Guarantee of the Robustness for the changes of road condition and vehicle velocity in ABS

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1 Introduction

ABS was born to prevent the flat phenomenon of railways or the burst of the tire at landing. Recently, almost vehicles are equipped with ABS as a safety device.

The control of ABS is required to prevent wheel lock and to short the braking distance keeping braking power maximum at braking in all road conditions. The dynamic behavior of ABS depends on the vehicle velocity and the friction coefficient of the road surface. Because of this, some research papers of the robust control design for the vehicle velocity or the friction coefficient of the road surface are reported [5]. As well, the dynamic behavior of ABS has strong non-linearity. Because of this, some research papers of the PID control design or SMC (Sliding Mode Control) design for the strong non-linearity of ABS are reported [9]-[14]. The vehicle velocity can be measured from the wheel velocity, but the friction coefficient of the road surface can't be measured in ABS.

Therefore, in this research, the method of the control design for the changes of the vehicle velocity and the friction coefficient of the road surface is proposed to satisfy the two conditions required for ABS "all road conditions" and "to prevent wheel lock at braking". The first characteristic of this research is to design the observer to estimate the friction coefficient of the road surface. The equation of the friction coefficient of the road surface is derived from the motion equation which obtained in the formulation of the ABS model. The second characteristics of this research are to implement the optimum descriptor variables and to extract the fluctuating parameters usefully with LFT (Linear Fractional Transformation). Because of this, polytopic LPV (Linear Parameter Varying) system is composed without ignoring the non-linearity of the fluctuating parameter. In the case of doing the GS control design in a framework of the state space expression, the design result tends to be conservative because it is difficult to polytypically express strictly the fluctuating parameter which appears non-linearly in the state equation. Some research papers that the conservatism of the design result is reduced or the treatment of Lyapunov function which depends on parameters is made easy by implementing descriptor variables are reported [2],[7],[8]. The third characteristic of this research is to propose the method of the GS control design based on Lyapunov function which depends on parameters considering the time varying of the vehicle velocity and the friction coefficient of the road surface while estimating the friction coefficient of the road surface with designed observer. Because of this, to be able to construct the stable control system for all changes of the vehicle velocity and the friction coefficient of the road surface is proved in theory. As well, the effectiveness of the proposed method is verified through simulation and experiment.

2 Controlled Object

In this research, the brake control is done with the ABS experimental unit like Figure 1. This unit has two wheels. The upper wheel simulates the vehicle wheel, and the lower wheel simulates the road surface. As well, the upper wheel has a hydraulic disk brake. This unit can measure the wheel velocity and the vehicle velocity.

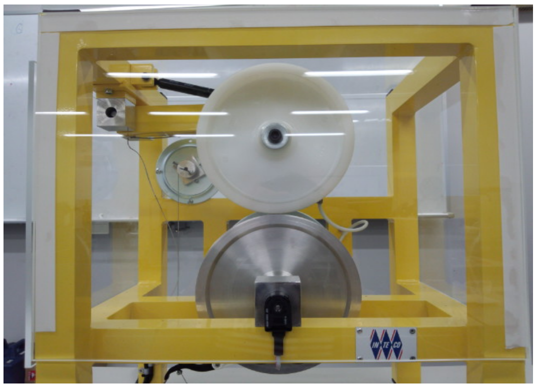

Figure 1 ABS Experimental Unit

2.1 Formulation of the model

The physical model of this unit is shown in Figure 2.

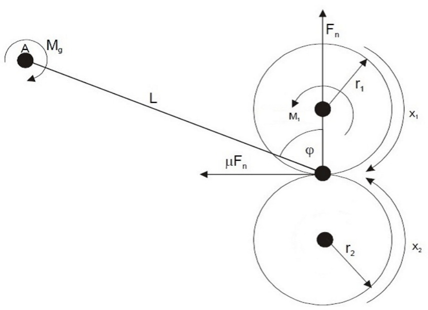

Figure 2 Physical Model of ABS

The definition of the parameters are shown as follows.
$x_1(t)$:angular velocity of the upper wheel[rad/s]
$x_2(t)$:angular velocity of the lower wheel[rad/s]
r_1:radius of the upper wheel[m]
r_2:radius of the lower wheel[m]
J_1:moment of inertia of the upper wheel[$\mathrm{kg \cdot m^2}$]
J_2:moment of inertia of the lower wheel[$\mathrm{kg \cdot m^2}$]
F_n:total force generated by the upper wheel and pressing on the lower wheel[N]
M_1:braking torque[Nm]
M_g:gravitational and shock absorber torques acting on the balance lever[Nm]
μ:friction coefficient of the road surface

