Robust Control of Active Suspension to Improve Ride Comfort with Structural Constraints

2012SE275 Fumiaki Yamada
Supervisor : Isao Takami

1 Introduction

In this study, the robust H_2 controller for the active suspension is proposed. The purpose of the study is to improve the ride comfort with satisfying the structural constraints. Based on ISO 2631[1], the frequency bands 4-8[Hz] for the vertical acceleration and 0.63-0.8[Hz] for the pitch angular acceleration are suppressed by using frequency shaping. They are constraints on the vertical force of the wheel, the suspension stroke constraint, and the control input constraints are considered. In addition, the robust stability for the perturbation of the front and rear weights of the car body is guaranteed. The robust stability for the perturbation is guaranteed by using the polytopic representation. The robust H_2 controller is designed by solving a finite set of Liner Matrix Inequalities (LMIs) to achieve these purposes. The effectiveness of the proposed controller is verified by simulations and experiments.

2 Modeling

The half car model is shown in Figure 1. The motion

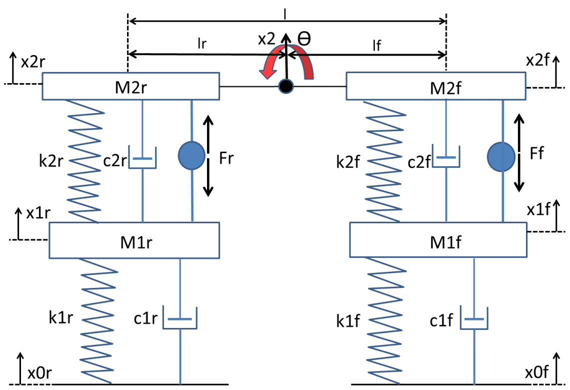

Figure 1 Half Car Model

equations of the front wheel position x_{1f}, the rear wheel position x_{1r}, the car body position x_2, and the pitch angle θ are derived as (1) to (3), respectively($k = f, r$).

$$M_{1k}\ddot{x}_{1k} = k_{2k}(x_{2k} - x_{1k}) + c_{2k}(\dot{x}_{2k} - \dot{x}_{1k}) - k_{1k}(x_{1k} - x_{0k}) - c_{1k}(\dot{x}_{1k} - \dot{x}_{0k}) - F_k \quad (1)$$

$$(M_{2f} + M_{2r})\ddot{x}_2 = -k_{2f}(x_{2f} - x_{1f}) - c_{2f}(\dot{x}_{2f} - \dot{x}_{1f}) - k_{2r}(x_{2r} - x_{1r}) - c_{2r}(\dot{x}_{2r} - \dot{x}_{1r}) + F_f + F_r \quad (2)$$

$$M_{2f}M_{2r}l\ddot{\theta} = -k_{2f}M_{2r}(x_{2f} - x_{1f}) - c_{2f}M_{2r}(\dot{x}_{2f} - \dot{x}_{1f}) + k_{2r}M_{2f}(x_{2r} - x_{1r}) + c_{2r}M_{2f}(\dot{x}_{2r} - \dot{x}_{1r}) + M_{2r}F_f - M_{2f}F_r \quad (3)$$

The motion equations of the front car body position x_{2f} and the rear car body position x_{2r} are derived as (4) by using (2) and (3).

$$\ddot{x}_{2f} \cong \ddot{x}_2 + l_f\ddot{\theta}, \ \ddot{x}_{2r} \cong \ddot{x}_2 - l_r\ddot{\theta} \quad (4)$$

The state vector $x(t)$, the output vector $y(t)$, and the disturbance vector $w(t)$ are defined as (5) to (7).

$$x(t) = [\ x_{2f} - x_{1f} \quad x_{2r} - x_{1r} \quad x_{1f} - x_{0f} \quad x_{1r} - x_{0r} \quad \dot{x}_{2f} \quad \dot{x}_{2r} \quad \dot{x}_{1f} \quad \dot{x}_{1r}\ ]^T \quad (5)$$

$$y(t) = [\ y_1(t) \quad y_2(t) \quad y_3(t)\ ]^T = [\ \ddot{x}_{2f} \quad \ddot{x}_{2r} \quad \ddot{\theta}\ ]^T \quad (6)$$

$$w(t) = [\ \dot{x}_{0f} \quad \dot{x}_{0r}\ ]^T, \ u(t) = [\ F_f \quad F_r\ ]^T \quad (7)$$

3 Controller synthesis

3.1 Frequency filter

The frequency filters for the body acceleration and the pitch angular acceleration are designed to improve the ride comfort. The transfer functions, $W_1(s)$, $W_2(s)$, and $W_3(s)$, are introduced for the frequency filter of the body acceleration $\ddot{x}_{2f}$ and $\ddot{x}_{2r}$, and the pitch acceleration $\ddot{\theta}$, respectively. The gain diagram of $W_1(s)$, $W_2(s)$, and $W_3(s)$ are shown in Figure 2. Then, the new state space

