \ldots & \vdots \end{bmatrix}$$

(i = 1, ..., 3)

$$\theta_1[X_{k.1-k} \ldots X_{k.10-k}] = 0 \quad (k = 7, 8)$$ (17)

$$\theta_2[X_{9.1-9} \ldots X_{9.10-9}] = 0$$

$$\theta_3[X_{10.1-10} \ldots X_{10.10-10}] = 0$$

3.3 LQ control specification

For Eq.(9), an evaluated function $J_z$ is given by Eq.(18).

$$J_z = \int_0^\infty (x_c^TQx_c + u^TRu)dt$$ (18)

Where, $Q \geq 0$ and $R > 0$. Matrices $A_f(\theta), X_f(\theta), X(\theta), Y_f(\theta), Y(\theta)$ and $\dot{X}_f(\theta)$ are affine to the scheduling parameter and its derivative. Matrices can be represented by parameter box $\Theta_i$.

$$\Theta_1 = (\theta_1, \theta_2, \theta_3, \theta_4), \Theta_2 = (\bar{\theta}_0, \theta_1, \theta_2, \theta_3, \theta_4), \ldots \Theta_{31} = (\bar{\theta}_0, \bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3, \bar{\theta}_4)$$

For stabilizing the LPV system Eq.(10), LMI conditions which derive GS controller minimize evaluated function $J_z$ from w to z can be obtained as Eq.(19)-(22).

minimize $\gamma$ subject to

$$X(\Theta_i) > 0$$

$$\begin{bmatrix} \text{He}(A_f(\Theta_i)X_f(\Theta_i) + B_fY_f(\Theta_i)) - E_fX_f(\Theta_i) \\ R^T \dot{Y}_f(\Theta_i) \\ Q_fX_f(\Theta_i) \end{bmatrix} < 0$$ (20)

$$\begin{bmatrix} W & I_{f1} & I_{f3} \\ I_{f1} & X(\Theta_i) \end{bmatrix} > 0$$ (21)

$$\text{Trace}(W) < \gamma^2$$ (22)

(i = 1, ..., 32)

Where, $I_{f1}, I_{f2}, I_{f3}$ are unit matrices which have the appropriate dimensions. $X(\theta)$ and $Y(\theta)$ are obtained by $X(\Theta_i), Y(\Theta_i)$ that satisfy Eq.(19)-(22). GS controller $K(\theta)$ is assigned as $K(\theta) = [Y(\theta) \ X(\theta)^{-1}]$.

3.4 Estimation of derivative

In Eq.(19)-(22), GS controller via parameter dependent Lyapunov function is synthesized. However, there is a Lyapunov matrix derivative. The controller guarantees robustness of variation of $U_p$ which does not include in dynamics. A estimation method of Lyapunov matrix derivative is adopted[5]. Matrix polynomials are define as Eq.(23).

$$\text{Co}[\alpha](M_1, \ldots, M_q) := \sum_{i=1}^q \alpha_i M_i : \alpha_i \geq 0, \sum_{i=1}^q \alpha_i = 1$$ (23)

Let upper bound of Lyapunov matrix $\bar{X}_0$ be Eq.(24).

For vertexes of $\bar{\theta}_i < 0$, $0 \leq X_i \leq \bar{X}_0$, $\forall i \in I_D$ (24)

Where, $I_D$ is set of indexes of $\bar{\theta}_i < 0$. $\bar{X}$ is guaranteed by an upper bound of Lyapunov function as Eq.(25).

$$-\bar{X} = \sum_{i \in I_f} (-\bar{\alpha}_i X_i) + \sum_{i \in I_D} (-\bar{\alpha}_i X_i) \leq \mu_0 \bar{X}_0$$ (25)

Where, $\mu_0$ is max $\sum_{i \in I_D} (-\bar{\alpha}_i)$. $\bar{\alpha}_i (i = 1, \ldots, 8)$ are derived as Eq.(26).