L:distance between the contact point of the wheels and the rotational axis of the balance lever[m]
φ:angle between the normal in the contact point and the line L[rad]

The physical expressions of the upper wheel and the lower wheel are given as eq.(1),(2) by Newton's motion equation.

$$J_1\dot{x}_1(t) = F_n r_1 \mu - M_1 \quad (1)$$
$$J_2\dot{x}_2(t) = -F_n r_2 \mu \quad (2)$$

As well, by the sum of torques around the point A,

$$F_n = \frac{M_g + M_1}{L(\sin\varphi - \mu\cos\varphi)} \quad (3)$$

Here, eq.(3) is substituted into eq.(1),(2),

$$J_1\dot{x}_1(t) = (M_g + M_1)r_1 S - M_1 \quad (4)$$
$$J_2\dot{x}_2(t) = -(M_g + M_1)r_2 S \quad (5)$$

where,

$$S = \frac{\mu}{L(\sin\varphi - \mu\cos\varphi)} \quad (6)$$

Here, the slip ratio λ is defined as a control indicator.

$$\lambda = \frac{r_2 x_2(t) - r_1 x_1(t)}{r_2 x_2(t)} \quad (7)$$

Next, the differential equation of the slip ratio λ is derived.

$$\dot{\lambda} = f(\lambda, x_2(t), S) + g(\lambda, x_2(t), S)M_1 \quad (8)$$

where,

$$f(\lambda, x_2(t), S) = \frac{1}{x_2(t)}\{-\frac{r_1}{r_2}c_{11}S + (1-\lambda)c_{14}S\}$$
$$g(\lambda, x_2(t), S) = \frac{1}{x_2(t)}\{-\frac{r_1}{r_2}(c_{12}S + c_{13}) + (1-\lambda)c_{15}S\}$$

($c_{11}, c_{12}, c_{13}, c_{14}, c_{15}$ are constants.)

Here, linearization is done around the equilibrium point (λ^*, M_1^*).

$$\dot{\lambda} - \dot{\lambda}^* \cong \frac{\partial\dot{\lambda}}{\partial\lambda}(\lambda - \lambda^*) + \frac{\partial\dot{\lambda}}{\partial M_1}(M_1 - M_1^*) \quad (9)$$

$$(c_{22}S - \beta_1)x_2(t)\dot{\lambda} \cong (\alpha_{11}S^2 + \beta_2 S)(\lambda - \lambda^*) + (\alpha_{21}S^2 + \alpha_{22}S + \beta_1^2)(M_1 - M_1^*) \quad (10)$$

($\alpha_{11}, \alpha_{21}, \alpha_{22}, \beta_1, \beta_2$ are constants.)

2.2 Derivation of the friction coefficient of the road surface μ

μ is derived to estimate the friction coefficient of the road surface. By eq.(2),

$$\mu = -\frac{J_2}{F_n r_2}\dot{x}_2(t) \quad (11)$$

2.3 State Equation

In this research, the error of the slip ratio and the integration of the error are added to a state variable to let the slip ratio follow the target value. The state variable $x(t)$ is given as $x(t) = [\int(\lambda - \lambda^*)dt \quad \lambda - \lambda^*]$, and the input of the system is given as $u(t) = M_1 - M_1^*$.

$$E\dot{x} = Ax + Bu \quad (12)$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & \alpha_{11}S^2 + \beta_2 S \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \alpha_{21}S^2 + \alpha_{22}S + \beta_1^2 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & x_2(c_{22}S - \beta_1) \end{bmatrix}$$

3 Control Design

The GS control design is done regarding the vehicle velocity and the friction coefficient of the road surface as the scheduling parameters. The friction coefficient of the road surface is expressed with S which has μ. The roles of ABS are to prevent wheel lock and reduce the vehicle velocity while letting the braking force keep maximum at braking. To perform this, the slip ratio is needed to follow the ideal slip ratio which can maximize the braking force.