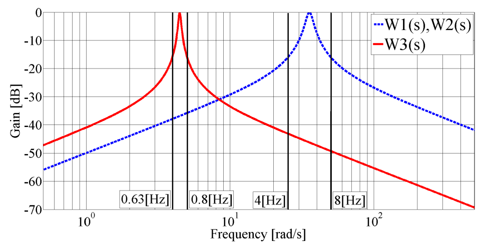

Figure 2 The gain diagram of $W_1(s)$, $W_2(s)$, and $W_3(s)$

representation is derived as (8).

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}_w w(t) + \tilde{B}_u u(t) \quad (8)$$

The block diagram of the extended system is shown in Figure 3. The weighted evaluation output $z_1(t)$ is de-

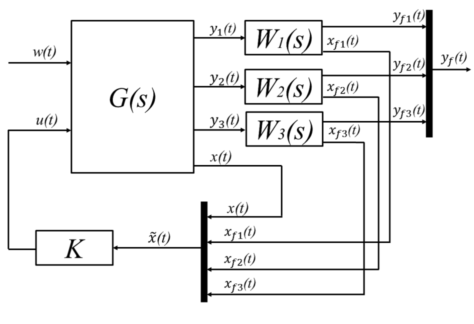

Figure 3 Block Diagram of the Extended System

fined as (9) by using the weight matrix W_z.

$$z_1(t) = W_z y_f(t) = W_z C_{z1}\tilde{x}(t) + W_z D_{z1}u(t) \quad (9)$$

3.2 Time-domain constraint

To avoid the jump of the wheels, the constraints for the vertical force of the wheels are given by suppressing the vertical force of the wheels less than the entire weight of vehicle as (10) [2].

$$\begin{cases} k_{1f}(x_{1f} - x_{0f}) & < & (M_{2f} + M_{1f})g \\ k_{1r}(x_{1r} - x_{0r}) & < & (M_{2r} + M_{1r})g \end{cases} \quad (10)$$

To prevent the suspension bottoming out, the constraints are given as (11) by the constraints of suspension strokes S_f and S_r.

$$\left\{ \begin{array}{lll} |x_{2f} - x_{1f}| & < & S_f \\ |x_{2r} - x_{1r}| & < & S_r \end{array} \right. \tag{11}$$

The evaluation output $z_2(t)$ is derived by using (10) and (11) as follows.

$$\begin{aligned} z_2(t) &= C_{z2}x(t) \\ &= \left[\begin{array}{cccc} \frac{x_{2f}-x_{1f}}{S_f} & \frac{x_{2r}-x_{1r}}{S_r} & \frac{k_{1f}(x_{1f}-x_{0f})}{(M_{2f}+M_{1f})g} & \frac{k_{1r}(x_{1r}-x_{0r})}{(M_{2r}+M_{1r})g} \end{array} \right]^T \end{aligned} \tag{12}$$

To prevent the saturation of the control inputs, the constraints are given as (13) by defining the constraint of input voltage as U_{maxf} and U_{maxr}.

$$\left\{ \begin{array}{lll} |F_f| & < & U_{maxf} \\ |F_r| & < & U_{maxr} \end{array} \right. \tag{13}$$

3.3 Robust H_2 controller

The robust stability for the perturbation of the car body weight M_{2f} and M_{2r} are considered. However, there are the nonlinear terms of the uncertain parameters M_{2f} and M_{2r} in matrices $\tilde{A}$, $\tilde{B}_u$, C_{z1}, D_{z1}, and C_{z2}. The nonlinear terms of the matrices are transformed by introducing $\alpha_f, \beta_f, \alpha_r$, and β_r.

$$\alpha_f = \frac{1}{M_{2f}},\ \beta_f = \frac{1}{M_{2f}+M_{1f}} = \frac{\alpha_f}{\alpha_f M_{1f}+1},$$
$$\alpha_r = \frac{1}{M_{2r}},\ \beta_r = \frac{1}{M_{2r}+M_{1r}} = \frac{\alpha_r}{\alpha_r M_{1r}+1}$$

Therefore, the LMI conditions of the Robust H_2 controller with constraint are given as Corollary 1[2].