$$\alpha_1 = \frac{\bar{\theta}_1 - \bar{\theta}_2}{\bar{\theta}_1 - \bar{\theta}_2}$$

$$\alpha_2 = \frac{\bar{\theta}_1 - \bar{\theta}_2}{\bar{\theta}_1 - \bar{\theta}_2}$$

$$\alpha_3 = \frac{\bar{\theta}_1 - \bar{\theta}_2}{\bar{\theta}_1 - \bar{\theta}_2}$$

For stabilizing the LPV system Eq.(10), LMI conditions which derive GS controller minimize evaluated function $X_f$ in Eq.(20) is shown as Eq.(12). We should derive upper bound of $X$. LMI conditions which include the
estimation method of Lyapunov matrix derivative are shown as Eq.(27)-(31).

\[
\begin{align*}
\text{minimize} & : \gamma \\
\text{subject to} & \\
X_0 - X(\Theta_i) > 0 & \quad (27) \\
X(\Theta_i) > 0 & \quad (28) \\
& \\
\begin{bmatrix}
He\{A_{lf}(\Theta_i)X_{lf}(\Theta_i) + B_{lf}Y_{lf}(\Theta_i)\} + \mu_0X_0 \\
R^\top Y_{lf}(\Theta_i) \\
Q^\frac{1}{2}X_{lf}(\Theta_i)
\end{bmatrix} \\
Y_{lf}^T(\Theta_i)R^{\frac{1}{2}T}X_{lf}(\Theta_i)Q^\frac{1}{2}Y_{lf} & < 0 \\
\begin{bmatrix}
W \\
I_{lf3} \\
X(\Theta_i)
\end{bmatrix} & > 0 \\
\text{Trace}(W) & < \gamma^2 \\
(i = 1, \ldots, 16)
\end{align*}
\]

4 Simulations

Upper and lower bounds of the varying parameters are assigned as \(\theta_0 \in [0.147, 1.470], \theta_1 \in [0.1, 0.7], \theta_2 \in [-0.23, 0.23], \theta_3 \in [-0.292, 0.292], \theta_4 \in [-1.3745, 1.3745]\). Results of controllers are shown by several simulations. The reference is assigned 1.0 [m] for the horizontal position \(y\) of the load with hoisting movement. The reference is assigned 1.0 [m] for the horizontal position \(y\) of the load with hoisting movement. Results of input and output considering variation of the mass of the load and considering fixed mass \((m_p = 0.735 \text{ [kg]})\) are shown in Fig. 2 and 3. These controllers are not adopted estimation of derivative and are assigned the weight matrix as \(Q = [1, 1, 1, 1, 1, 1], R = 1\). From Fig. 2 and 3, the oscillation is eliminated by considering the varying parameters of the load. Results of input and output adopting descriptor representation, LFT and adopting only descriptor representation are shown in Fig. 4 and 5. The Fig. 4 and 5 are not adopted estimation of derivative and are assigned the weight matrix as \(Q = [1, 1, 1, 1, 1], R = 10\), the mass of the load as 0.147, \(\gamma\) achieved by adopting LFT and descriptor representation is 6.87, only descriptor representation is 7.23. From Fig. 5 and \(\gamma\), the controller adopting LFT and descriptor representation can suppress the control input and make upper bound of evaluated function lower than adopting only descriptor representation. Results of input and output adopting estimation of derivative and not adopting estimation of derivative are shown in Fig. 6 and 7. The Fig. 6 and 7 are not considering \(l_p, l_q\) and are assigned the weight matrix as \(Q = [10, 1, 1, 1, 1], R = 10\), the mass of the load as 0.147. \(\gamma\) achieved by adopting estimation of derivative is 10.9, not adopting the method is 11.6. From

\[
\begin{align*}
\begin{bmatrix}
\frac{1}{2}X_{lf}(\Theta_i)
\end{bmatrix} & > 0 \\
\text{Trace}(W) & < \gamma^2
\end{align*}
\]

5 Conclusion

This paper synthesizes Gain-Scheduling control for crane which guarantees variation of the mass of the load, rope length, its velocity and acceleration. The controller is synthesized with parameter-dependent Lyapunov function. For improve performance of controller, descriptor representation, Linear Fractional Transformation and estimation of Lyapunov matrix derivative are adopted. By several simulations, it is shown that these methods are effective for the crane.

References