3.1 GS Control Design

Parameter boxes that upper and lower bounds of the scheduling parameter θ and its changing speed $\dot{\theta}$ are tops are defined.

$$\Theta = \{\theta = [\theta_1, \theta_2, \theta_3 : \theta_i \in \{\underline{\theta}_i, \overline{\theta}_i\}\}$$
$$\Theta_d = \{\dot{\theta} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 : \dot{\theta}_i \in \{\underline{v}_i, \overline{v}_i\}\}$$
$$\theta_1 = S, \theta_2 = S^2, \theta_3 = x_2(t)\ (i = 1, 2, 3)$$

In eq.(12), the matrixes A, B, E have the scheduling parameters. Being able to derive the computable LMI condition by implementing the redundant descriptor variable for the system like this is known [2]. In this research, the scheduling parameters are gathered into the matrix A_G by giving a descriptor variable as $x_G(t) = [\int(\lambda - \lambda^*)dt \quad \lambda - \lambda^* \quad \dot{\lambda} \quad M_1 - M_1^*]^T$.

$$E_G\dot{x}_G = A_G x_G + B_G u \quad (13)$$

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \alpha_{11}\theta_2 + \beta_2\theta_1 & (-c_{22}\theta_1 + \beta_1)\theta_3 & \alpha_{21}\theta_2 + \alpha_{22}\theta_1 + \beta_1^2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ B_G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

The scheduling parameters have been gathered into the matrix A_G from eq.(13). There is a polynominal expression of the scheduling parameters in the matrix A_G. Therefore, a descriptor variable $x_{\acute{G}}(t) = [\int(\lambda - \lambda^*)dt \ \ \lambda - \lambda^* \ \ \dot{\lambda} \ \ M_1 - M_1^* \ \ \lambda - \lambda^* \ \ \dot{\lambda} \ \ M_1 - M_1^* \ \ M_1 - M_1^*]^T$ is given to express polynominal expressions of the scheduling parameters monominally.

$$E_{\acute{G}}\dot{x}_{\acute{G}} = A_{\acute{G}}x_{\acute{G}} + B_{\acute{G}}u \quad (14)$$

$$A_{\acute{G}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_{11}\theta_2 & -c_{22}\theta_1\theta_3 & \alpha_{21}\theta_2 & \beta_2\theta_1 & \beta_1\theta_3 & \alpha_{22}\theta_1 & \beta_1^2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_{\acute{G}} = \begin{bmatrix} E_G & 0 \\ 0 & 0 \end{bmatrix},\ B_{\acute{G}} = \begin{bmatrix} B_G \\ 0 \end{bmatrix}$$

The matrix $A_{\acute{G}}$ has a bilinear term of the scheduling parameters $\theta_1\theta_3$. Here, the matrix $A_{\acute{G}}$ is transformed equally to the form which doesn't have $\theta_1\theta_3$ to derive the computable LMI condition in the case of using Lyapunov function which depends on parameters. The method to extract the scheduling parameters with LFT in order to transform like this is adopted [7]. In this research, a scheduling parameter Δ is given as eq.(16) and the matrix $A_{\acute{G}}(\theta)$ is given as eq.(15) with Δ considering the form of the scheduling parameters in the matrix $A_{\acute{G}}$.

$$A_{\acute{G}}(\theta) = A_n + B_\delta\Delta(I - D_\delta\Delta)^{-1}C_\delta \tag{15}$$

$$\Delta = \sum_{i=1}^{3} \theta_i\Delta_i \tag{16}$$

$$\Delta_1 = diag(1,1,1,1,0,0,0,0)$$
$$\Delta_2 = diag(0,0,0,0,1,1,0,0)$$
$$\Delta_3 = diag(0,0,0,0,0,0,1,1)$$

Then, a system which equals to eq.(14) is expressed as eq.(17)-(19). In this research, considering the design of the GS controller which based on Lyapunov function which depends on parameters, the matrixes A_n, C_δ are transformed without including the scheduling parameters.