Corollary 1 If there exist X and Y satisfying following LMI conditions (14) to (17), the closed-loop system is stabilized with satisfying the structural constraints. Also, the upper bound of H_2 norm is less than γ and the state feedback gain is $K = YX^{-1}$. Here $w_{\max}$ is the upper bound of disturbance energy calculated as $\int_0^\infty \parallel w(t) \parallel^2 dt$.

$$minimize\ \gamma^2\ subject\ to$$

$$\left[\begin{array}{cc} He\{\tilde{A}_{ij}X + \tilde{B}_{u\_ij}Y\} & M^T \\ M & -I \end{array} \right] < 0 \tag{14}$$

$$(M_{ij} = W_z C_{z1\_ij}X + W_z D_{z1\_ij}Y)$$

$$\left[\begin{array}{cc} Z & \tilde{B}_w^T \\ \tilde{B}_w & X \end{array} \right] > 0,\ Trace(Z) < \gamma^2 \tag{15}$$

$$\left[\begin{array}{cc} \frac{1}{w_{\max}}I & C_{z2\_ij}X \\ (C_{z2\_ij}X)^T & X \end{array} \right] > 0 \tag{16}$$

$$(i = 1,\ 2,\ 3,\ j = 1,\ 2,\ 3)$$

$$\left[\begin{array}{cc} \frac{1}{w_{\max}}U_{maxf}^2 & Y_1 \\ Y_1^T & X \end{array} \right] > 0, \left[\begin{array}{cc} \frac{1}{w_{\max}}U_{maxr}^2 & Y_2 \\ Y_2^T & X \end{array} \right] > 0 \tag{17}$$

Where Y_1 and Y_2 are the first row and the second row of the matrix Y, respectively.

4 Simulation and Experiment

Figure 4 and Figure 5 indicate the gain diagram of the front car body acceleration $\ddot{x}_2$ and the pitch angular acceleration $\ddot{\theta}$ when the front and rear weights of the car body are considered as $M_{2f} = 1.45$[kg] and $M_{2r} = 1.45$[kg]. As can be seen from the Figure

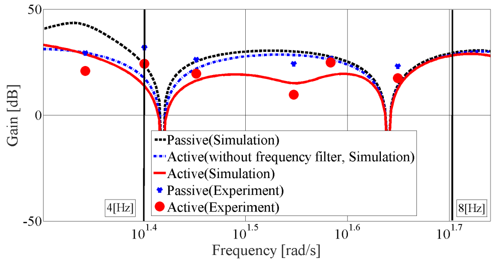

Figure 4 Frequency Response of $\ddot{x}_2$

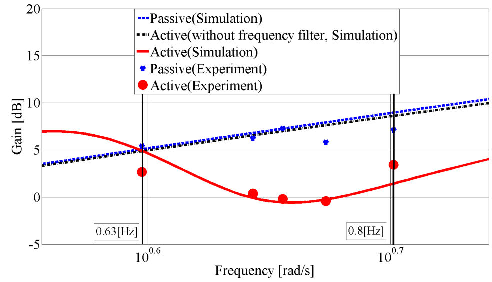

Figure 5 Frequency Response of $\ddot{\theta}$

4, the body acceleration is suppressed to about 82% at 5.61[Hz] by the proposed controller in comparison with the passive suspension. The pitch acceleration is suppressed to about 57% at 0.72[Hz] in Figure 5. It can be said that the ride comfort is improved by the proposed controller. Figure 6 indicates the vertical force of the front wheel and the front suspension stroke($M_{2f} = 1.45[kg], M_{2r} = 1.45[kg]$). As can be seen

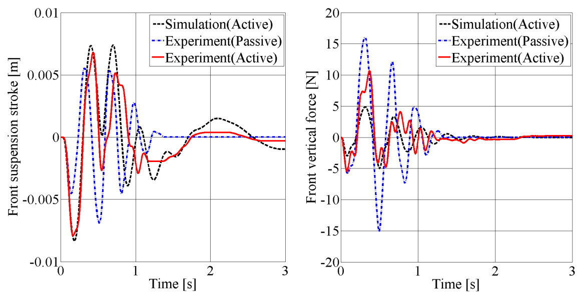

Figure 6 Suspension Stroke and Vertical Force (Front)

from the Figure 6, these value are less than constraint of the entire weight of the vehicle (24.03[N]), the front suspension stroke (0.02[m]). From the results of these simulations and experiments, it can be said that the ride comfort is improved with satisfying the structural constraints by the proposed controller.

5 Conclusion

In this study, the robust H_2 controller with the structural constraints on the active suspension by using the half car model is proposed by solving the finite set of LMIs. The effectiveness of the proposed controller is verified by the simulations and experiments.

References

[1] ISO2631-1, Mechanical vibration and shock evaluation of human exposure to whole body vibration Part 1 General requirements Geneva International Organization for Standardization, (1997)

[2] M. Ma, H. Chen, Constrained H_2 Control of Active Suspensions using LMI optimization, 25th Chinese Control Conference, pp.702-707, (2006)