$$E_{\acute{G}}\dot{x}_{\acute{G}} = A_n x_{\acute{G}} + B_\delta w_\delta + B_{\acute{G}}u \tag{17}$$
$$z_\delta = C_\delta x_{\acute{G}} + D_\delta w_\delta \tag{18}$$
$$w_\delta = \Delta z_\delta \tag{19}$$

$$A_n = \begin{bmatrix} A_{n11} & A_{n12} \\ A_{n21} & A_{n22} \end{bmatrix}, B_\delta = \begin{bmatrix} B_{\delta1} \\ B_{\delta2} \end{bmatrix}$$

$$C_\delta = [C_{\delta1} \quad C_{\delta2}]$$

Finally, the coefficient matrix A_d of x_d is transformed to the form which doesn't have the term $(I - D_\delta\Delta)^{-1}$ by giving a new descriptor variable $x_d(t) = [x_{\acute{G}}(t)^T \ \ z_\delta(t)^T]^T$. From the above, the computable LMI condition can be derived. The system after this transformation is given as eq.(20).

$$E_d\dot{x}_d = A_d(\theta)x_d + B_{du}u \tag{20}$$

$$A_d = \begin{bmatrix} A_n & B_\delta\Delta \\ C_\delta & D_\delta\Delta - I \end{bmatrix} = \begin{bmatrix} A_{n11} & A_{n12} & B_{\delta1}\Delta \\ A_{n21} & A_{n22} & B_{\delta2}\Delta \\ C_{\delta1} & C_{\delta2} & D_\delta\Delta - I \end{bmatrix}$$

$$E_d = \begin{bmatrix} E_{\acute{G}} & 0 \\ 0 & 0 \end{bmatrix},\ B_{du} = \begin{bmatrix} B_{\acute{G}} \\ 0 \end{bmatrix}$$

3.2 Condition to discriminate stability

The condition to discriminate stability for the descriptor system which expressed by eq.(20) is shown as follows[7]. Considering the structure of the matrix E_d in this research, the candidates of the Lyapunov matrix $X_d(\theta)$ and the variable matrix $Y_d(\theta)$ are limitted as follows.

$$X_d(\theta) = \begin{bmatrix} X(\theta) & 0 & 0 \\ X_{21}(\theta) & X_{22}(\theta) & X_{23}(\theta) \\ X_{31}(\theta) & X_{32}(\theta) & X_{33}(\theta) \end{bmatrix},\ X(\theta) > 0$$

$$E_d\dot{X}_d(\theta) = \begin{bmatrix} \dot{X}(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ Y_d(\theta) = [Y(\theta) \quad 0 \quad 0]$$

By the reference [7], the sufficient condition to make the system of eq.(20) stable is to be $X(\theta) > 0, X_{21}(\theta), X_{22}(\theta), X_{23}(\theta), X_{31}(\theta), X_{32}(\theta), X_{33}(\theta), Y(\theta)$ which satisfied with eq.(21). Here, $He\{M\} = M + M^T$ is adopted.

$$He\{A_d(\theta)X_d(\theta) + B_{du}Y_d(\theta)\} - E_d\dot{X}_d(\theta) < 0 \tag{21}$$

Eq.(22) is obtained by multiplying $[I \ \ B_\delta\Delta(I - D_\delta\Delta)^{-1}]$ and its transposition into the left side and the right side of eq.(21).

$$He\{A_{\acute{G}}(\theta)X(\theta) + B_{\acute{G}}Y(\theta)\} - \dot{X}(\theta) < 0 \tag{22}$$

There is the product of the scheduling parameter Δ and the Lyapunov matrix which depends on parameters $X(\theta)$ in eq.(22). But there is not this product in eq.(21). In eq.(21), there is the product of Δ and $X_{31}(\theta), X_{32}(\theta), X_{33}(\theta)$. By giving $X_d(\theta)$ as eq.(23) and giving limitation as eq.(24) for $X_{31}(\theta), X_{32}(\theta), X_{33}(\theta)$, eq.(21) becomes multi affine for θ. This means that the stability is guaranteed by resolving the LMI condition at the endpoint nodes of the scheduling parameter θ and its changing speed $\dot{\theta}$.

$$X_d(\theta) = \hat{X}_{d0} + \sum_{i=1}^{3} \theta_i\hat{X}_{di} \tag{23}$$

$$\Delta_i[X_{31i} \ X_{32i} \ X_{33i}] = 0 \tag{24}$$

In the case of considering H_2 control specification for eq.(20), the generalized controlled system is given as eq.(25)-(26).

$$E_d\dot{x}_d = A_d(\theta)x_d + B_{dw}w + B_{du}u \tag{25}$$
$$z = C_d x_d + D_d u \tag{26}$$

$$B_{dw} = \begin{bmatrix} B_w \\ 0 \\ 0 \end{bmatrix}, C_d = [C \quad 0 \quad 0], C = \begin{bmatrix} Q^{\frac{1}{2}} \\ 0 \end{bmatrix}, D_d = \begin{bmatrix} 0 \\ R^{\frac{1}{2}} \end{bmatrix}$$

Here, w means a disturbance input, z means a evaluation output, Q means a weighting matrix for the state variable $x_G(t)$, R means a weight for a control input. The matrixes A_d, X_d, Y_d are expressed as eq.(27).

$$[A_d(\theta) \ X_d(\theta) \ Y_d(\theta)] = \sum_{i=1}^{8} \beta_i(\theta)\,[A_{di} \ X_{di} \ Y_{di}] \tag{27}$$

$$\beta_i \geq 0, \ \sum_{i=1}^{8} \beta_i = 1$$

The LMI conditions are given as eq.(28)-(33) to seek the state feedback style GS controller in order to make the system which expressed by eq.(25)-(26) stable in the variation ranges of the scheduling parameter θ and its changing speed $\dot{\theta}$. The problem is to minimize H_2 norm from w to z.

$$minimize\ :\ \gamma \tag{28}$$

$$s.t.\ :\ X(\theta) > 0 \tag{29}$$

$$\begin{bmatrix} He\{A_d(\theta)X_d(\theta) + B_{du}Y_d(\theta)\} - S_k & B_{dw} \\ B_{dw}^T & -I \end{bmatrix} < 0 \tag{30}$$

$$\begin{bmatrix} X(\theta) & (CX(\theta)+DY(\theta))^T \\ CX(\theta)+DY(\theta) & W \end{bmatrix} > 0 \tag{31}$$

$$Trace(W) < \gamma^2 \tag{32}$$

$$S_k = \sum_{i=1}^{3} \dot{\theta}_i E_d X_d(\theta_i) \tag{33}$$

The GS controller $K_d(\theta)$ in Descriptor Expression is given as eq.(34) by obtaining $X(\theta), Y(\theta)$ which satisfied with eq.(28)-(33).

$$K_d(\theta) = Y(\theta)X(\theta)^{-1} \tag{34}$$

Moreover, if $X(\theta), Y(\theta), W$ are single and $S_k = 0$ in eq.(28)-(33), the robust controller of a fixed gain considering the time varying of the scheduling parameter is obtained.

4 Simulation

Whether designed GS controller with designed observer can control correctly is tested on the following conditions.

- The vehicle velocity slows from 50 km/h.
- Simulation is done with designed observer.

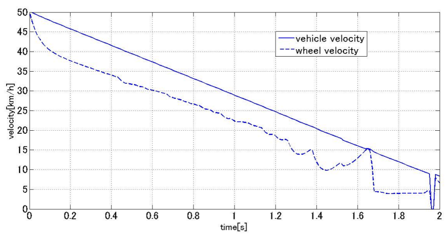

Figure 3 Vehicle Velocity and Wheel Velocity

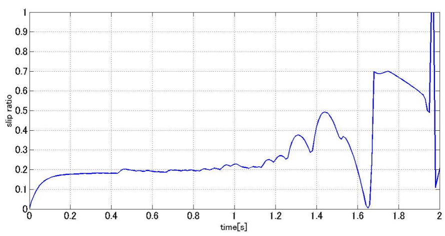

Figure 4 Slip Ratio

5 Conclusion

Results are shown as follows.

- The observer to estimate μ correctly has been designed.
- The GS controller for the friction coefficient of the road surface μ and the vehicle velocity $x_2(t)$ with designed observer has been designed.

Problems are shown as follows.

- To follow the ideal slip ratio more perfectly.
- To do simulation and experiment considering the friction behavior of the hydraulic disk brake system.
- Only S and $x_2(t)$ are adopted as the scheduling parameters.

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